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NUMERICAL MODELING OF CENTRIFUGAL CASTING OF FUNCTIONALLY-GRADED ALUMINUM MATRIX COMPOSITES REINFORCED WITH DIBORIDE PARTICLES

Gustavo Gutierrez[a], O. Marcelo Suárez[b], and Mauricio A. Giordano[a]

[a]*Mechanical Engineering Department, University of Puerto Rico Mayaguez, PR 00681-9045, Puerto Rico, gguttierr@me.uprm.edu*
[b]*Engineering Science & Materials Department, University of Puerto Rico Mayaguez, PR 00681-9044, Puerto Rico, msuarez@ece.uprm.edu*

Abstract. Surface modification and coating techniques are widely used to improve tribological properties of metallic materials for a wide range of manufacturing, transportation, defense, and consumer industries. However, in all these techniques a surface treatment is applied after the part has been fabricated, and this adds significantly to the overall costs. An alternative, cost-effect method has been, proposed via functionalization of aluminum matrix composites uniformly reinforced with hard particles. A larger volume fraction of those particles can be attained near the wear surface via centrifugal casting,. The volume fraction of the heavier borides is controlled by inertial forces upon centrifugal processing the semisolid composite.
In this study, boride particles are modeled as spherical particles subject to a drag force in a Stoke flow in the liquid aluminum matrix. This equation of motion for the particles under the applied centrifugal forces is solved numerically assuming a gaussian diameter size distribution with a spatial uniform random distribution of particles in the sample. The effect of temperature on the viscosity is also considered by solving the energy equation. From parametric studies in the numerical model, it is possible to better understand and control the experimental conditions to obtain an appropriate functionally-graded aluminum matrix for high wear resistance applications.

Keywords: Centrifugal Casting, Functionally-Graded aluminum alloy.

INTRODUCTION

A Functionally Graded Material (FGM) is a class of composite material, consisting of two or more phases, which is fabricated with its composition and/or microstructure varying in some spatial direction (Suresh and Mortensen 1998). This allows us to control physical and/or chemical properties. Metal matrix composite FGMs feature gradual compositional variation from ceramic at one end to metal at the other, leading to unique advantages of smooth transition in thermal stresses across the thickness and minimized stress concentration at the interface of dissimilar materials. There are several methods to achieve the functionally graded properties. One of them is the centrifugal method proposed by Fukui (Fukui,1991) where a centrifugal force is used for segregating the particles. This is, probably the most economical and attractive processing route to get a FGM. The extent of segregation depends on various process parameters including cast geometry, pouring temperature of the melt, solidification time, rotational speed, and density difference between reinforcement particles and matrix. Because of the higher density of the reinforcement particles compared to the density of the molten matrix, particles move in the radial direction.

In this work the molten matrix, with the reinforcement particles, is put into a cylindrical mold and then exposed to a centrifugal action with its central axis along the radial direction. Some previous work was done by (Panda, Mazumdar et al. 2006) and (Watanabe, Kawamoto et al. 2002) but they used a hollow cylinder rotating vertically around the central axis. Because of this configuration, they had to consider the effect of gravity. In this work the effect of gravity in the movement of particles is neglected because the radial acceleration is much higher than the gravity acceleration.

The particle size distribution plays an important role in controlling the mechanical properties of the sample. A mathematical model for the motion of the particles is formulated, and the model divided into different cases, each one corresponding to different assumptions up to the more general case that relax the assumptions made. This will help us in noticing how each parameter could affect the results and if a simplification could be made or not. Solidification plays an important role in the final volume fraction of particles obtained so energy equation comes into account in the model proposed. When the Biot number is less than unity, the energy equation takes a simple form because the sample can be modeled as a Lumped Heat-Capacity System which means that the sample is at uniform temperature as it gets cold.

The aim of this work is to get insights about how the parameters involved in the process affect the volume fraction distribution and hence the features of the sample obtained. Experimental work is carried out and this will allow us, according the properties measured to the sample, to check whether the numerical results are a good prediction or not.

MATHEMATICAL FORMULATION

Particles are assumed uniform distributed spatially in the liquid aluminum matrix with a particles normal size distribution diameter. The sample is rotated at a constant centrifugal speed ω. Diboride particles have a higher density than the density of the liquid matrix. A schematic of the centrifugal casting sample is shown in Figure 1.

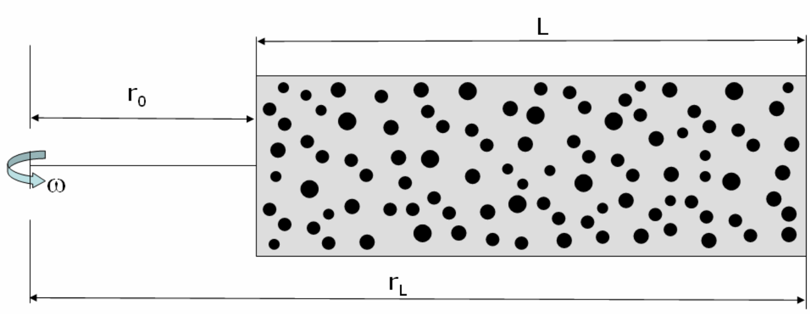

Figure 1: Schematic of the centrifugal casting sample.

Considering spherical particles in an aluminum matrix, the equation of motion for the particles in a rotational stoke flow is:

$$m_p\left[\left(\ddot{r}-r\dot{\theta}^2\right)\mathbf{e_r}+\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)\mathbf{e_\theta}\right]=\underbrace{-6\pi\mu\, a\underbrace{\left[\dot{r}\,\mathbf{e_r}+r\left(\dot{\theta}-\omega\right)\mathbf{e_\theta}\right]}_{\mathbf{v}_{rel}}}_{drag\ force} \quad (1)$$

Thus, the two equations of motion for each coordinates direction are given by:

$$m_p\ddot{r}=-6\pi\mu\, a\,\dot{r}+m_p r\dot{\theta}^2 \quad (2)$$

$$m_p r\ddot{\theta}=-6\pi\mu\, a\, r\left(\dot{\theta}-\omega\right)-m_p 2\dot{r}\dot{\theta} \quad (3)$$

Subject to the initial conditions: $at\ \ t=0\quad r=r_0\ \ ;\ \ \dot{r}=0\,;\quad \theta=0\ \ ;\ \ \dot{\theta}=\omega$

Dimensionless form of the equations of motion

In the radial direction:

$$m_p\ddot{r}=-6\pi\mu\, a\,\dot{r}+m_p r\dot{\theta}^2$$

$$m_p=\rho_p\forall=\rho_p\frac{4}{3}\pi a^3 \qquad \text{assuming a spherical particle}$$

$$\frac{d^2r}{dt^2}=-\frac{6\pi\mu\, a}{\rho_p\forall}\frac{dr}{dt}+r\dot{\theta}^2=-\frac{9}{2}\frac{\mu}{\rho_p a^2}\frac{dr}{dt}+r\dot{\theta}^2$$

Defining the following dimensionless parameters: $\tau=\dfrac{\mu_0\,\mathrm{t}}{\rho\, a^2}$, $\mathrm{r}^*=\dfrac{\mathrm{r}}{\mathrm{r}_0}$ and $\mu^*=\dfrac{\mu(T)}{\mu_0}$ and doing a change of variables, a dimensionless equation is obtained:

$$\frac{d^2r^*}{d\tau^2}=-\frac{9}{2}\frac{\rho}{\rho_p}\mu^*\frac{dr^*}{d\tau}+r^*\left(\frac{d\theta}{d\tau}\right)^2 \quad (4)$$

The dimensionless forms of the initial conditions are:

$$at\ \ \tau=0\quad r^*=1\ \ ;\ \ \frac{dr^*}{d\tau}=0 \quad (5)$$

In the θ-direction:

In the same way, the dimensionless of equation (3) in the circunferencial direction is:

$$r^* \frac{d^2\theta}{d\tau^2} = -\frac{9}{2}\frac{\rho}{\rho_p}\mu^* r^* \left(\frac{d\theta}{d\tau} - \underbrace{\frac{\rho\omega a^2}{\mu_0}}_{\mathrm{Re}_a} \right) - 2\frac{dr^*}{d\tau}\frac{d\theta}{d\tau} \tag{6}$$

With the following dimensionless form of the initial conditions:

$$at \ \ \tau = 0 \quad \theta = 0 \quad ; \ \frac{d\theta}{d\tau} = \frac{\rho\omega a^2}{\mu_0} \tag{7}$$

From equations (4) and (6), three dimensionless parameters are identified:

$$\rho^* = \frac{\rho}{\rho_p} \tag{8}$$

$$\mathrm{Re}_a = \frac{\rho\omega a^2}{\mu_0} \tag{9}$$

And the third one, for the case of temperature dependent viscosity:

$$\mu^* = \frac{\mu(T)}{\mu_0} = \frac{\mu(t)}{\mu_0} \tag{10}$$

The last parameter takes account of the dependency of temperature with the viscosity.

Effect of cooling on the viscosity of the matrix

Due to the high thermal conductivity of the aluminum and the small convection coefficient, a small Biot number can be assumed and the sample can be treated as a lumped capacitance system. For a lumped system, the temperature of the sample will be only function of the time and given by:

$$T(t) = \left(T_i - T_\infty\right)\exp\left(-\frac{h\,A}{\rho\forall c_P}t\right) + T_\infty \tag{11}$$

By knowing the variation of the viscosity with the temperature, the viscosity as a function of time can be calculated:

$$\mu = f(T(t)) = f_1(t) \tag{12}$$

For parametric studies, the convection coefficient can be estimated from a Nusselt correlation for a cylinder under forced convection.

ANALYTICAL AND NUMERICAL SOLUTIONS

The equations of motion for the more general case form an initial value problem that can be solved using Runge-Kutta algorithms. If we assume that the tangential velocity $r \times \dot{\theta}$ of the particles is approximated by $r \times \omega$, $\left(\dot{\theta} \approx \omega\right)$, the trajectories of the particles can be calculated analytically for the linear case (constant viscosity independent of the temperature).
Parametric studies can be carried out to understand the importance of the dimensionless

parameter R_{ea} and ρ/ρ_p. Also, the effect of temperature in the viscosity can be analyzed numerically.

Analytical solution

Particles are assumed with a Gaussian size distribution with a uniform spatial distribution. For a constant viscosity and neglecting the differences between $\dot{\theta}$ and ω (that is, $\dot{\theta} = \omega$) the equation of motion (4) becomes:

$$\frac{d^2 r_i}{d\tau^2} = -\frac{9}{2}\frac{\rho}{\rho_p}\frac{dr_i}{d\tau} + \left(\frac{\rho\omega a_i^2}{\mu_0}\right)^2 r_i \tag{13}$$

Here, $\frac{d\theta}{d\tau} = \frac{\rho a_i^2 \omega}{\mu_0} = \text{constant}$ and the asterisk (*) has been dropped for convenience. a_i is the radius of the particle *i* and r_i is its radial position. The initial conditions are:

$$at\ \ \tau = 0 \quad r = 1 + \frac{(r_L - r_0)}{r_0} \times rand(N,1) \quad ; \quad \frac{dr}{d\tau} = 0 \tag{14}$$

Where N is the number of particles. N equations are solved, one for each particle. Since the flow is laminar, each particle follows its own path line without colliding with other particles. So, under these assumptions, the following second order linear differential equation is solved:

$$\frac{d^2 r_i}{d\tau^2} + \underbrace{\frac{9}{2}\frac{\rho}{\rho_p}}_{b}\frac{dr_i}{d\tau} - \underbrace{\left(\frac{\rho a^2 \omega}{\mu_0}\right)^2}_{c^2} r_i = 0 \tag{15}$$

The roots of the characteristic equation are:

$$\alpha_1 = -\frac{b}{2}\left(1 + \sqrt{1 + 4(c/b)^2}\right) \text{ and } \alpha_2 = -\frac{b}{2}\left(1 - \sqrt{1 + 4(c/b)^2}\right)$$

And the general solution is:

$$r(\tau) = A_1 \exp(\alpha_1 \tau) + A_2 \exp(\alpha_2 \tau) \tag{16}$$

Applying the initial conditions, $at\ \ \tau = 0 \quad r_{init} = 1 + \frac{(r_L - r_0)}{r_0} \times rand(N,1)\ ;\ \frac{dr}{d\tau} = 0$, the analytical solution is obtained:

$$\frac{r(\tau)}{r_{init}} = \frac{\left(1+\sqrt{1+4(c/b)^2}\right)}{2\sqrt{1+4(c/b)^2}} \exp\left(-\frac{b}{2}\left(1-\sqrt{1+4(c/b)^2}\right)\tau\right) - \frac{\left(1-\sqrt{1+4(c/b)^2}\right)}{2\sqrt{1+4(c/b)^2}} \exp\left(-\frac{b}{2}\left(1+\sqrt{1+4(c/b)^2}\right)\tau\right) \tag{17}$$

Where $b = \frac{9}{2}\frac{\rho}{\rho_p}$ and $\frac{\mathrm{c}}{\mathrm{b}} = \frac{2}{9}\frac{\rho_p a^2 \omega}{\mu_0}$

Some typical values of the parameters such as density of the matrix and particles, radius of the particles, dimension of the sample are given in Table 1.

Distance of the cylinder side (end), r_0 to the axis of rotation	19.50cm
Distance of the cylinder side (end), r_L farther away from the axis of rotation	32.00cm
Density of the particle ρp (AlB2)	3.19g/cm3
Density of the liquid ρ (Al melt) (at 660 ºC)	2.4g/cm3
Rotational speed (rpm)	250 RPM
Viscosity of the liquid (at 660 ºC)	1.38 (mPa-s)
Average diameter of the particles	7.5 mμ
Variance	0.5 mμ

Table 1: Some typical values of the parameters.

Numerical solution

For the more general case, the angular velocity $\dot{\theta}$ of the particles could be different from the rotational velocity ω of the molten matrix. For this case, equation of motion and initial conditions are:

$$\begin{aligned}
&\frac{d^2 r_i}{d\tau^2} = -\frac{9}{2}\frac{\rho}{\rho_p}\frac{dr_i}{d\tau} + \left(\frac{d\theta_i}{d\tau}\right)^2 r_i\,, \\
&at\ \ \tau = 0\ ;\ r_i = 1 + \frac{(r_L - r_0)}{r_0}\times rand(N)\ ;\ \frac{dr}{d\tau} = 0.
\end{aligned} \tag{18}$$

$$\begin{aligned}
&\frac{d^2 \theta_i}{d\tau^2} = -\frac{9}{2}\frac{\rho}{\rho_p}\left(\frac{d\theta_i}{d\tau} - \frac{\rho\omega a_i^2}{\mu_0}\right) - \frac{2}{r_i}\frac{dr_i}{d\tau}\frac{d\theta_i}{d\tau}\,, \\
&at\ \ \tau = 0\quad \theta = 0\ ;\ \frac{d\theta}{d\tau} = \frac{\rho\omega a_i^2}{\mu_0}.
\end{aligned} \tag{18}$$

This is a system of two coupled non-linear ordinary differential equations of second order that can be transformed to a system of four differential equations of first order. Doing $x_1 = r$ and $x_3 = \theta$:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -bx_2 + x_4^2\, x_1 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -bx_4 + b\,c - 2\frac{x_2\, x_4}{x_1}
\end{aligned} \tag{19}$$

Subjected to the following initial conditions at $\tau = 0$:

$$\begin{aligned}
x_1 &= 1 \\
x_2 &= 0 \\
x_3 &= 0 \\
x_4 &= \frac{\rho\omega a^2}{\mu_0}
\end{aligned} \tag{20}$$

With these considerations, only the equation of motion in the r-direction has to be solved. From the numerical solution of the system, it can be shown relative velocity between particles and the molten matrix is negligible and $\dot{\theta}$ can be assumed equal to ω. If the temperature effect in the viscosity is considered, the equation of motion becomes non linear and has to be solved numerically. Hence, the following equations are solved:

$$\frac{d^2 r_i}{d\tau^2} = -\frac{9}{2}\frac{\rho}{\rho_p}\mu^* \frac{dr_i}{d\tau} + \left(\frac{\rho\omega a^2}{\mu_0}\right)^2 r_i$$
$$at \;\; \tau = 0 \quad r = 1 + \frac{(r_L - r_0)}{r_0} \times rand(N) \quad ; \quad \frac{dr}{d\tau} = 0 \tag{21}$$

Where

$$\mu = f(T(\tau)) = f_1(\tau) \tag{22}$$

And

$$T(t) = \left(T_i - T_\infty\right)\exp\left(-\frac{h\,Aa_i^2}{\rho\forall c_P \mu_0}\tau\right) + T_\infty \tag{23}$$

RESULTS AND DISCUSSION

In order to control the final particles distribution, the angular velocity or the time that the sample is exposed to the centrifugal effect can be tuned. Results for different values of these parameters will be presented and discussed here. The considered sample will be composed of four thousands particles with a spatial uniform random distributed and a Gaussian size distribution.

Figure 2a shows the particles distribution after 50 seconds for a rotational speed of $\omega = 250$ rpm. For this case, the viscosity is assumed constant and the analytical solution is used to calculate the results. It can be seen that the inner region of the sample does not have particles, which means that even the smaller particles moved due to centrifugal effect. To be able to maintain certain number of particles at the inner region, the rotational speed or the time that the sample rotates should be reduced. The average diameter of the particles and its variance are parameters more difficult to control, but by reducing the diameter and increasing the variance (poly-disperse particles) will maintain particles at the inner region providing better mechanical properties to the sample.

Figure 2b shows the effect of the viscosity on the particle distribution. In this case, a fourth order Runge-Kutta algorithm is used to integrate numerically the equation of motion. A third order polynomial was curve fitted to compute the viscosity as a function of the temperature. Since the viscosity is increasing with time due to the cooling of the sample, particles are exposed to a bigger viscous force. Then, particles are moving slower and a more uniform distribution is observed.

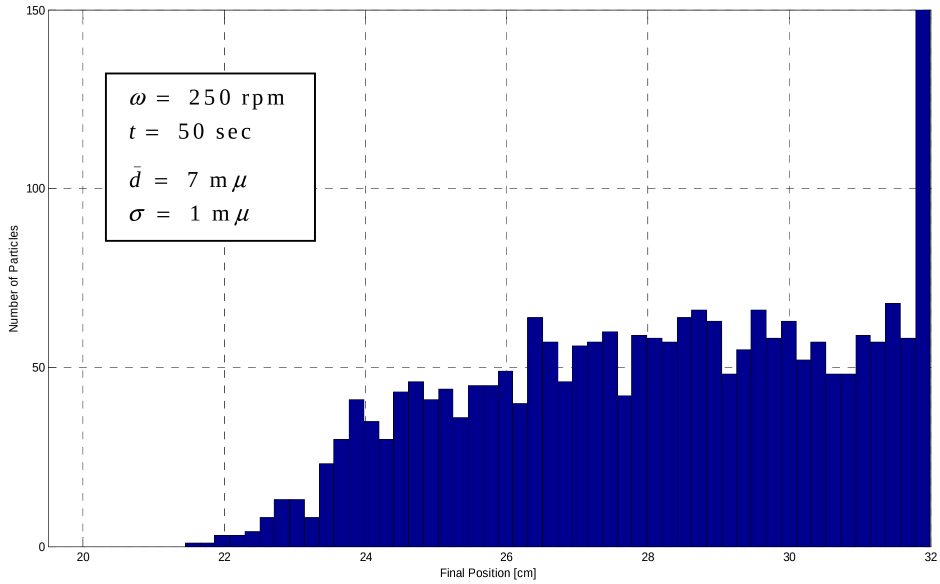

Figure 2a: Particle distribution for constant viscosity

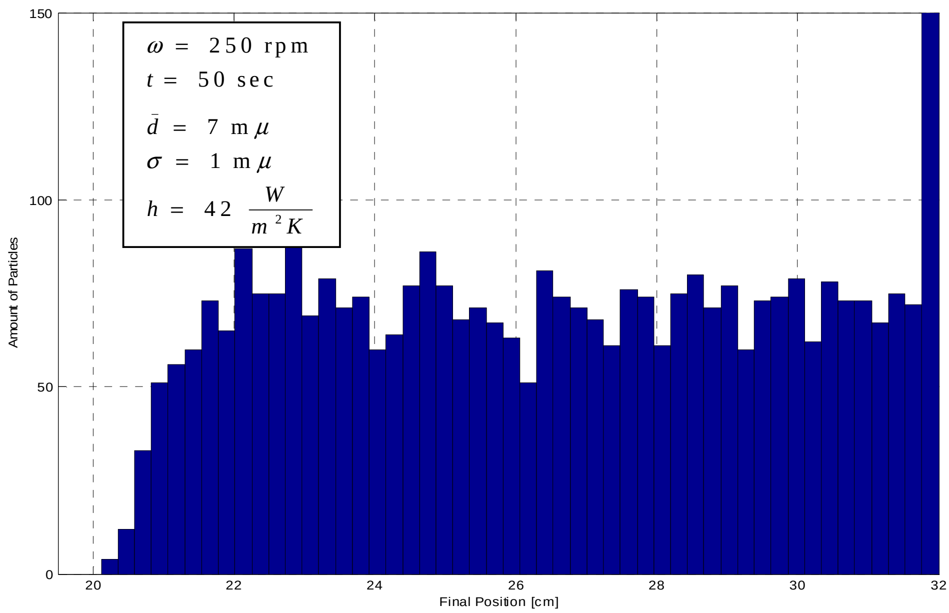

Figure 2b: Particle distribution for variable viscosity

Figure 3 shows a comparison between the constant viscosity case and the viscosity-temperature dependent case for a particle diameter of d =7 μm. From the figure, it can be seen the strong effect of the viscosity on the particle motion. For the linear case, the viscosity is smaller and the particle moves faster than for the non-linear case. From the above consideration, the effect of the variable viscosity can not be neglected. However, the increase in the viscosity contributes to a more uniform particle distribution.

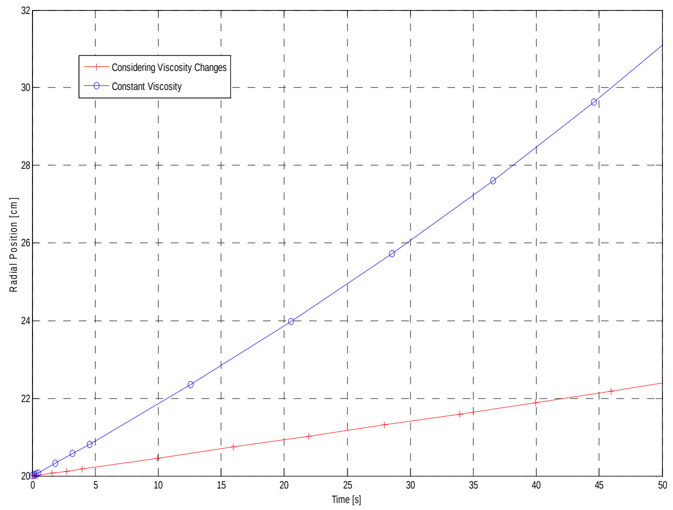

Figure 3: Comparison between the constant viscosity and the variable viscosity case

Figure 4 shows the effect of the diameter on the motion of the particles. As it is expected bigger particles move faster. Just for parametric purposes to particles are simulated: d = 20 µm and d = 10 µm. It can be seen that if the diameter is double the average speed of the particles is more than quadruple. Then, most of the bigger particles will concentrate on the outer region of the sample and a decreasing particles concentration from the outer to the inner region of the sample will be obtained after the centrifugal casting.

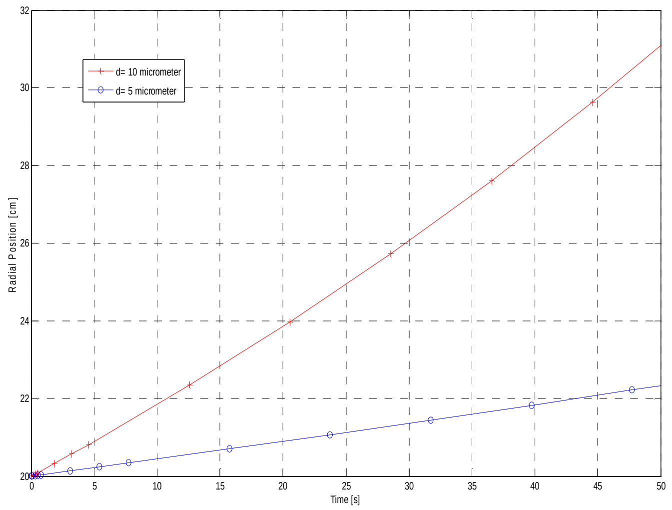

Figure 4: Effect of the size of the particles

CONCLUSIONS

An analytical and a numerical model of a centrifugal casting process for diboride particles in an aluminum matrix is developed and studied. Particles with a spatial uniform random distribution and a Gaussian diameter size distribution are assumed as initial condition of the casting process. From parametric studies, the viscosity dependence of the temperature has a strong effect and can not be assumed constant. The variable viscosity tends to generate a more uniform particle distribution as compared with the constant viscosity case. The effect of the average particle diameter is also important. The bigger the diameter of the particles the faster their motion. The two important control parameters of the centrifugal casting process are the rotation speed and the time of centrifugation. This parameter can be estimated from numerical experiment to maintain an appropriate particle concentration inside the sample.

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APPENDIX

NOMENCLATURE

$m_P =$ mass of a particle

$a =$ radius of a particle

$r =$ radial position of a particle

$r^* =$ dimensionless radial coordinate

$r_0, r_L =$ inner and outer radius of the sample

$t =$ time

$\tau =$ dimensionless time

$\dot{\theta} =$ angular velocity of the particle

$\omega =$ angular velocity of the sample

$\rho =$ density of the aluminum matrix

$\rho_P =$ density of the particles

$\mu =$ dynamic viscosity

$\mu_0 =$ reference dynamic viscosity

h = heat transfer coefficient

c_p = Specific heat

T = Temperature

T_i, T_∞ = initial temperature of the sample and ambient temperature, respectively

$A, \forall$ = area and volume of a particle