VISCO-HYPOPLASTIC MODEL FOR PAMPEAN LOESS

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Abstract. The visco-hypoplastic constitutive model is a powerful tool to represent the stress-strain behavior of clayey soils in geotechnical engineering. The model is able to describe a non-linear stress dependent stiffness, even for very high amplitudes of loading and unloading cycles. The visco-hypoplastic equation for monotonic load-unload stress paths are physically meaningful and the required soil-parameters can be determined from routine laboratory test. The model is briefly described herein. The main purpose of this work is to explore the capability of the viscohypoplastic model to approximate the stress-strain behavior of a collapsible loess from the center of Argentina under oedometric and triaxial type of loading. The experimental results were obtained for undisturbed samples of loess tested in a well instrumented triaxial apparatus by means of local displacement transducers. The results showed that the model approximated very well the experiments.

INTRODUCTION

Loess is one of the most abundant soil formations on the continental surface of the world. The Argentinean deposit is the largest one in the southern hemisphere, with a thickness that varies between 20 and 60 meters. Loess can experience high volume changes when loaded or wetted, thus it is classified as collapsible in the unstable soils group (Aitchinson, 1973).

Research efforts in the past decades have focused on understanding collapse mechanisms, the relationship between collapse and soil structure, the evaluation of collapsibility potential and the modeling of the stress-strain response. Concepts rooted in the field of unsaturated soil mechanics seem to explain correctly the behavior of loess under static load conditions (Alonso and Gens 1994, Rinaldi et al. 2006, and Rinaldi and Capdevila 2006). The small-strain dynamic shear modulus and damping ratio of loess are less known; available data for the Argentinean loess can be found in Rinaldi, et al. (1998 and 2006).

The stress-strain behavior of loess is of main importance to asses the potential settlement of structures designed in this type of soils. Thus, the scope of the paper is to evaluate the capability of the visco-hypoplastic model to describe the stress-strain behavior of loess under oedometric and triaxial loading. Undisturbed and saturated samples of loess were tested in the oedometer cell and in a triaxial apparatus. Instrumentation icludes local displacement transducers (LDT) to measure strains without the influence of end-cap effects. The agreement of the model to the experimental results is described herein.

THE VISCO-HYPOPLASTIC MODEL

Niemunis (1996, 2003) developed a visco-hypoplastic model for clay-like soils which can describe the viscous behavior of soft soils like creep, relaxation and rate dependence. Niemunis modified the hypoplastic equation proposed by Wolffesdorff (1996) in order to describe the stiffness upon oedometric or isotropic and unloading or reloading conditions and introduced an expression that depends on the true time increment. The assumptions of the visco-hypoplastic constitutive law are:

- The state of the soil is defined by the stress state and the void ratio
- Considers fully saturated clayey soils at slow rates of deformation
- The overconsolidated ratio must be smaller than 2. For larger values of OCR the formula of creep intensity becomes less precise
- Change of temperature and ion concentration in pore water are ignored
- Primary consolidation is disregarded because it is not the subject of constitutive modeling

The visco-hypoplastic model has some advantages compared to other constitutive models that turn it into a very useful tool for the geotechnical engineer. The main advantages are that the material constants of the model are closely related to standard soil parameters and so that the model could be easily implemented to a FE program (Niemunis 1996; Niemunis 2003).

The basic visco-hypoplastic equation is1:

$$\overset{\circ}{\mathbf{T}} = f_b \mathsf{L} : \left(\mathbf{D} - \mathbf{D}^{vis} \right),$$
(1)

where $\overset{\circ}{\mathbf{T}}$ is the effective stresses-rate, L is the hypoplastic fourth order stiffness tensor, f_b is barotropy factor and $\mathbf{D}$ is the rate deformation.

The stiffness tensor L from hypoplasticity (Wolffesdorff, 1996) was modified for the visco-hypoplastic model in order to reduce the change of volume when the deviator stress increase. The tensor L for the visco-hypoplastic equation is:

$$\mathsf{L} = a^2 \left[\left(\frac{F}{a} \right) \cdot \mathsf{I} + \hat{\mathbf{T}} \otimes \hat{\mathbf{T}} \right], \tag{2}$$

where I is the four order identity tensor, $\hat{\mathbf{T}}$ is defined as the normalized Cauchy stress tensor $\mathbf{T}$ $\left(\hat{\mathbf{T}} := \mathbf{T}/\mathrm{tr}\mathbf{T} \right)$ and a is calculated as a function of the critical friction angle with

$$a = \frac{\sqrt{3}\left(3 - \sin\varphi_c\right)}{2\sqrt{2}\sin\varphi_c}. \tag{3}$$

The factor F in equation (2) considered the Matsuoka-Nakai failure criterion and is defined as

$$F = \sqrt{\frac{1}{8}\tan^2\psi + \frac{2 - \tan^2\psi}{2 + \sqrt{2}\tan\psi\cos(3\theta)}} - \frac{1}{2\sqrt{2}}\tan\psi \tag{4}$$

where $\tan\psi = \sqrt{3}\left\|\mathbf{T}_c^*\right\|$ with $\mathbf{T}_c^* = \mathbf{T}_c - \frac{1}{3}\mathrm{tr}\mathbf{T}_c \cdot \mathbf{1}$, being $\mathbf{T}_c$ the stress tensor in the critical state, and $\cos(3\theta) = -\sqrt{6}\dfrac{\mathrm{tr}\hat{\mathbf{T}}_c^*}{\left[\mathrm{tr}\hat{\mathbf{T}}_c^*\right]^{3/2}}$. The angles ψ and θ (Lode's angle) are displayed on the Figure 1.
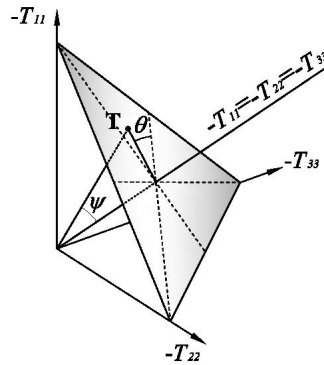

Figure 1.- Definitions of the ψ and θ in equation (4)

The barotropy factor f_b was modified by Niemunis in order to describe the stiffness upon unloading and reloading for isotropic and oedometric conditions, and volume changes for constant modulus of strain rate. The barotropy factor is defined according to the condition of the experiment. For isotropic conditions yields:

$$f_b = -\frac{\mathrm{tr}\,\mathbf{T}}{[1 + a^2/3]\kappa} = -\beta_b\,\mathrm{tr}\,\mathbf{T} \tag{5}$$

and for oedometric conditions

$$f_b = -\frac{\text{tr } \mathbf{T}}{[1 + a^2 / (1 + 2K_0)]\kappa^o} = -\beta_b \text{ tr } \mathbf{T}. \tag{6}$$

The parameters κ, κ^o, and K_0 are necessary to calculate the barotropy factor. Using the Butterfield (1979) compression law, the parameters κ and κ^o are the unloading or reloading slope of the isotropic and oedometric test respectively. The parameter K_0, defined as the earth pressure coefficient, is calculated as:

$$K_0 = \frac{-2 - a^2 + \sqrt{36 + 36a^2 + a^4}}{16}. \tag{7}$$

A critical state is defined by shearing of soil going on indefinitely without change of effective stress and volume.

The viscous rate $\mathbf{D}^{vis}$ in equation (1) is represented by the following equation which is described analogously to Norton's law:

$$\mathbf{D}^{vis} = D_r \vec{\mathbf{B}} \left(\frac{1}{\text{OCR}} \right)^{1/Iv}, \tag{8}$$

Here, D_r is the reference creep rate; $\vec{\mathbf{B}}$ is the direction of the creep rate and could be seen as a hypoplastic flow rule; I_v is the viscosity index of Leinenkugel (1976); the OCR is the overconsolidated ratio that can be calculated from OCR $= p_e / p'$ where p' represents the current effective mean stress and p_e is the equivalent isotropic pressure by Hvorslev (1960).

The visco-hypoplastic model has the eight model parameters and reference values listed on Table 1. The calibration of the visco-hypoplastic model is extensively described elsewhere (see for example Punlor, 2004; Garcia, 2005).

Table 1. Visco-hypoplastic model parameters and reference values

Reference model	Laboratory test
e_{100}	oedometric test
λ	oedometric test
κ	oedometric test
β_R	undrained test
I_v	oedometric creep test
D_r	oedometric test
φ_c	triaxial test
OCR	oedometric test

TESTING PROGRAM

Block samples of Loess were obtained at the campus of the National University of Córdoba from a 4 meters deep open trench. Table 2 shows the most significant physical parameters of the soil tested. Undisturbed (structured) specimens saturated were prepared for testing in the triaxial cell. Triaxial specimens were 50 mm in diameter and 100 mm in height.

Table 2. Geotechnical relevant parameters of the loess tested in this work.

Soil Unified Clasification	Average Natural Water Content (%)	Plasticity Index	Dry Unit Weight [kN/m^3]	Initial Degree of Saturation	Percent Passing Sieve Nº 200	Natural Matric Suction [kPa]
ML	18	3.6 %	12.5	55 %	92.4 %	75

The undisturbed specimens were obtained by trimming. Triaxial test were performed in consolidated drained condition. Table 3 display the soil parameters and testing conditions of the different specimens prepared here. The triaxial cell allows the measurement of strains by means of three local displacement transducers (LDTs) placed on the specimen under test. An additional displacement transducer was attached at the center of the sample to determine the local radial strains by measuring changes in specimen diameter. The LDTs used here are similar to those described by Goto et al. (1991).

Table 3. Soil parameters and testing conditions of the prepared soil specimens.

Specimen	Testing Condition	Confining Pressure [kPa]	Dry Unit Weight [kN/m^3]	Moisture Content [%]
U	Saturated Undisturbed	10	12.4	43.20
		20	12.4	43.20
		40	12.2	44.50
		80	12.6	41.90

A similar sample was trimmed and tested in saturated condition in the oedometer test. In this case the sample was trimmed manually and saturation was induced by applying a constant setting load of 10 kN/m^2. After saturation, the sample was loaded. Each applied load was keep constant until the fully consolidation of the sample usually attained after 30 minutes.

The grain size distribution curve of the structured specimens were obtained by sieving analysis, following the conventional sieving test (ASTM D 422) but here, a block of the structured sample was placed on the coarsest sieve of the series and gently washed until the water extracted from last sieve No 200 become clear indicating the absence of fine particles detached form the aggregates retained in the upper sieves. No energy (e.g. vibration, rotation) was applied to the soil during the test. The distribution curve corresponding to the destructured specimen was obtained in a similar fashion but the aggregates in this case were broken manually. Figure 2 displays the results. The difference between the distribution curves indicates the presence of a real slightly cemented structure that remains even in saturated conditions.

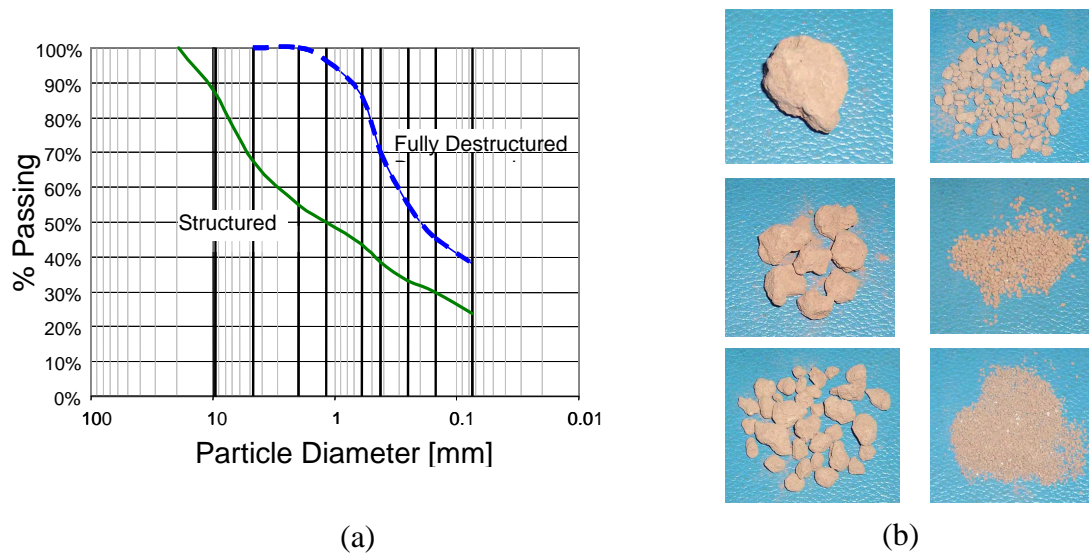(a)   (b)

Figure 2.- a) Sieve analysis performed on structured and fully destructured loess specimens, b) cemented aggregates obtained from sieving analysis of a loess sample without structure of the soil by means of any mechanical action (Capdevila and Rinaldi 2004).

RESULTS

Model parameters

The parameter $\lambda = 0.084$ was determined as slope of the virgin compression curve in the oedometer curve plotted in the double logarithm space $\ln(1+e)$ vs. $\ln(\sigma)$ showed on Figure 3. Because the oedometer test was performed without unloading-reloading the parameter κ was obtained from the results of the triaxial test and the visco-hypoplastic model. The slope of the reloading portion of the curve was determined as $\kappa = 0.006$ and being aproximately 10% of λ. Using the recommendation of Leinenkugel (1976) the viscosity index I_v can be estimated from the liquid limit (that for loess is $w_L = 32\%$) as:

$$I_v = 0.05 + 0.026\ln\!\left(w_L\right) \tag{9}$$

Figure 4 describe the triaxial result for the sample tested at the confining pressure of 80 kPa. The critical condition was extrapolated to a larger deformation since the result does not show a critical condition even at strains larger than 10 %. The critical friction angle for loess at this confining pressure yield a value of $\varphi_c = 29.8°$, which was obtained from the triaxial tests results and the visco-hypoplastic model. This value can be considered very high for loess, but as observed in the same Figure 4, the presence of cementation in the sample generate a structure composed of coarser-cemented equivalent particles that remain with little changes during the test and even after large strains. At small strains the stress-strain curve breaks due to a sudden collapse of the structure, however as vertical strain increases, the breakdown of the structure still continues (Rinaldi et al., 2006). Figure 5 shows the same result of Figure 4 in a different scale to enhance the yielding or collapse of the structure.
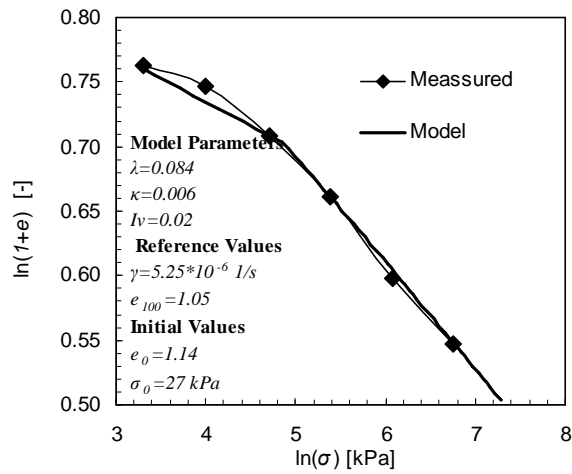
Figure 3.- Oedometer curve for a saturated sample of loess. Agreement within the visco-hypoplastic model and the measured result.
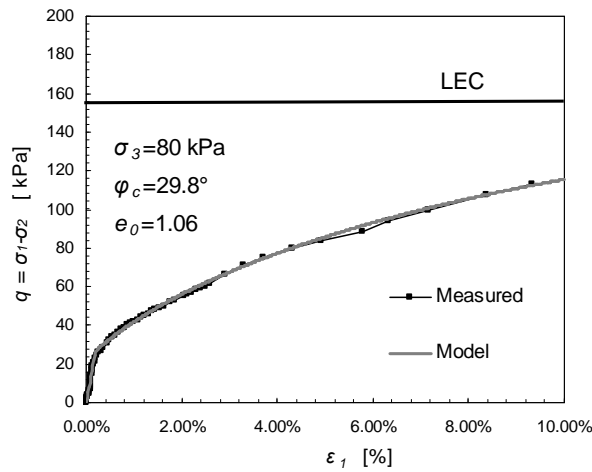

Figure 4.- Triaxial test result for the saturated sample of loess tested at the confining pressure of 80 kPa. Agreement of the visco-hypoplastic model and the measured result.
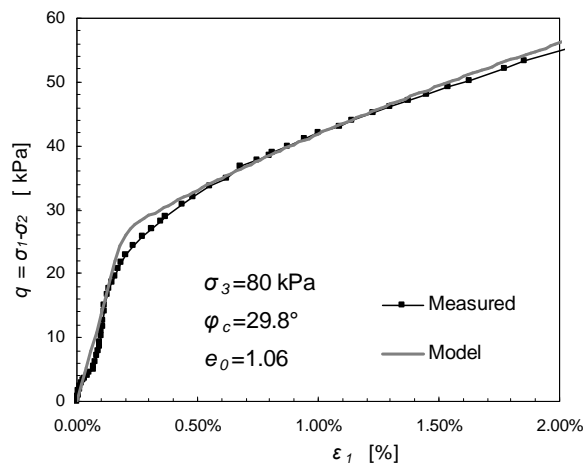

Figure 5.- Triaxial test result for the saturated sample of Figure 4. Agreement of the visco-hypoplastic model and the measured result.

Figure 6 describes de variation in the shape of the yielding surface with respect to the parameter β_R. For the case of loess a value of $\beta_R = 0.8$ was adopted assuming that yielding occurs at low strains.
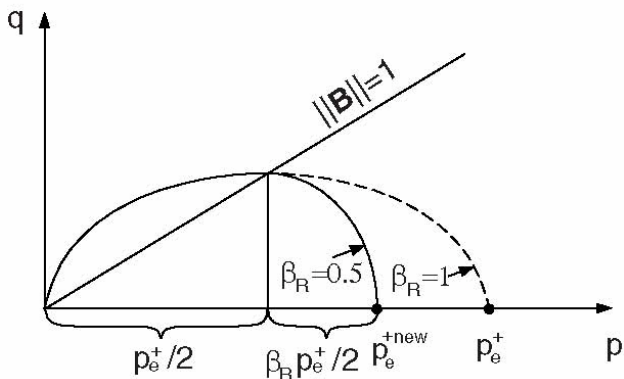

Figure 6.- Variation of yielding ellipsoidal surface with the prameter β_R.

Reference parameters

The void ratio $e_{100} = 1.05$ is determined for the loading pressure of 100 kPa (see Figure7) assuming a constant loading velocity $D_r = 5.25 \ 10^{-5}$ 1/s.
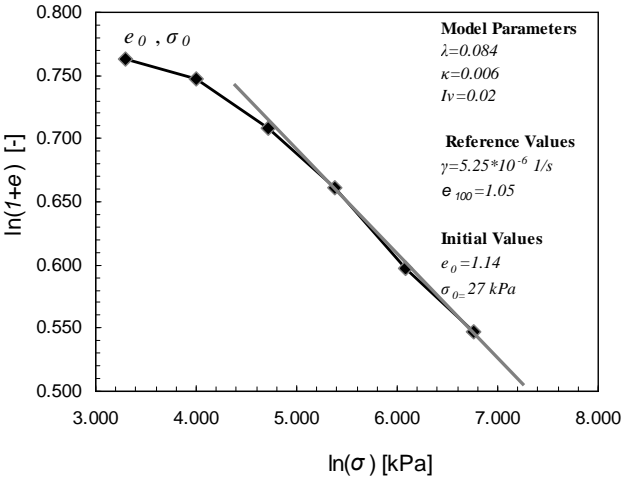

Figure 7.- Model parameters and reference values from oedometer curve for a saturated sample of loess. Agreement of the visco-hypoplastic model and the measured result.

The fitting of the visco-hypoplastic model to the measurements is displayed on the same Figures 3 to 5. From these Figures we can see that the model is able to reproduce accurately the behavior of Pampean loess under oedometric and triaxial compression loading. Notice that the model describes also the yielding of soil structure in both oedometer and triaxial compression tests.

In Figure 5 the high stiffness of loess at low strains is evident as can be expected from cemented soil. The selection of a low value for the parameter κ contributes with the simulation of this high stiffness portion. The yielding point is determined by the initial void ratio and the viscous deformation rate traduced in an initial equivalent pressure. The parameter β_R contributes also to the extension of the stiffens portion of the curve. After yielding the soil stiffness decreases. The stress-strain curve after beyond the yielding point is well simulated by a critical friction angle of 29.8° (Figure 4). The simulated and experimental curves show very good agreement.

CONCLUSIONS

From this work, the following conclusions can be derived:
- The stress-strain behavior of loess is markedly influence by an open structure and the presence of weak cementing agents bonding the structure. The collapse of the structure is observed in both oedometric and triaxial loading
- The visco-hypoplastic model has been extensively used to model fine grained soils and can be described in terms of soil parameters easily determined in laboratory tests of common practice in geotechnical engineering. Additionally, the model is able to describe phenomena observed in fine grained soils as creep, relaxation and viscose effects
- The visco-hypoplastic model is able to reproduce accurately the stress-srain behavior of Panpean loess in saturated condition under oedometric and triaxial compression loading
- Further results are required to evaluate this model with higher accuracy.
- It would be very interesting to extend the application of the visco-hypoplastic model to describe the behavior of loess in unsaturated conditions.

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