

1)

$$x_1 = a_1$$

$$x_2 = \frac{e^t(a_2 + a_3)}{2} + \frac{e^{-t}(a_2 - a_3)}{2}$$

$$x_3 = \frac{e^t(a_2 + a_3)}{2} - \frac{e^{-t}(a_2 - a_3)}{2}$$

(1)

$$v_1 = \frac{dx_1}{dt} = 0$$

$$v_2 = \frac{dx_2}{dt} = \frac{e^t(a_2 + a_3)}{2} - \frac{e^{-t}(a_2 - a_3)}{2}$$

$$v_3 = \frac{dx_3}{dt} = \frac{e^t(a_2 + a_3)}{2} + \frac{e^{-t}(a_2 - a_3)}{2}$$

(2)

((b) → v_i(a, t))

Notas:

$$v_2 = x_3$$

$$v_3 = x_2$$

$$v_1 = 0$$

(3)

((a) → v_i(x, t))

También (más largo, proc estándar):

De (1), sumando y restando:

$$a_2 = x_1$$
$$a_2 = \frac{[e^{-t}(x_2 + x_3) + e^t(x_2 - x_3)]}{2}$$

$$a_3 = \frac{[e^{-t}(x_2 + x_3) - e^t(x_2 - x_3)]}{2}$$

Reemplazando en (2), se logra el resultado (3).

$$2) \phi(x,y) = ax^3 + bx^2y + cxy^2 + dy^3$$

$$\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2} = 2cx + 6dy$$

$$\sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2} + V = 6ax + 2by + \rho_w g y$$

$$\sigma_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} = -2bx - 2cy = \sigma_{yx}$$

Equilibrio:

$$\sigma_{ij,j} + X_i = 0$$

$$\underline{X} = \begin{pmatrix} 0 \\ -\rho_w g \end{pmatrix} \quad (\text{cargas de volume})$$

$$\sigma_{xx,x} + \sigma_{xy,y} + X_x = 2c - 2c + 0 = 0$$

$$\sigma_{yx,x} + \sigma_{yy,y} + X_y = -2b + 2b + \rho_w g - \rho_w g = 0$$

(d)

(1) Eq fuer horiz

$$F_{x \text{ Agua}} = \rho_w g h \times \frac{h}{2}$$

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -\sigma_{xy} \\ -\sigma_{yy} \end{bmatrix}$$

~~$$\int_0^h -\sigma_{xy} dx = \int_0^h +2bx dx = +bx^2 \Big|_0^h = +bh^2$$

$$\therefore b = \frac{\rho_w g}{2} \left(\frac{h}{l} \right)^2$$~~

$$\sigma_{xy}(x, -h) = 2ch - 2bx \quad (3)$$

$$\int_0^l (-\sigma_{xy}) dx = -2chx + bx^2 \Big|_0^l = bl^2 - 2chl$$

$$\therefore bl^2 - 2chl = \frac{\rho_w g}{2} h^2 \quad (1)$$

II) E_g force vert:

$$F_v = -\frac{\rho_w g l h}{2} \Rightarrow R_v = \frac{\rho_w g l h}{2} \quad \begin{array}{l} \text{(reacción} \\ \text{vertical} \\ \text{sobre la} \\ \text{presa} \end{array}$$

$$\sigma_{yy}(x, -h) = 6ax - 2bh - \rho_w gh$$

$$\int_0^l (-\sigma_{yy}) dx = 2bhx + \rho_w ghx - 3ax^2 \Big|_0^l =$$

$$= 2bhl + \rho_w gh l - 3al^2$$

$$\therefore -3l^2 a + 2hlb = -\frac{\rho_w gh l}{2} \quad (2)$$

III) $\sigma_{xx}|_A =$ pres hidrostáticas

$$p = -\rho_w gh \Rightarrow \sigma = -p = \rho_w gh \quad \begin{array}{l} \text{(Acción sobre} \\ \text{la presa} \end{array}$$

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -\sigma_{xx} \\ -\sigma_{xy} \end{pmatrix}$$

$$-\sigma_{xx}|_A = -6d(-h) = 6dh$$

$$6dh = \rho_w gh \Rightarrow d = \frac{\rho_w g}{6}$$

