Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXVI, pp.2384-2394
Sergio A. Elaskar, Elvio A. Pilotta, Germán A. Torres (Eds.)
Córdoba, Argentina, Octubre 2007

ON THE VARIATION OF DYNAMIC PROPERTIES OF A FULL-SCALE 3-STORY REINFORCED CONCRETE FLAT-PLATE BUILDING

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Keywords: dynamic properties, damage detection, forced vibration, free vibration.

Abstract. A full-scale three-story reinforced concrete flat-plate building is to be subjected to a series of quasi-static load cycles. At various stages of the quasi-static test corresponding to different damage levels, dynamic tests in the forms of free and forced vibrations, ambient vibrations and modal impact-hammer tests will be performed to collect low-amplitude dynamic response data from the building. These data will be used to estimate linear dynamic characteristic parameters of the building. This paper presents some preliminary results from the structure in its initial, undamaged condition. In particular, data on natural frequencies, modal shapes, shear wave velocity, story stiffnesses and damping ratios are provided and discussed. It is also shown that at the very low displacement levels, the undamaged structure exhibits non-linear elastic behavior. In the future when the testing part is completed, the experimental data will be used to try to establish a relationship between damage sustained by the building and change in its dynamic properties. Dynamic parameter(s) that correlate best with damage will be investigated as well.

INTRODUCTION

Not all properties of a structure that affect its dynamic characteristics provide useful information about the state of the structure in terms of damage. Typically, uncertainties in parameterization and numerical modeling of the structure, such as those in mechanical properties of the materials, degree of fixity of the members, mass distribution in the structure, and soil-structure interaction, among others, make it impossible to infer about the health of a given structure by using a single observation of its dynamic properties. Instead, damage may be estimated by observing variations in those properties. To be able to make such observations, it is desirable to have good quality data (i.e. high signal to noise ratio), and accurate measurement methods and sensors.

Often, parameters used in estimating/detecting damage in a structure (Doebling et. al. 1996) relate directly or indirectly to the fundamental dynamic characterization of the structure, i.e. its natural frequencies, mode shapes, or damping properties. These fundamental model parameters are associated with the physical properties of stiffness, mass and damping of the structure. However, difficulties abound even in calculation of these fundamental parameters. For example, from a practical point of view, computation of natural frequency presents difficulties regarding to the frequency resolution of the data which is related to the duration of the record ($\Delta_f = 1/t_d$); thus, ability to observe sensitivity of natural frequency to damage is controlled by the efficiency of the measurement methods. Moreover, changes observed in natural frequencies may be due to loss of non-structural elements and/or due to soil-structure interaction (Todorovska et. al. 2004), which technically do not represent structural damage even though they may, ultimately, compromise the use of the structure. Therefore, experimental observation of structural damage in a structure and recording of changes in its dynamic properties, provide critical data for calibration of damage-detection techniques.

This paper discusses the dynamic tests to be performed on a full-scale 3-story flat-plate building shown in Figure 1. The building will be subjected to quasi-static loading cycles to investigate its load-displacement relationship (Fick 2007). Dynamic tests in the forms of ambient vibration, free and forced vibration, and impact-hammer modal tests, are to be carried out at different stages of damage in the structure. Comparison of the dynamic response will be used to try to establish relationships between the damage sustained by the building and changes in its dynamic properties. Identification of parameter(s) more sensitive to damage in this type of structure is to be investigated as well. Comparison of results from different types of tests is expected to lead to more accurate estimates. Absence of non-structural elements in the building and the presence of support conditions simulating a rigid foundation should allow relating changes in dynamic properties directly to changes in the state of the structural system itself.

Quasi-static and dynamic tests on the building are to be performed along one of the principal directions of the building and will engage two rows of columns. Dimensions of the building are given in Figure 2. Average mechanical properties of the concrete and the steel reinforcement were measured as $f_c' = 4000\,psi$ (28 days) and $f_y = 67\,ksi$, respectively. The building was built in the Robert L. and Terry L. Bowen Laboratory for Large Scale Civil Engineering Research at Purdue University.

Following sections illustrate preliminary results regarding the dynamic properties of the structure in its current, undamaged condition. Three different types of tests are presented: Free vibration test, Forced vibration test and Modal Impact-hammer test. Although a given dynamic property can be obtained using data collected through more than one type of test, its computation is shown only once for illustration purposes and sake of brevity. Finally, an observation about the non-linear elastic behavior of the structure on the undamaged condition is presented. A comparison of the non-linear elastic behavior observed by different types of tests is given.

Figure 1: Overall view of the full-scale 3-story flat-plate building test specimen.

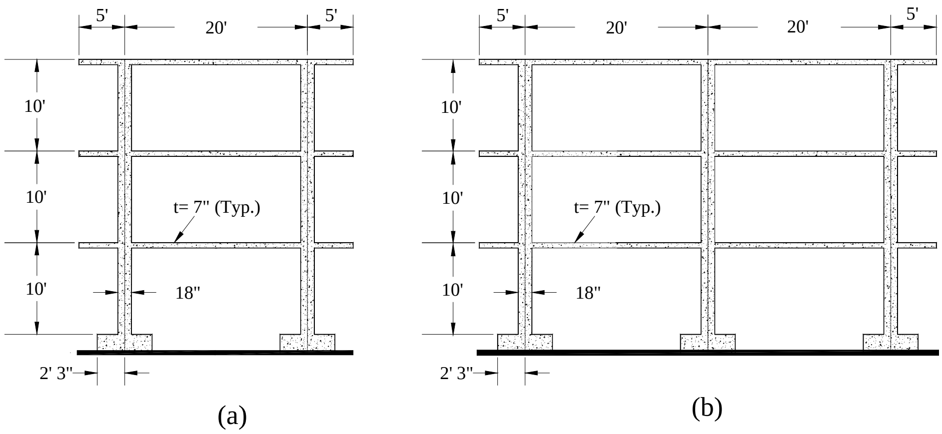

Figure 2: Elevation of the building: (a) north elevation; (b) east elevation.

1 FREE VIBRATION TEST

In order to perform the free vibration test, the building is pulled and released at a given floor level by using a mechanism attached to the strong wall. In each floor level one LVDT, mounted on a steel frame supported by the strong wall, is placed at the edge of the slab and one accelerometer is placed at the center of the slab. Additionally, eight accelerometers are spaced uniformly over the full height of one of the center columns. All sensors are aligned along the direction in which the building will be loaded during the quasi-static tests. Computation of natural frequencies, damping ratios and shear-wave velocity from this test is described in the following sections.

1.1 Natural Frequencies and Damping Ratios

Natural frequency is computed using the Fourier Spectrum of the response at a given location. Damping ratio is computed using the logarithmic decrement method, i.e. damping ratio is approximated as $\xi = \ln\left(u_j / u_{j+n}\right) / 2\pi n$ where u_j and u_{j+n} are the peak displacement amplitudes in cycles *j* and *j+n*, respectively. Figure 3 shows (a) the free vibration displacement response of the building at its third story recorded by the corresponding LVDT when the building is pulled at the third story, and (b) the Fourier Spectrum of the response.

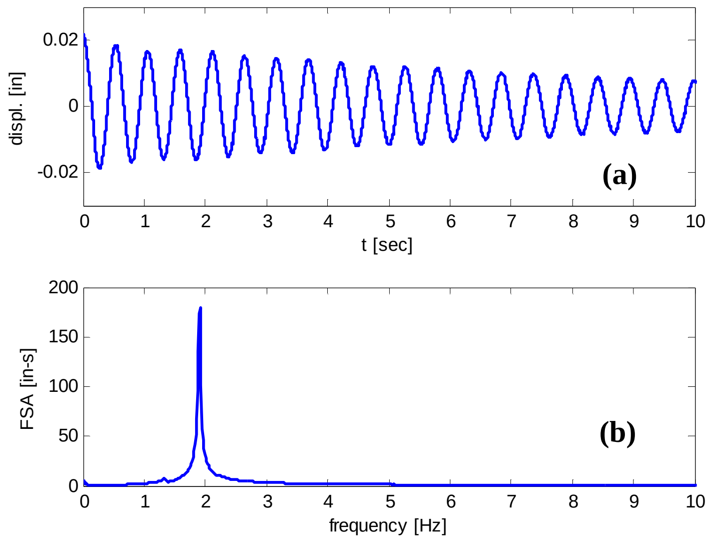

Figure 3: Free vibration response at the third floor: (a) displacement; (b) Fourier Spectrum.

1.2 Shear-Wave Velocity

The shear-wave velocity can be obtained by tracking the propagation of a distortion, initially input to the building at the floor level where it is pulled, over the building. Figure 4 shows (a) the entire displacement records at each floor for a distortion input to the building at the 3rd floor (i.e. at the top of the structure), and (b) the initial parts of the responses showing the time lag in the first arrival of distortion at each floor level. The good signal-to-noise ratio of the LVDT record permits measurement of the shear-wave velocity directly from such a plot without any signal processing.

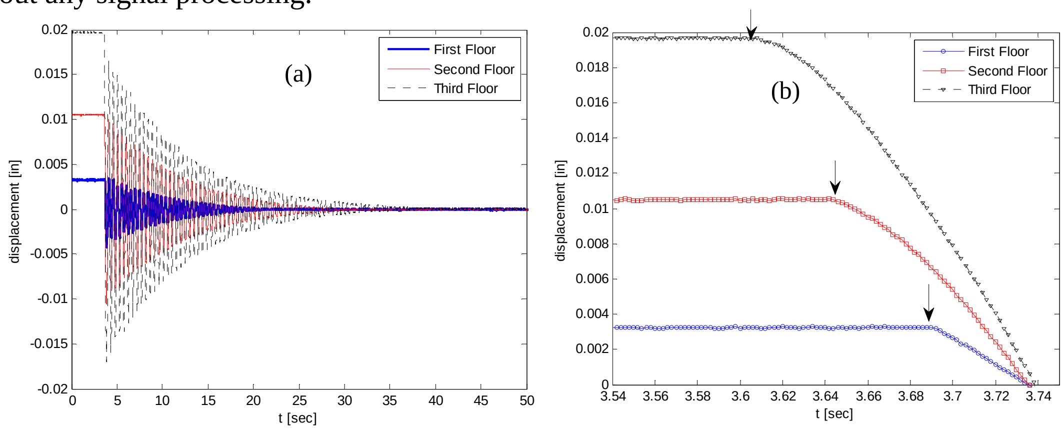

Figure 4: Free vibration response: (a) entire records; (b) initial parts of the records showing the arrival of distortion at different floor levels.

2 FORCED VIBRATION TEST

A linear actuator placed at the top floor applies a sinusoidal force to a cart that moves freely back and forth. The inertial force of the entire system is transmitted to the building by friction between the actuator and the slab. The steady-state response comparison under different excitation frequencies permits estimating the natural frequencies and corresponding modal damping ratios of the building. For this test, in addition to the accelerometers installed on the structure, an accelerometer is attached to the cart. The following section describes how story stiffnesses are computed using the forced vibration test.

2.1 Story Stiffnesses

Equation (1) gives the equation of motion written for the third story when one uses the simple discrete model shown in Figure 5 (a). Similar expressions can be written for other stories.

$$m_3 \ddot{x}_3 + \underbrace{c_3 (\dot{x}_3 - \dot{x}_2) + k_3 (x_3 - x_2)}_{\text{Re}\,sisting\,force} = P_o \sin \Omega t \quad \textbf{(1)}$$

Average slope of the *hysteresis loops*, relating the resisting force to the interstory-displacement, represent the story stiffness estimates. Figure 5 (b) shows the hysteresis loops obtained using data from the forced vibration test done on the undamaged building. Note that the hysteresis loops are very tight as there is very little energy dissipation (i.e. damping) in the building.

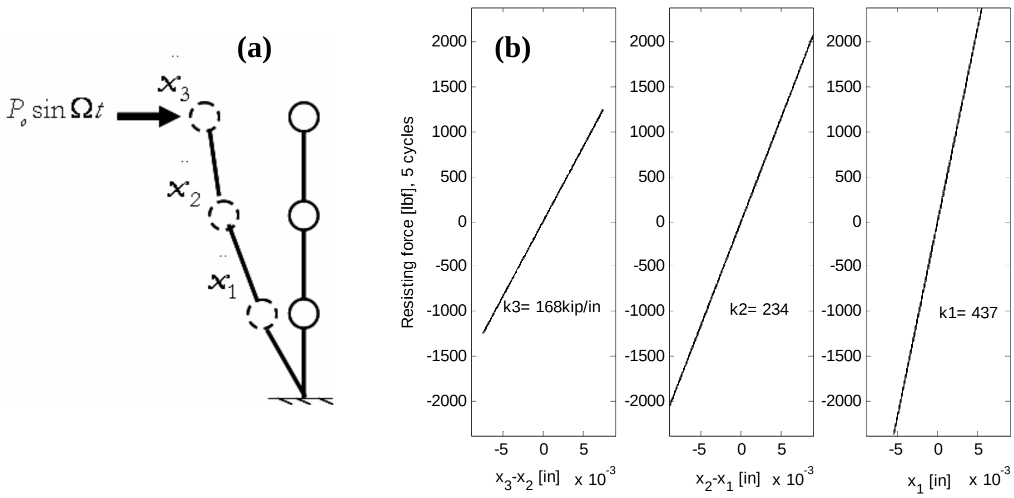

Figure 5: Story stiffness: (a) simple discrete model (b) hysteresis loops.

3 MODAL IMPACT-HAMMER TEST

Modal impact-hammer testing in the form of multiple input-multiple output (MIMO) has been performed on the building. A sledge hammer is used to impact the edge of each floor-slab and output accelerations are recorded at the center of each slab. Natural frequencies and modal shapes of the building are computed using frequency response functions (FRF).

3.1 Mode Shapes

The analytical expression for FRF between the output-acceleration $\ddot{X}$ and the input-force F for a SDOF is given by

$$\mathrm{FRF}(\Omega)=\frac{\ddot{X}(\Omega)}{F(\Omega)}=\frac{-\Omega^2}{-m\Omega^2+ci\Omega+k} \tag{2}$$

Where *m*, *c*, *k*, and Ω are the mass, damping coefficient, stiffness and circular frequency, respectively; and $i=\sqrt{-1}$. Right at a natural frequency where the real part of FRF becomes zero, comparison of FRFs obtained using output-acceleration at different degrees of freedom give a representation of the corresponding mode shape (Adams 2007). Figure 6 shows the imaginary part of the FRFs when the impact load is applied at the third (i.e. top) floor and Table 1 shows the computed mode shapes.

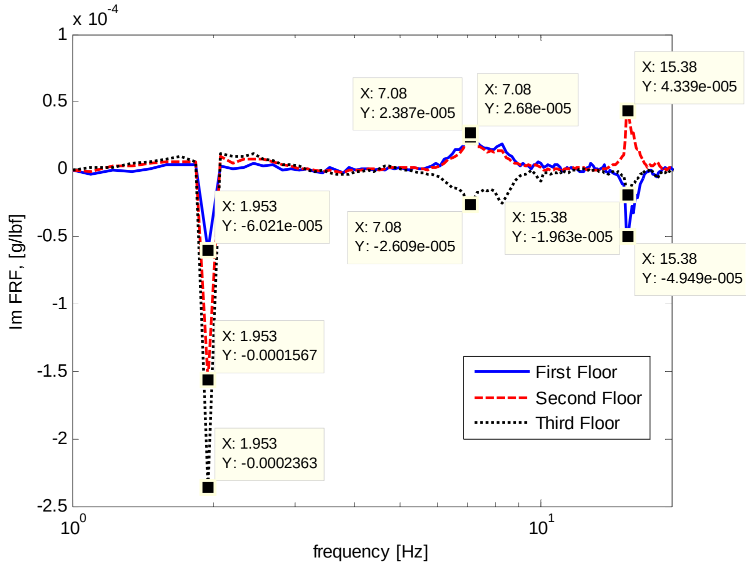

Figure 6: Imaginary part of FRF for impact load applied at the top floor.

Table 1: Mode shapes from modal impact-hammer test.

	First Mode		Second Mode		Third Mode	
Story	**Absolute**	**Normalized**	**Absolute**	**Normalized**	**Absolute**	**Normalized**
1	-6.021E-05	**0.25**	2.680E-05	**1.00**	-4.949E-05	**1.00**
2	-1.567E-04	**0.66**	2.387E-05	**0.89**	4.339E-05	**-0.88**
3	-2.363E-04	**1.00**	-2.609E-05	**-0.97**	-1.963E-05	**0.40**

4 NON-LINEAR ELASTIC RESPONSE OF THE UNDAMAGED BUILDING

During the tests explained in sections 1 and 2 it was observed that, in its undamaged state, the building may respond in a non-linear elastic fashion. The Cohen's class time-frequency representation of building response permits to identify variation of the vibration frequency of the building as a function of time. In its most general form, a time-frequency distribution (TFD) in Cohen's class can be represented as (Bradford 2006)

$$TFD_c(t,\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x(u+\tau/2)x^*(u-\tau/2)\Gamma(\theta,\tau)e^{-i(\theta t+\omega t-\theta u)}du\,d\tau\,d\theta \tag{3}$$

where t and ω denote time and frequency respectively; x is the analytical signal of the original time-series $s(t)$ and is defined as $x(t) = s(t) + i\tilde{s}(t)$, with $\tilde{s}(t)$ being the Hilbert transform of $s(t)$ (Bendat et. al. 1986); x^* represents the conjugate of the analytical signal; and, Γ is the smoothing kernel function. For the smoothing kernel function, binomial function (O'Neill et. al. 1999) was chosen. The current TFD (Cohen's class) and the mentioned smoothing function were selected to detect even very small changes in the natural frequencies of the building. (Spectrogram approach cannot provide the required frequency resolution and Wigner-Ville distribution approach disturbs the information due to the interference terms (Huang et. al. 1998)). Figure 7(b) shows the Cohen's class time-frequency representation of the building displacement response, Figure 7(a), as recorded in the third floor during the free vibration test. It can be seen in Figure 7(b) that as the vibration decays the vibration frequency increases. Similar observations can be made in forced vibration tests.

Figure 8 shows the FRF of the building at the third floor. Comparison of analytical (linear) model FRF and results from two different tests is given. *Test 1* represents the *forced vibration test* performed with a constant stroke of actuator, which results in non-constant force and non-constant displacement response at each excitation frequency. *Test 2* is the *forced vibration test* performed by setting the stroke of actuator to get a constant displacement-response of about 0.010in at the third floor of the building regardless the excitation frequency. Thus, while *test1* provides a non-uniform displacement response setting, *test 2* is intended to result in a uniform displacement response in the building. It is seen that *test 2* matches the analytical (linear) curve better than *test 1*. This comparison suggests presence of stiffness change as a function of displacement-amplitude of the building even at very low displacement levels: during *test 1*, the structure is responding with slightly different stiffness (or different natural frequency) depending on the amplitude of each vibration frequency-step. However, these changes are recoverable and as such, the response could be considered as non-linear elastic. In fact, a TFD similar to the one in Figure 7 was observed in other *free vibration tests* performed at similar initial amplitude displacement.

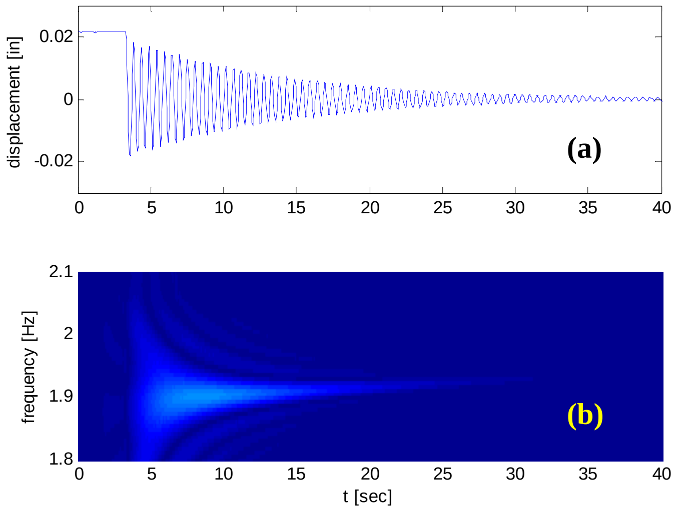

Figure 7: Free vibration response of the test building: (a) time domain; (b) cohen's class TFD with binomial kernel function.

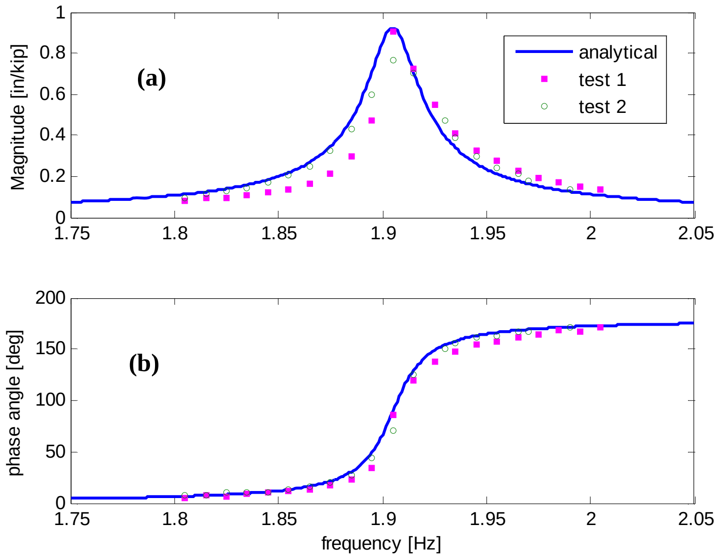

Figure 8: FRF from forced vibration test: (a) magnitude; (b) phase angle.

Computation of instantaneous frequency via Hilbert Transform (Yang et. al 2004) is also used to investigate the variation of natural frequency of the building as a function of displacement amplitude. In this approach, the analytical signal $x(t)$ is expressed as a function of instantaneous amplitude *A(t)* and phase angle θ*(t)*, as $x(t) = s(t) + i\tilde{s}(t) = A(t)e^{i\theta(t)}$. Instantaneous frequency, obtained as time derivative of the phase angle, permits estimation of the vibration frequency at each time step. Figure 9 (a) shows the third floor displacement response during a constant-frequency *forced vibration test* and during its free vibration phase after the forcing is stopped. Figure 9 (b) shows the corresponding instantaneous frequency. It is seen that when the actuator is stopped (at about 200 seconds into the record shown in Figure 9) the building moves into free vibration response very rapidly and the instantaneous frequency becomes the natural frequency of the building.

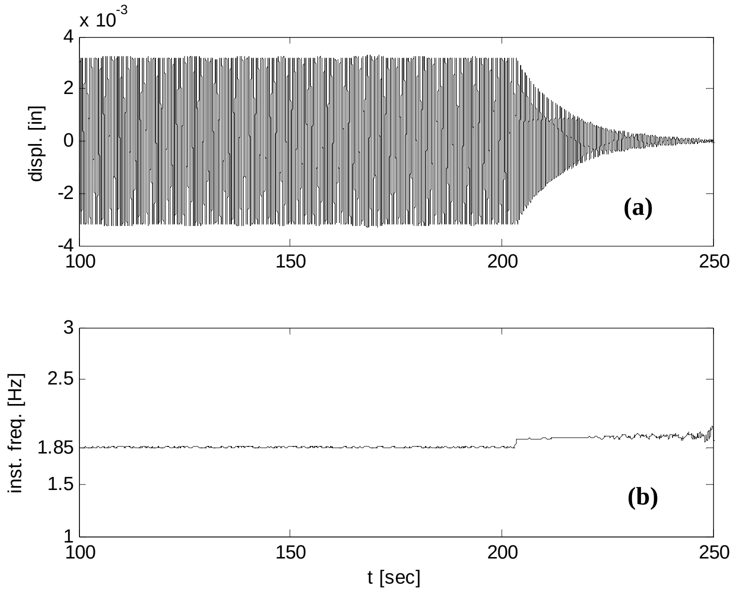

Figure 9: Forced vibration response of building at excitation frequency $f = 1.85Hz$: (a) displacement recorded at third floor; (b) instantaneous frequency.

Frequency shifts observed during two forced vibration-free vibration tests is shown in Figure 10. In these tests, the forcing frequency was identical but the forcing amplitudes were different. These two tests are taken from the constant-stroke and constant-response amplitude tests explained earlier and, hence, named *test 1* and *test 2*. Steady-state top floor response amplitudes observed during the forced vibration phases of *test 1* and *test 2* were 0.003in and 0.010in, respectively. It is seen that instantaneous frequency after the actuator is stopped is different for each test. At the end of *t*est *1,* i.e. during the free vibration phase, the building exhibits a natural frequency higher than that observed during the free vibration phase in *test 2*. Once again, the change in stiffness as a function of the vibration-amplitude is evidenced.

It should be noted that data from a series of tests in the form of constant-stroke (*test1*) and constant-response (*test2*) were used to identify the natural frequency of the building (Figure 8) and that both series resulted in identical natural frequency estimates (about 1.91Hz). The top story displacement response at the natural frequency in *test1* and *test2* were 0.02in and 0.01in, respectively. Compare this pair of peak displacements with the 0.003in and 0.01in pair at 1.85 Hz (i.e. the values from the tests used to illustrate the frequency shift in Figure 10). The mismatch in free vibration response frequencies suggests that the stiffness change is present for very small displacement amplitudes (below 0.01in at top floor). For larger amplitude displacements, the system behaves according to the natural frequency identified in Figure 8. This variation in natural frequency estimate at very small response levels may limit accuracy of ambient vibration based or low-level forcing-based structural health monitoring methods particularly at no-damage or low-level damage states.

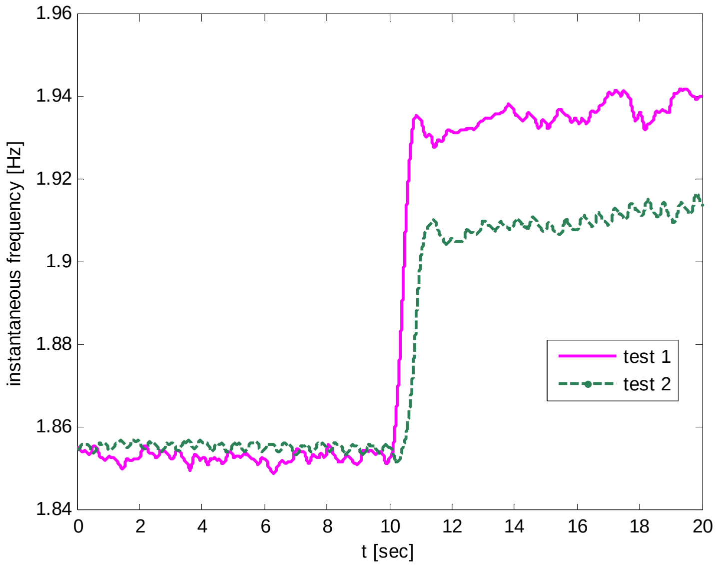

Figure 10: Frequency shift for *test1* and *test2* with an excitation frequency of $f = 1.85Hz$.

5 CONCLUSIONS

The dynamic tests performed on the undamaged full-scale 3-story reinforced concrete flat-plate building allowed estimation of base dynamic properties such as natural frequencies, damping ratios, shear wave velocity, story stiffnesses and mode shapes. Comparison of these quantities with those to be obtained from the structure at various damage states is expected to permit establishing, if any, relationships between sustained damage and changes in linear dynamic parameters. Investigation of non-linear behavior of the building in very low amplitude response conditions, as found in ambient vibration or low-amplitude forced vibration tests, may also provide useful information regarding the utility of field testing of flat-plate structures to detect absence or presence damage.

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