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AN ITERATIVE PROCEDURE FOR COMPUTING WATER SURFACE PROFILES OF RETURN FLOWS IN GROUNDWATER MODELING

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Abstract. *This communication describes the application of a methodology designed to improve the representation of water surface profiles along drainage canals within the frame of regional groundwater modeling. It is an iterative procedure that combines the use of two public domain computational codes, MODFLOW and HEC-RAS. MODFLOW is a quasi-three-dimensional finite difference model to simulate groundwater flow. The model possesses a modular structure, each module representing a feature of the hydrologic system such as return flow to drains, stream-aquifer interactions, recharge, etc. In spite of its versatility, modeling water surface profiles in drainage canals presents some limitations. The Drains Module available with MODFLOW simulates groundwater flow to drain canals as a linear function of the difference between the aquifer hydraulic head and the drain hydraulic head. The main disadvantage of this module is that considers a static representation of water surface profiles along drains. Therefore, the proposed methodology uses HEC-RAS, a 1-D code for surface water calculations, to iteratively estimate hydraulic profiles along drains in order to improve the aquifer/drain interaction process. The procedure was applied to the grounwater/surface water system of the Choele Choel Island, Río Negro, Argentina. Although more testing is needed, preliminary results show the feasibility of the approach. Smooth and realistic hydraulic profiles along drains were obtained while backwater effects were clearly represented.*

1 INTRODUCTION

Irrigation is used all around the world either to improve agricultural production or to allow crop growth in semiarid regions. Drain tiles and canals are constructed in irrigated fields to remove water and evapoconcentrated salts from the root zone to maintain a suitable growing environment for the crops. Normally, the drainage effluent or return flow is released to downstream surface waters.

Subsurface flow, overland flow, surface flow, water surface storage, evapotranspiration and surface water/groundwater interactions are just some of the processes involved in complex drainage/irrigation systems. Numerical modeling is a commonly used tool to address a wide variety of issues regarding irrigated agriculture. For instance, models can be developed for design and management purposes[1,2,3] or they can be used to analyse hydrological and environmental impacts of drainage and/or irrigation projects[4]. They have also been developed for basin irrigation analysis[5] or simply to investigate groundwater flow to drains[6,7].

In this work, results of a regional groundwater flow model that include the main hydrologic features of an irrigated area are discussed. Some of the limitations encountered for the representation of open drains on the context of the regional groundwater model used are discussed. Finally, an alternative approach to overcome those limitations is proposed and tested on the same hydrologic setting in order to investigate the feasibility of the approach.

2 MATHEMATICAL MODEL

In general, the problem of the interaction between groundwater and surface water can be mathematically posed as follows

$$\nabla \cdot \mathbf{K} \nabla h - W = S \frac{\partial h}{\partial t} \tag{1}$$

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial s} = \sigma \tag{2}$$

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial s}\left(\frac{Q^2}{A}\right) + g\frac{\partial H}{\partial s} - g(S_o - S_f) = 0 \tag{3}$$

$$\sigma = \begin{cases} C_r\left[h - (H + \eta)\right], \; \textit{on rivers} \\ C_d\left[h - (d + \eta)\right], \; \textit{on drains} \end{cases} \tag{4}$$

where

- ***K*** : hydraulic conductivity tensor (LT^{-1});
- *S* : specific storage of the porous media (L^{-1});

W : volumetric flux per unit volume (T^{-1});
h : hydraulic head in the aquifer (L);
t : time (T);
σ : surface water/groundwater interaction term ($L^2\,T^{-1}$);
Q : stream discharge ($L^3\,T^{-1}$);
A : cross sectional area of the stream (L^2);
s : arc length along the stream (L);
g : gravity acceleration (LT^{-2});
S_f : energy slope;
S_o : bottom slope;
C_r : riverbed conductance (LT^{-1});
C_d : drains conductance (LT^{-1});
H : water depth in rivers (L);
η : streambed elevation (L);
d : water depth in drains (L).

Equation (1) describes three dimensional (3D) groundwater flow of constant density through heterogeneous and anisotropic porous media, Equations (2) and (3) are the conservation form of the Saint Venant equations for open channel flow, and Equation (4) represents either the interaction between aquifers and streams or aquifer and drain canals. The specific storage S and the hydraulic conductivity tensor $\boldsymbol{K}$ are space dependent physical parameters, while the source/sink term W may be a function of space and time.

Except for very simple cases, analytical solutions of the system of Equations (1)-(4), or even simplifications of it are rarely possible, so numerical methods must be employed to obtain approximate solutions.

3 GROUNDWATER MODEL DISCRETIZATION

The public domain computational code MODFLOW, developed by the U.S. Geological Survey[8] solves Equation (1) based on the discretization of a continuous aquifer system using the finite difference method. The code possesses a modular structure, each module representing a specific sink/source of the hydrologic system being simulated.

The finite difference discretization consists on replacing Equation (1) by a finite set of discrete points or cells in space and time where aquifer head values are calculated. The cells location is described in terms of rows, columns and aquifer layers in a 3D setting. An i, j, k indexing system is used, where i is the row index, j is the column index and k is the layer or vertical index. The vertical discretization can either correspond to horizontal aquifer units or follow the geometry of aquifer layers. Layers are numbered from top to bottom, therefore an increment in the k index corresponds to a decrease in elevation. In the same way, rows are considered parallel to the x coordinate axis, so increments in the row index i correspond to increases in the y axis; and columns are considered parallel to the y coordinate axis, so that increments in the column index j correspond to increases in the x axis. Figure 1 illustrates a cell and its six adjacent aquifer cells.

Two formulations are available. In the block-centered formulation, the cells are the blocks formed by the sets of parallel lines and the nodes are at the center of each cell. In the point-centered formulation, the nodes are at the intersection points of the sets of parallel lines, and cells are drawn around the nodes with faces halfway between nodes.

The finite difference equation is derived upon the application of the continuity equation in the cell. According to the Darcy´s law[9], the flow into cell *i,j,k* in the row direction from cell *i,j-1,k* is given by

$$q_{i,j-1/2,k} = KR_{i,j-1/2,k}\,\Delta C_i \Delta V_k \frac{(h_{i,j-1,k} - h_{i,j,k})}{\Delta R_{j-1/2}} \qquad (5)$$

where

$q_{i,j-1/2,k}$: volumetric discharge through the face between cells *i,j,k* and *i, j-1,k* ($L^3 t^{-1}$),

$KR_{i,j-1/2,k}$ hydraulic conductivity along the row between nodes *i,j,k* and *i,j-1,k* (Lt^{-1})

$\Delta C_i \Delta V_k$: area of the cell faces normal to the row direction (L^2);

$\Delta R_{j-1/2}$: distance between nodes *i,j,k* and *i, j-1,k* (L).

A similar Expression to equation (5) can be written to approximate the flow into cell *i, j, k* through the remaining five faces. To account for flows into the cell from external features or processes such as streams, drains, areal recharge, evapotranspiration or wells, additional terms are required. In general, if there are *N* external sources or stresses affecting a single cell, the combined flow is expressed by

$$QS_{i,j,k} = \sum_{n=1}^{n=N} p_{i,j,k,n}\, h_{i,j,k} + \sum_{n=1}^{n=N} q_{i,j,k,n} \qquad (6)$$

where $QS_{i,j,k}$ is the flow into the cell from external sources/sinks; and $p_{i,j,k,n}$ and $q_{i,j,k,n}$ are constants ($L^2\ t^{-1}$) and ($L^3\ t^{-1}$), respectively, dependent on the sources/sinks characteristics.

The finite difference approximation for the time derivative in Equation (1) is expressed with the help of the head hydrograph at node *i,j,k* shown in Figure 2. According to this hydrograph, the backward difference approximation to the time derivative of head at time t_m is obtained as

$$\left[\frac{\Delta h_{i,j,k}}{\Delta t}\right]^m = \frac{h^m_{i,j,k} - h^{m-1}_{i,j,k}}{t_m - t_{m-1}} \qquad (7)$$

In summary, the backward-finite-difference equation for cell *i,j,k* is

$$q^m_{i,j-1/2,k} + q^m_{i,j+1/2,k} + q^m_{i-1/2,j,k} + q^m_{i+1/2,j,k} + q^m_{i,j,k-1/2} + q^m_{i,j,k+1/2} + QS^m_{i,j,k} = S_{i,j,k}(\Delta R_j \Delta C_i \Delta V_k)\frac{h^m_{i,j,k} - h^{m-1}_{i,j,k}}{t_m - t_{m-1}} \qquad (8)$$

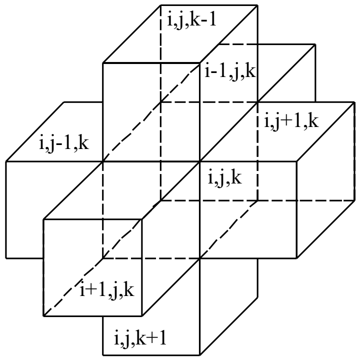

Figure 1: Model cells scheme[8]

Then, upon introducing (5) into (8), the entire system of equations may be written in matrix form as

$$\boldsymbol{A}\boldsymbol{h}=\boldsymbol{b} \tag{9}$$

where $\boldsymbol{A}$ is the matrix of the head coefficients, $\boldsymbol{h}$ is the vector of unknown head values at the end of the time step m, and $\boldsymbol{b}$ is a vector of constant terms. In MODFLOW, solvers for the system (9) include the Strongly Implicit Procedure, the Slice-Successive Overrelaxation and the Preconjugate Gradient methods.

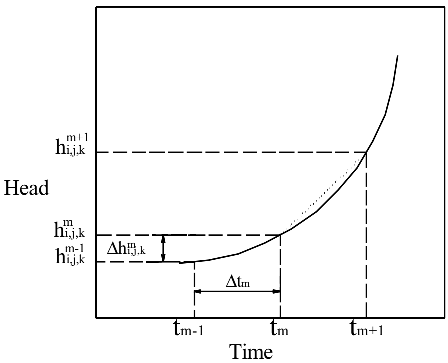

Figura 2: Time discretization

3.1 Stream-aquifer interaction

MODFLOW is strictly a groundwater flow model, therefore streams are represented in a simplified way by means of the so-called Stream Module or Stream Package[10]. In fact, the Stream Module is simply a streamflow accounting through the Manning´s expression, used to calculate water surface elevations along streams. The interaction between streams and aquifers takes place through the coupling term σ of Equation (4). In spite of its simplifications, this approach has been successfully implemented in many engineering applications. Abundant literature exist on this topic, where models with different degrees of complexity are described. However, a revision of the literature on this topic is beyond the scope of this work.

3.2 Drains-aquifer interaction

The interaction between drain canals and the aquifer are of particular interest in this work. In MODFLOW, Equation (4) for drains is handled on a different module. However, this module presents some limitations. The drain elevation has to be calculated externally by the user based on field data, and provided to the model. When data is limited, interpolation and/or extrapolation is the only way to compute initial water elevations. Moreover, the model does not handle surface flow routing nor backwater effects at drains discharge points. Then, one of the main disadvantages of the Drain Package is that considers a static representation of water surface profiles along drains.

An attempt was made to achieve a more sound representation of those profiles by means of an iterative procedure that combines the use of MODFLOW and HEC-RAS.

4 SURFACE WATER MODEL PRINCIPLES/DISCRETIZATION

This section briefly describes the main characteristics of a public domain code HEC-RAS (Hydrologic Engineering Center´s River Analysis System), developed by the U.S. Army Corps of Engineers[11], to simulate open channel flow.

HEC-RAS contains three one-dimensional analysis components: 1) steady flow water surface profile computations; 2) unsteady flow simulations; and 3) movable boundary sediment transport computations. All three components can perform on a full network of natural as well as artificial channels.

In this work only the steady state component is revised, which is intended for calculating water surface profiles for steady gradually varied flow. As mentioned above, the model can handle a full network of canals, a dendritic system or a single river reach. Subcritical, supercritical and mixed flow regime water surface profiles can be calculated. Rivers are discretized into reaches which are divided into segments by cross sections.

Water surface profiles are computed from one cross section to the next by solving the energy equation below with an iterative procedure called the standard step method.

$$d_2 + \eta_2 + \alpha_2 \frac{v_2^2}{2g} = d_1 + \eta_1 + \alpha_1 \frac{v_1^2}{2g} + h_e \tag{10}$$

where d_1, d_2 are the depths of water at two cross sections; η_1, η_2 are the topographic elevations of main canals; v_1, v_2 are the average velocities; a_1, a_2 are the velocity weighting coefficients; and h_e is the energy head loss. Moreover, energy losses between two consecutive cross sections are computed as the sum of friction losses (Manning´s equation) and contraction/expansion losses (coefficient multiplied by the change in velocity head). The corresponding calculation equation is

$$h_e = L\,\overline{Sf} + C\left|\frac{\alpha_2 v_2^2}{2g} - \frac{\alpha_1 v_1^2}{2g}\right| \tag{11}$$

where L is the discharge weighted reach length; $\overline{Sf}$ is the representative friction slope between two sections; and C is the expansion/contraction loss coefficient.

The calculation of the total conveyance and the velocity coefficients for a given cross section requires that flow be subdivided into units for which the velocity is uniformely distributed. HEC-RAS subdivides flow in overbank areas using the input cross section n-value breakpoints (locations where n values change in a compound channel). Conveyance within each subdivision is calculated by means of the Manning´s equation. The main channel conveyance is normally computed as a single conveyance element.

Basic geometric data consist on the connectivity of the river system, cross sections data, reach lengths, energy loss coefficients, stream junctions information and hydraulic structures data. The river system schematic defines how the various river reaches are connected. The connectivity of the reaches is key to the model in order to define how the computations proceed from one reach to the next. Cross sections are required at representative locations throughout a stream reach and at locations where changes occur in discharge, slope, shape or roughness. On the other hand, stream junctions are defined as locations where two or more streams come together or split apart.

Profile computations begin at a cross section with known or assumed starting conditions and proceed upstream for subcritical flow or downstream for supercritical flow. An iterative solution of Equations (10) and (11) is used. In case the flow regime changes from subcritical to supercritical or vice versa, the model can also run in a mixed flow regime mode. Boundary conditions are also necessary to establish the starting water surface at the stream system endpoints (upstream and downstream). On the other hand, discharge information is required at each cross section in order to compute the water surface profile.

5 GROUNDWATER MODEL APPLICATION TO A STUDY CASE

This section describes a MODFLOW application in an aquifer located in Patagonia, Argentina, and some of the limitations faced when trying to represent hydraulic conditions in open channels that collect return flows, i.e. drain channels.

The Negro River is born upon the confluence of the Neuquén River and the Limay River. It runs from west to east through its Upper Valley, and enters the Middle Valley approximately 283 km downstream where it bifurcates into the North Branch and the South Branch, that join again about 70 km downstream. Entrapped between these two branches is the Choele Choel Island (Figure 3). The North Branch conducts more than 90 % of the Negro River flows, which are largely controlled by a series of dams located on the Upper Valley.

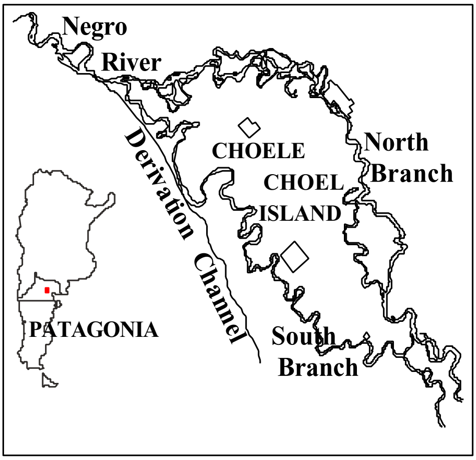

Figure 3: Study area

Alluvial deposits constitute the main aquifer, characterized by high hydraulic conductivity and high heterogeneities. The alluvial aquifer is about 20 m thick and is underlain by deposits of very low hydraulic conductivity. It is considered an unconfined aquifer in most of its extent. Therefore, from a mathematical point of view, it is represented by the depth-average form of Equation (1)[9].

The island and some of the adjacent land is one of the main fruit production regions of Argentina. Agricultural activities are sustained thanks to a drainage and irrigation system, composed by an intricate network of channels. Moreover, both river branches naturally drain groundwater along approximately 150 km of aquifer-stream contact. The artificial drainage network is about 73 km long, and contributes to evacuate excess water, which is discharged at downstream locations.

A numerical simulation was performed aimed at getting a better understanding of the behavior of the coupled stream-aquifer and drain-aquifer system. MODFLOW with the Stream Module[9] was used to represent the groundwater flow in the island and flows/water elevations in both branches of the river. Discretization of the area resulted in 4415 active nodes within the island, including border stream nodes (Figure 4). The inset in the Figure shows a detail of the grid nodes. A key feature to this modeling application was the representation of main drainage canals, accomplished by means of the Drain Module.

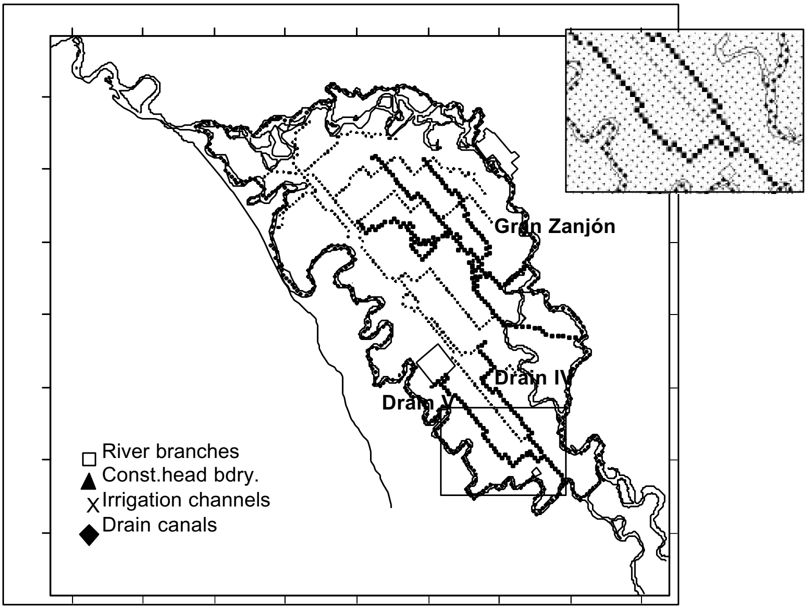

Figure 4: Model grid, sinks and sources

According to Equation (4), input parameters to this package include the water elevation in the drain *d*, the location of the drain and the drain conductance C_d. Two separate networks of drains canals were simulated: the first composed by Drains IV and V, and the second called the "Gran Zanjón" system, both shown in Figure 4. In expression (4), *h* is calculated by the model, however, water elevation along drain channels $h_d = d + \eta$ (or simply *d*) must be externally estimated by the user based on field data. In the case under analysis, scarce drain related information was available. Some field indicators were used to define a first estimate of *d* at drain nodes. Those values were adjusted later during the model calibration.

A simulation was performed under no stress or non-irrigation conditions. The only internal boundary active during this simulation were drain canals, represented as line sinks, carrying very low return flows. The objective of this simulation was twofold: on one hand to obtain a set of model parameters that would yield an acceptable fit between simulated and observed values of hydraulic heads (i.e. the model calibration) and, on the other hand, to define initial conditions for a second simulation carried over a complete 10 month irrigation season.

For the second simulation, losses through 110 km of unlined secondary and primary irrigation canals, i.e. 349 model cells, were added as line source terms (Figure 4). An effective recharge rate resulting from evapotranspiration and losses in irrigated fields was introduced as an aerial source term over the whole study area. Results for the 10 months simulation as well as for the non-irrigation simulation fit observed hydraulic head values satisfactorily well for the modeling objectives. The results of these simulations have been published elsewhere[11] and are not shown here for the sake of brevity.

In general, drain flows or return flows were underestimated. For the first simulation the total return flow was 0.37 m^3/s, while during the 10 months simulation this variable reached a maximum of 0.78 m^3/s.

6 MODFLOW – HEC-RAS APPLICATION TO THE STUDY CASE

The proposed procedure works as follows: in the first iteration MODFLOW is run and calibrated with a manually estimated drain elevation (d_0). Return flows σ calculated by MODFLOW become lateral flows along the drainage network. These lateral flows plus appropriate geometry data at different cross sections are input to HEC-RAS in order to obtain a more sound hydraulic profile along drain canals The profile so obtained is introduced back into MODFLOW as the new drain elevations, and a new set of aquifer hydraulic heads and drain flows are obtained. HEC-RAS is run once again with the new drain flows as lateral flows to get a new hydraulic profile. The procedure continues until the difference between two successive iterations is less than a prescribed tolerance.

On a first stage, the feasibility of the approach was tested only for steady state conditions on the network composed by Drains IV and V. Drain IV was divided into two reaches 11,189 m and 2,077 m long, respectively. Both reaches contained 18 cross sections. Drain V was represented by a single 13,154 m long reach, and 13 cross sections. Figures 5 and 6 show the hydraulic profiles along Drain IV and V, respectively, calculated in successive iterations. Convergence was reached after the third iteration, with a convergence criteria of 0.02 m. At the junction of Drain IV and the South Branch of the stream, i.e. at cumulative distance equal zero, backwater is noticeable due to high river elevations present during the simulation in tune with natural conditions. The Figures also show the successive MODFLOW calculated aquifer head. The irregular nature of these curves is due to the position of the drain channel with respect to the direction of groundwater flow.

When the drain elevations obtained by the conventional approach (MODFLOW) are compared to those of the iterative approach, the differences do not seem very significant. However, they are different enough to produce an increment of 11 % of the previously estimated return flows for the non-irrigation period. The new value fits field observations more closely.

Simulations are on the way first to apply the iterative procedure to the whole system of drain canals, and second to simulate transient conditions using this approach.

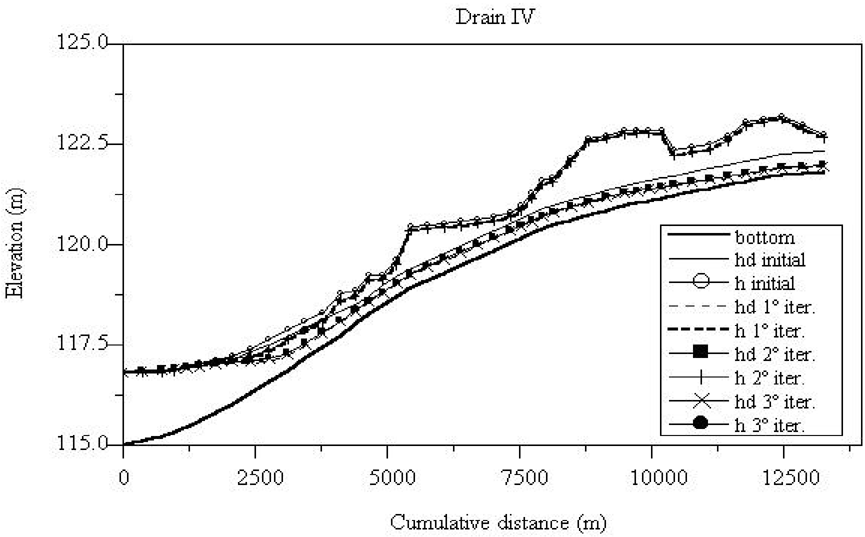

Figure 5 – Successive hydraulic profiles along Drain IV

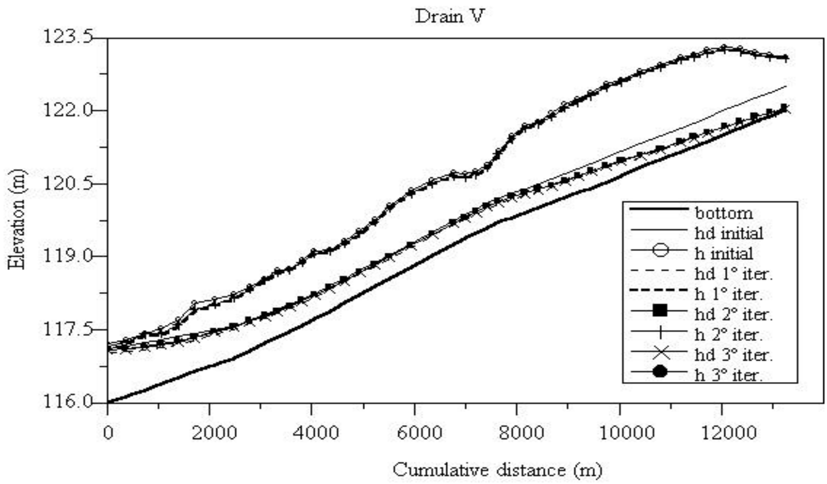

Figure 6 – Successive hydraulic profiles along Drain V

7 CONCLUSIONS

MODFLOW is a valuable tool for regional groundwater flow simulations in drainage/irrigation systems, including stream aquifer interactions and flow to drain channels. An alternative iterative procedure combining MODFLOW and HEC-RAS in order to get a more physical representation of surface flows along drains has been presented. Preliminary results showed the feasibility of the proposed methodology. Additional model runs and verification with field data are needed in order to validate the procedure and extend it to the whole drainage network as well as to transient conditions.

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