Mecánica Computacional Vol. XXI, pp. 2493–2504
S. R. Idelsohn, V. E. Sonzogni and A. Cardona (Eds.)
Santa Fe–Paraná, Argentina, October 2002

TRIDIMENSIONAL SIMULATIONS OF BLOOD FLOW THROUGH MODIFIED BLALOCK-TAUSSIG PULMONARY SHUNTS

Dimitrius C. Pereira.*, Cleber Spode* , Sérgio Frey* e
João Ricardo Sant'anna†

*Federal University of Rio Grande do Sul, Laboratory of Computational and Applied Fluid Mechanics (LAMAC), 90050-170 Porto Alegre/RS – Brazil
frey@mecanica.ufrgs.br web page: http://www.mecanica.ufrgs.br/lamac

† Cardiology Institute of RS, Porto Alegre/RS – Brazil

Key words: hemodynamics; SUPG; Biomechanics.

Resumo. *Systemic pulmonary procedure is a palliative surgical technique and is indicated to patients with congenital heart disease with reduction of the arterial blood flow in the pulmonary arteries. In this surgical procedure, known as Modified Blalock-Taussig Shunts, prosthetic graft material is interposed between the subclavian artery and the pulmonary artery with purpose to increase blood flow in pulmonary artery. The modified Blalock-Taussig Shunt can be performed on either side, and subclavian blood supply to the arm is preserved. Kinking of the subclavian artery is not a problem, but distortion of the pulmonary artery may still occur with the child's growth. Early survival of the Modified Blalock-Taussig palliation is dependent of pulmonary and systemic flows what is directly connected with geometric factors involved in this surgery. In this article, a tridimensional geometric model, employing a Streamline Upwind/Petrov-Galerkin formulation with trilinear lagrangean finite elements, was created to approximate the blood flow through systemic pulmonary shunts, in order to study the effects of the shunt's geometry (angle, diameter) and pulsatility of the flow.*

1 INTRODUCTION

This procedure is a palliative surgical technique and is indicated to patients with congenital heart disease with reduction of the arterial blood flow in the pulmonary arteries. In this surgical procedure, known as Modified Blalock-Taussig Shunts (see Fig.1), prosthetic graft material is interposed between the subclavian artery and the pulmonary artery with purpose to increase blood flow in pulmonary artery [1]. The modified Blalock-Taussig Shunt can be performed on either side, and subclavian blood supply to the arm is preserved. Kinking of the subclavian artery is not a problem, but distortion of the pulmonary artery may still occur with the child's growth. Early survival of the Modified Blalock-Taussig palliation is dependent of pulmonary and systemic flows what is directly connected with geometric factors involved in this surgery.

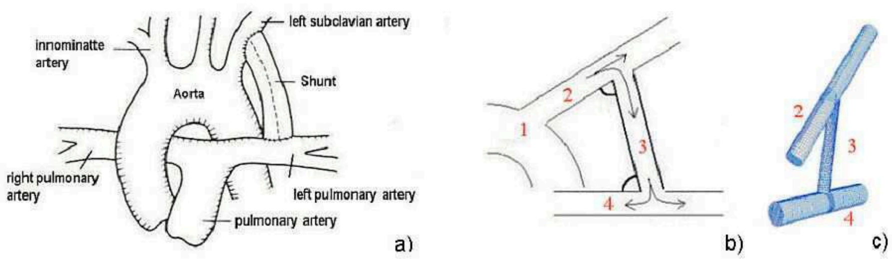


Figure 1. Problem statement: (a) Modified Blalock-Taussing shunt: (b) 1. Aorta, 2. Subclavian artery, 3. Shunt, 4.Pulmonar artery; (c) 3D model adopted.

2 BLOOD REOLOGY

The blood may be regarded as a viscous fluid consisting of cells and plasma which fills the vascular system. The plasma is formed by water, leaves minerals and organic molecules. The cells belong the three categories: red cells (erythrocytes), white cells (leukocytes) and platelets. The red cells represent 99% of the total of cells, meaning that they are the only cells that present significant influences in the physical features of the blood.

The per cent of the blood that is cells is called the hematocrit. The hematocrit of normal man averages about 42, while that of normal woman averages about 38. Blood viscosity does not have same value in different parts of the vascular system. The viscosity of the blood ranges from 3 to 4 times of the water depending upon hematocrit, temperature, and flow rate. As hematocrit increases, there is an increase in viscosity although the relationship is not linear. Temperature also has a significant effect on viscosity. As temperature decreases, viscosity increases. Viscosity increases approximate 2% for each °C decrease in temperature. The flow rate of blood also effects viscosity. At very low flow states in the microcirculation as occurs during circulatory shock, the blood viscosity can increase quite significantly. This occurs because at low flow states there are increased cell-to-cell and protein-to-cell adhesive interactions that can cause erythrocytes to adhere to one another (formation of rouleaux) and increase the blood viscosity. But another effect can occurs in microcirculation called Fahraeus-Lindqvist effect, when the red cells, instead of moving randomly, line up and move through the vessels as a single plug, thus eliminating the viscous resistance that occurs internally in the blood itself. Concentration and types of proteins in the plasma are others factors that affects blood viscosity, but these effects are so much less important and they are not considerated in most hemodynamic studies.

Various empirical, or semi-empirical, equations regarding relative viscosity to volume fraction in more concentrated suspensions have been developed. Most of them have been used to describe the relation between the relative viscosity of the plasma and the protein concentration, and the relation between the relative viscosity of the blood and the red cell concentration (hematocrit value): none of them, however, does so perfectly. Since the viscosity of blood is shear-dependent, it is obvious that the effect of changing the hematocrit value must be observed in some defined conditions of shear, although this has not always been done.

3 MECHANICAL MODELING

Employing the Mechanical Continuous Theory's, where the fluid is a continuous distribution of the matter, the pressure and velocity fields of the blood can be describe by the conservative principles of mass and momentum.

3.1 Continuity Equation

We suppose that the fluid possesses a density function $\rho = \rho(\mathbf{x}, t)$, which serves by the means of the expression,

$$M = \int_{\Omega} \rho(\mathbf{x}, t) d\Omega \tag{1}$$

to determine the mass M of fluid occupying a region Ω. We naturally assume $\rho > 0$, and assign to ρ the physical dimension mass per unit volume. Turning now to the physical significance of the concept of mass, we postulate the principle of conservation of mass: the mass of fluid in a material volume Ω does not change as Ω moves with the fluid. The following statement otherwise expresses the principle of conservation of mass,

$$\dot{M} = \frac{\partial}{\partial t} \int_{\Omega} \rho(\mathbf{x}, t) d\Omega = 0 \tag{2}$$

Applying Reynolds theorem and using the localization theorem we achieve the spatial form - or euclidean - of continuity equation,

$$\dot{\rho}(\mathbf{x}, t) + \rho(\mathbf{x}, t) \nabla \cdot \mathbf{u}(\mathbf{x}, t) = 0 \tag{3}$$

3.2 Fluid Dynamics

Interactions between a moving fluid and the exterior are described by the forces acting on the fluid, which in turn are related to its motion by Euler axioms: Linear and Angular Momentum Conservation Principles [3].

I. Principle of conservation of linear momentum: the time rate of change of linear momentum of a fluid volume Ω equals the resultant force on the volume. This principle is otherwise expressed by the statement.

$$\frac{\partial}{\partial t}\int_{\Omega}\rho(\mathbf{x},t)\mathbf{u}(\mathbf{x},t)d\Omega = \int_{\Omega}\mathbf{f}(\mathbf{x},t)d\Omega + \int_{\Gamma}\mathbf{t}(\mathbf{n};\mathbf{x},t)d\Gamma \tag{4}$$

where Γ denotes the boundary of the volume Ω.

II. Principle of conservation of angular momentum: the time rate of change of angular momentum of a fluid volume equals the total moment of all forces acting on the volume. Mathematically, this principle assumes the form.

$$\frac{\partial}{\partial t}\int_{\Omega}\mathbf{r}(\mathbf{x},t)\times\rho(\mathbf{x},t)\mathbf{u}(\mathbf{x},t)d\Omega = \int_{\Omega}\mathbf{r}(\mathbf{x},t)\times\mathbf{f}(\mathbf{x},t)d\Omega + \int_{\Gamma}\mathbf{r}(\mathbf{x},t)\times\mathbf{t}(\mathbf{n};\mathbf{x},t)d\Gamma \tag{5}$$

where $\mathbf{r}$ is the position vector. In order to establish the governing equation of the fluid motion we introduce the Cauchy theorem, whose main assertion is the linearity of the stress tensor [1].

Cauchy theorem: Let $(\mathbf{t},\mathbf{f})$ be a system of forces acting on a moving body. A necessary and sufficient condition to satisfy the momentum conservation laws is the existence of a symmetric tensor field $\mathbf{T}$ - denoted as Cauchy tensor - such that: $\mathbf{t}(\mathbf{n}) = \mathbf{T}\mathbf{n}$, satisfying the equation:

$$\rho(\mathbf{x},t)\dot{\mathbf{u}}(\mathbf{x},t) = \nabla\cdot\mathbf{T} + \mathbf{f}(\mathbf{x},t) \tag{6}$$

3.3 Constitutive Hypothesis

Euler axioms defined by Eq.(4) and Eq. (5), although being valid for most bodies, are unable to characterize them completely, because of their incapacity of distinguish among different materials behavior. For fluid, friction phenomenon is expressed by shearing forces, which retard fluid's particles relative motion. A measure of this motion is given by the velocity gradient tensor $\nabla(\mathbf{u})$, suggesting constitutive equations of the following form,

$$\mathbf{T} = -p\mathbf{I} + \mathbf{C}(\mathbf{u}) \tag{7}$$

where, $\mathbf{C}(\mathbf{u})$, to newtonian incompressible flows is characterized by a single constant: its viscosity,

$$\mathbf{C}(\mathbf{u}) = 2\mu\mathbf{D}(\mathbf{u}) \tag{8}$$

where $\mathbf{D}(\mathbf{u})$ denotes de symmetric part of the velocity gradient tensor.

The blood, because its nonlinear behavior, is characterized, in this article, by two generalized newtonian models. These models introduces a apparent viscosity [4], that dependent of shear rate of the fluid (γ), defined by:

$$\mathbf{y}_{xy} = -n(\gamma)n \tag{9}$$

where $\mathbf{y}_{xy}$ is the shear stress and n is the absolute viscosity.

Ostwald de Waele Model: or Power Law, where the apparent viscosity is defined by:

$$n(\gamma) = m\gamma^{n-1} \tag{10}$$

with the parameters m and n empirically determined. When n=1 and m=μ, the model return to the Newtonian model of incompressible fluids. This model is very used, but have serious problems for high and low shear rates.

Carreau Model: this models contains five constants, that can represent better the blood behavior.

$$\eta(\gamma) = \eta_\infty + (\eta_0 - \eta_\infty)[1 + (\lambda\gamma)^A]^{\frac{n-1}{A}} \tag{11}$$

where the parameter η_0 is the viscosity at null shear rate, η_∞ is the viscosity at infinite shear rate, n is the power-law exponent, λ is a time constant and A is a dimensionless parameter that describe transition between null shear rate region and the Power Law region. When A=2, the Eq. (11) is known at Carreau Model.

As Bird et al.[4], in low shear rates, the shear is proportional to the strain rate γ and viscosity is approaches to a constant value η_0. As the shear rate increases the viscosity decreases. For high shear rates the viscosity becomes again independent of the shear rate and is defined by η_∞.

4 FINITE ELEMENT MODELING

The finite element methodology we are concerned with, the so-called SUPG method [5], overcomes the well-know shortcoming of the classical Galerkin method adding mesh-dependent terms to it. These additional terms are designed to enhance stability of the original Galerkin formulation without upsetting its consistency [6]. Three-dimensional simulations of steady and pulsative newtonian incompressible blood flows through a shunt implant uniting the subclavian and pulmonary arteries may be modeled by the following set of equations:

$$
\begin{aligned}
&\rho\left(\frac{\partial \mathbf{u}}{\partial t}+[\nabla \mathbf{u}]\mathbf{u}\right)-2\eta(\gamma)\nabla\cdot\mathbf{D}(\mathbf{u})+\nabla p=\mathbf{f} \quad em\ \Omega\times(0,T)\\
&\nabla\cdot\mathbf{u}=0 \quad \text{in } \Omega\times(0,T)\\
&\mathbf{u}=\mathbf{u}_g \quad \text{on } \Gamma_g\times(0,T)\\
&\mathbf{Tn}=\mathbf{T}_h \quad \text{on } \Gamma_h\times(0,T)\\
&\mathbf{u}=\mathbf{u}_0 \quad \text{in } \ \Omega \text{ with } t=0
\end{aligned} \tag{12}
$$

where $\mathbf{u}$ is the fluid velocity field, p is its pressure field, the , $\eta(\gamma)$ the viscosity, $\mathbf{D}(\mathbf{u})$ the symmetric part of the velocity gradient tensor, $\mathbf{n}$ the outward unitary normal, $\mathbf{f}$ is the body force and $\mathbf{T}$ the Cauchy stress tensor defined by

$$\mathbf{T}=-p\mathbf{I}+2\eta(\gamma)\mathbf{D}(\mathbf{u}) \tag{13}$$

For Newtonian fluids, $\eta(\gamma)=\mu$, to non-newtonian fluids $\eta(\gamma)$ is modeled by Eqs. (10) and (11).

4.1 Streamline Upwind/Petrov-Galerkin Formulation

On the finite element approximation of Eq. (12) we will employ the current finite subspaces for Fluid Dynamics,

$$\mathbf{V}_h=\{\mathbf{v}\in H_0^1(\Omega)^N \,\big|\, \mathbf{v}_{|K}\in R_k(K)^N, K\in \mathcal{C}_h\} \tag{14}$$

$$P_h=\{p\in \mathcal{C}^0(\Omega)\cap L_0^2(\Omega) \,\big|\, p_{|K}\in R_l(K), K\in \mathcal{C}_h\} \tag{15}$$

$$\mathbf{V}_h^g=\{\mathbf{v}(\bullet,t)\in \mathrm{H}^1(\Omega)^{\mathrm{N}}, t\in[0,\mathrm{T}] \,\big|\, \mathbf{v}_{|\mathrm{K}}\in \mathrm{R}_k(\mathrm{K})^{\mathrm{N}}, \mathrm{K}\in \mathcal{C}_h, \qquad \mathbf{v}(\bullet,t)=\mathbf{u}_g \text{ on } \Gamma_g\} \tag{16}$$

where $\mathcal{C}_h$ is an usual partition of the computational domain Ω in finite elements [7] and R_k, R_l denote, respectively, polynomial spaces of degree k e l [7].

From the spaces definitions (14)-(16), we may write the following Streamline Upwind/Petrov-Galerkin formulation for system (12): find the pair $(\mathbf{u}_h, p_h)\in \mathbf{V}_h^g\times P_h$ such that

$$B(\mathbf{u}_h, p_h; \mathbf{v}, q)=F(\mathbf{v}, q), \quad (\mathbf{v}, q)\in \mathbf{V}_h\times P_h \tag{17}$$

where

$$
\begin{aligned}
B(\mathbf{u}_h, p_h; \mathbf{v}, q)=&\left(\frac{\partial \mathbf{u}}{\partial t}+[\nabla\mathbf{u}]\mathbf{u}, \mathbf{v}\right)+(2\eta(\gamma)\mathbf{D}(\mathbf{u}), \mathbf{D}(\mathbf{v}))-(\nabla\cdot\mathbf{v}, p)-(\nabla\cdot\mathbf{u}, q)\\
&+\sum_{K\in C_h}\left(\frac{\partial \mathbf{u}}{\partial t}+[\nabla\mathbf{u}]\mathbf{u}+\nabla p-2\eta(\gamma)\nabla\cdot\mathbf{D}(\mathbf{u}), \tau(\mathrm{Re}_K)([\nabla\mathbf{v}]\mathbf{v}-\nabla q)\right)_K
\end{aligned} \tag{18}
$$

and

$$F(\mathbf{v},q)=(\mathbf{f},\mathbf{v})+(\sigma_h,\mathbf{v})_{\Gamma_h}+\sum_{K\in C_h}(\mathbf{f},\tau(\mathrm{Re}_K)([\nabla\mathbf{v}]\mathbf{v}-\nabla q))_K \tag{19}$$

with the stability parameter τ is defined by:

$$\tau(\mathrm{Re}_K)=\frac{h_K}{2|\mathbf{u}|_p}\xi(\mathrm{Re}_K) \tag{20}$$

$$\xi(\mathrm{Re}_K)=\begin{cases}\mathrm{Re}_K, 0\le \mathrm{Re}_K<1\\ 1\ , \mathrm{Re}_K\ge 1\end{cases} \tag{21}$$

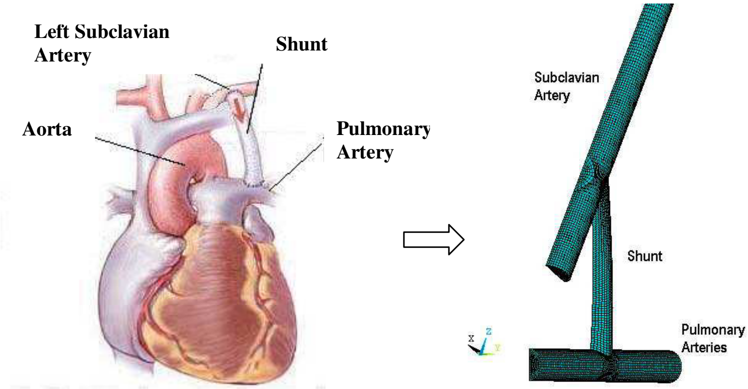
$$\mathrm{Re}_K=\frac{m_k|\mathbf{u}|_p h_K}{4\eta(\gamma)} \tag{22}$$

in which $|\mathbf{u}|_p$ denotes the p-norm on $\Re^N$ and the constant m_k for the SUPG formulation is defined as in Franca e Frey [8].

5 NUMERICAL RESULTS

In this section, a Streamline Upwind Petrov-Galerkin formulation was employed to simulate numerically blood flow in systemic to pulmonary shunts based on clinical measurement data from the Cardiology Institute of Rio Grande do Sul. The blood flow will be modeled as incompressible, generalized newtonian and pulsative, mathematically described by the system defined in Eq.(12). In the performed computations, we employ lagrangean trilinear interpolations to approximation velocity and pressure fields employing the finite element code Flotran (Ansys Inc.) and the post-processor Ensight software (CEI Inc.). All the numerical tests have been carried out at Laboratory of Computational and Applied Fluid Mechanics and National Supercomputer Center (CESUP) of Federal University of Rio Grande do Sul (UFRGS).

In Fig.2 (b), we illustrate a three-dimensional parametric model created in attempt to simulate computationally the blood flow through a Modified Blalock-Taussig Shunt (Fig.2 (a)). This model is based on direct intraoperative and angiocardiographic measurements of the associated vascularities. The employed model includes the left subclavian and pulmonary arteries and the shunt grafting them.



(a) (b)

Figure 2. (a) Modify Blalock-Taussing pulmonary shunt. (b) Tridimensional discretized model.

Approximately 65,000 trilinear lagrangean elements were used to approach the fields of velocity and pressure. Simulations were performed according to the following assumptions: the wall is rigid, blood is homogeneous a fluid, flow is laminar, no-slip boundary conditions at the artery walls, and uniform pressure at outlet sections of subclavian and pulmonary arteries. The subclavian artery inlet velocity profile was assumed to be flat in all simulations. To provide a pulsative boundary condition, the inlet velocity-time function was obtained from a Eco Doppler (Fig.3) from a patient who had previously undergone first stage of Modified Blalock-Taussig Shunt.

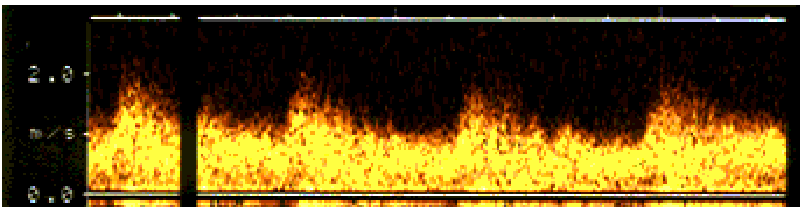


Figure 3. Pulsative inlet boundary condition(Eco Doppler exam)

Ten patients, with congenital cardiopathy and low pulmonary blood flow will be enclosed in the present study, submitted to the Modified Blalock-Taussig Shunt at Cardiology Institute of RS. This palliative surgery option is considered if repairman does not exist indication of a cardiac surgery. The inclusion of the patient was in accord with the pre-operatory diagnostic daily, the indication of the group of congenital cardiopathies, the clinical condition of the patient, the consent of responsible for the patient and the individual preference of the surgeon, who must consider adequate the systemic-to-pulmonary surgery of the type of Modified Blalock-Taussig Shunt.

In this article we are interested in the local phenomena of the hemodynamics in a Modified Blalock-Taussig Shunt. The Fig. 4 shows a local distribution of streamlines in the proximal shunt, where it is possible to identify the formation of vortices and vena contracta.

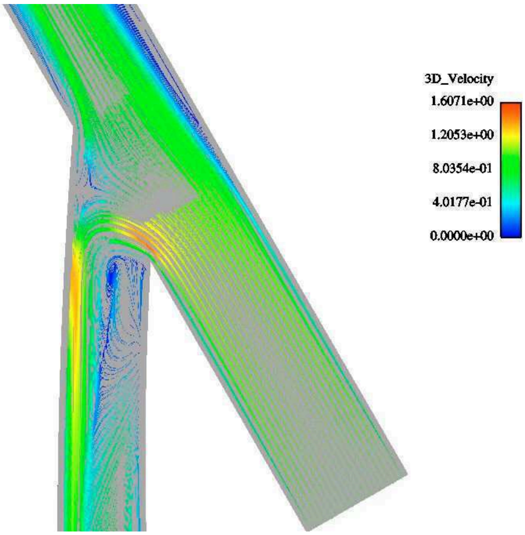


Figure 4. Streamlines in the proximal shunt with 30° between the subclavian artery and. Shunt

Figure 5 shows the influence of the angle between the subclavian and the shunt in the pressure field.

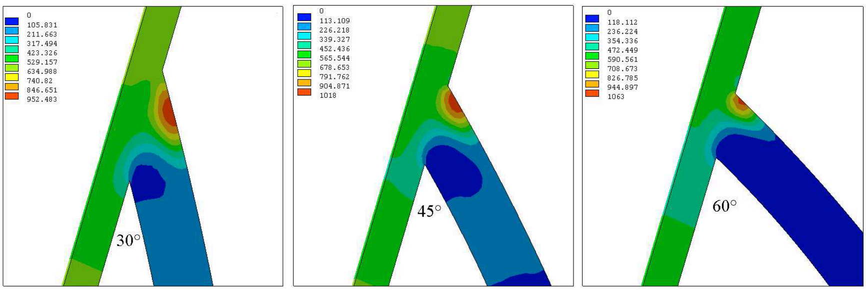


Figure 5. Pressure field for different configurations of angles

Table 1. Percentage of flow deviated by the shunt at different angles for P1, P3 and P4 patients.

Patients	Local	30°	45°	60°
P1	**Shunt**	**79,33%**	**77,96%**	**78,43%**
	RightPul.	40,64%	34,88%	29,92%
	Right Pul.	38,68%	43,09%	48,51%
P3	**Shunt**	**38,67%**	**38,45%**	**39,65%**
	RightPul.	21,30%	15,92%	11,77%
	Right Pul.	17,37%	22,52%	27,88%
P4	**Shunt**	**76,32%**	**75,41%**	**76,15%**
	RightPul.	36,65%	33,27%	31,02%
	Right Pul.	39,66%	42,14%	45,13%

Note that the angle between the subclavian and the shunt modify the stagnation point. It is not interesting that this point be on the union of the shunt with subclavian artery, increasing in this way the force at the joint. The influence of the angle in the flow deviated for the artery is insignificant if compared with the influence exerted for the alteration of the diameter of the anastomosis(shunt), when it is between 30° and 60°.

The increase in the diameter of the shunt results in a increase of flow deviated to the pulmonary artery, modifying only quantitatively the flow deviated, as shows in Fig 6.

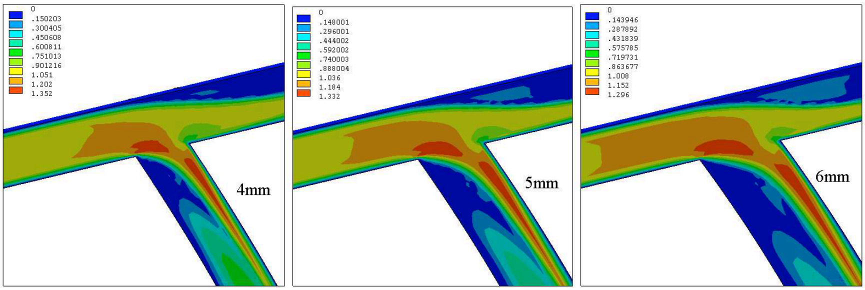


Figure 6. Quantitative modify of the flow at different shunt diameters.

The percentage of flow deviated is shows in Fig 7., where the **Dr** is defined by the ratio of the subclavian and shunt diameters. Note that the modify is significant and constitute most factor in the flow deviated.

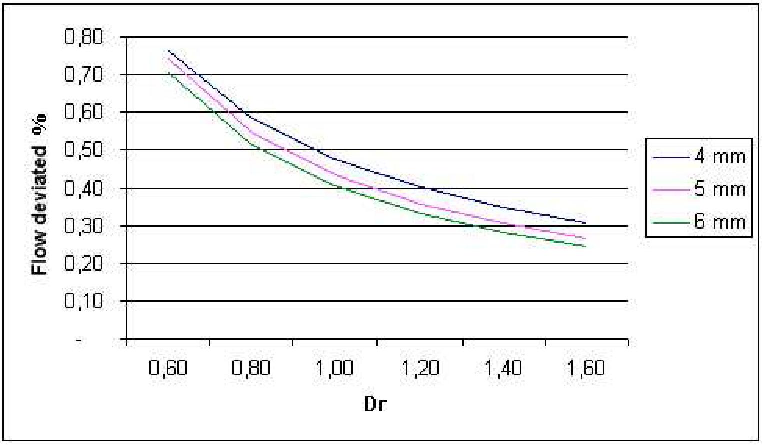


Figure 7. Percentage of flow deviated to shunt.

6 CONCLUSIONS

The present study evaluated the influence of geometric and flow parameters that influence in blood flow through a Modify Blalock-Taussig pulmonary shunt.

It was still observed that the angle of the proximal anastomosis has sensible influence in the amount of flow deviated to the shunt. This influence starts to diminish for the angles between 30° and 60°, situations which approximately correspond 80% of the raised sampling. Proximal anastomosis with angle of 110° had presented a bigger percentage of deviated flow. This angle, however, has the inconvenience to become extreme flow that reaches the left lung, in detriment of the supplied flow the right lung. The optimum angle for the proximal anastomosis is featured when the flow supplied for the two lungs is equal. The carried out simulations indicate that this excellent value is reached, in the majority of the times, between 30° and 60°.

For proximal anastomosis with concerned angles, the shunt diameter has the most influence in the percentage of flow deviated to the lung, and how bigger the diameter of the shunt, biggest will be the flow deviated.

7 ACKNOWLEDGEMENTS

The authors S. Frey, D.C. Pereira, and C. Spode thanks the financial support provided by the CNPq Proc. 350747/93-8 and Propesq-UFRGS, respectively.

8 REFERENCES

[1] Blalock, A. e Taussig, H., 1945, “Surgical treatment of malformations of the heart in which there is pulmonary stenosis or pulmonary atresia”. JAMA , Vol. 128, 189-202.

[2] Bayliss, L.E., 1973, “The rheology of Blood”, Handbook of physiology – Circulation I

[3] J. C. Slattery, *Advanced Transport Phenomena*, Cambridge University Press, U.S.A (1999).

[4] Bird, R. B., 1987, “Dynamics of polymeric liquids”, : John Wiley.

[5]A.N. Brooks e T.J.R. Hughes, *Streamline Upwind/Petrov-Galerkin formulations for convective dominated flows with particular emphasis on the incompressible Navier-Stokes equations*, Comput. Methods Appl. Mech. Engrg. Vol. 32, pp. 199-259 (1982).

[6] L.P. Franca, S. Frey e S. e T.R.J. Hughes, *Stabilized finite element methods: I. Application to the advective-diffusive model*, Comput. Methods Appl. Mech. Engrg., Vol. 95, pp. 253-276 (1992).

[7] Ciarlet, P.G., 1978, "The finite element method for elliptic problems", North-Holland, Amsterdam.

[8] L.P. Franca e S. Frey, *Stabilized finite element methods: II. The incompressible Navier-Stokes equations*, Comput. Methods Appl. Mech. Engrg., Vol. 99, pp. 209-233 (1992).