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IGNITION AND COMBUSTION OF METHANE FLOWING ALONG AN ARRAY OF THIN CATALYTIC PLATES

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Abstract. In this paper, the ignition and combustion of a methane/air mixture flowing along an infinite array of catalytic parallel plates has been studied by inclusion of the gas expansion effects and the finite heat conduction on the plates. The system of equations considers the full compressible Navier-Stokes equations coupled with the energy equations of the plates. The gas expansion effects which arises from temperature changes has been considered. The limits of large and small thermal conductivity of the plate material are analyzed and the critical conditions for ignition are obtained. The governing equations are solved numerically using finite differences.

1 INTRODUCTION

The catalytic ignition of fuel/air mixtures has received considerable attention in the literature in the past few years, due to its role in the start-up in the automobile catalytic converters and in the catalytic and catalytically assisted combustors [1-5]. Experimental and theoretical studies of catalytic ignition have been published increasing our knowledge about several aspects related with this type of critical nonlinear process. Veser and Schmidt (1996) published experimental results of the catalytic ignition of different hydrocarbons on platinum using an stagnation-point flow configuration. They showed that for methane, the critical ignition temperature decreases as the mixture becomes richer. From the theoretical point of view, the ignition process has been studied by either numerical simulations using elementary chemistry [6-8] or by large activation energy asymptotic analyzes using an one-step overall reaction mechanism [9-12]. The basic principles of heterogeneous catalysis has been described elsewhere Williams et al. (1992), Ertl (1982). Williams et al. (1992) presented a model for the catalytic combustion of hydrogen at high temperatures. They presented the rate parameters of a detailed surface chemistry. Warnatz et al. (1994) studied the catalytic combustion and ignition of hydrogen using detailed kinetic mechanisms for both surface and gas-phase reactions. Deutschmann et al. (1996) studied the catalytic ignition of different fuels on different catalyst materials. They indicated that the ignition process is an abrupt transition from kinetically controlled system to one controlled by mass transport and depends mainly on the adsorption - desorption reaction steps. In their numerical simulations they showed that one or the other reactant almost covered the surface prior to ignition. There is a need to obtain reduced kinetic schemes for the catalytic combustion, which help to build a bridge between the full numerical works and the theories developed using an overall one-step reaction for the surface kinetics. In these lines, Treviño (1999b) presented an asymptotic analysis for the catalytic ignition using a simplified model for the heterogeneous chemistry. The critical conditions for ignition has been deduced and obtained in a closed form the parametric influence on this critical process. He found that the most important reactions to predict ignition are the adsorption reactions for both reactants and the desorption reaction of the reactant which has a larger adsorption reaction rate. This analysis has been applied to methane and air mixtures on a platinum catalyst Treviño (1999a), with a closed form for the global reaction rate.

The objective of this work is to extend the previous analysis Treviño (1999a) to study the ignition and combustion of methane and air mixtures flowing along an infinite array of thin channels in an attempt to simulate the behavior in monolith combustors.

2 FORMULATION

The physical model under study is shown in Fig. 1. An uniform reactive mixture of methane and air gas flow, with velocity u_0^*, density ρ_0^*, temperature T_0^*, reactant concentrations Y_{i0} and pressure p_0^*, enters an infinite array of plates with a separation given by H^*. The thickness of the plates, h^*, is assumed to be very small compared with the separation H^*, $h^* << H^*$. The total length of the studied system is $L^* + L_i^* + L_f^*$, with an induction length L_i^* and a final length L_f^*. The array of plates begin at $x^* = 0$ and ends at $x^* = L^*$. The plates are heated internally by the catalytic reactions with a heat production rate per unit surface denoted by $\omega^*(x^*)$. A finite plate thermal conductivity, λ_w^*, enables heat to be transferred longitudinally thus changing in an important way the resulting temperature at the plate surface. For very low Mach number flows, the pressure variations, of order the dynamic pressure $\rho_0^* u_0^{*2}$, are very small compared with the pressure at the inlet conditions, p_0^*, thus reducing the equation of state of the gas to be written

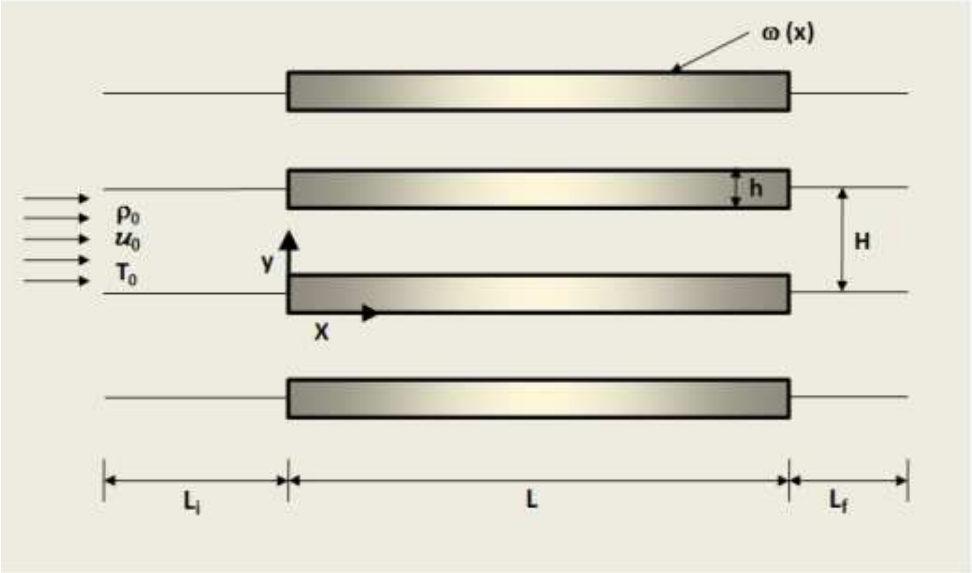

Figure 1: Schematics of the studied problem.

as $\rho^* T^* \simeq \rho_0^* T_0^*$. Scaling both Cartesian coordinates with H^* ($x = x^*/H^*$, $y = y^*/H^*$), all velocities with u_0^* ($u = u^*/u_0^*$, $v = v^*/u_0^*$), the temperature with that of the inlet condition T_0^* ($\varphi = T^*/T_0^*$), the density with ρ_0^* ($\rho = \rho^*/\rho_0^* = 1/\varphi$), the mass concentrations of reactants with those at the inlet, $Y = Y_F^*/Y_{F0}^*$, $Y_O = Y_O^*/Y_{O0}^*$, the viscosity μ^*, with that at the inlet ($\mu = \mu^*/\mu_0^*$), the thermal conductivity of the gas, k^*, with the corresponding value at the inlet conditions ($k = k^*/k_0^*$), the mass diffusivity for the fuel with those at the inlet conditions, $D = D_F^*/D_{F0}^*$ and the pressure with the dynamic pressure $\rho_0^* u_0^{*2}$ ($p = (p^* - p_0^*)/\rho_0^* u_0^{*2}$), the nondimensional equations for the gas flow are given by

$$(\rho u)_x + (\rho v)_y = 0 \tag{1}$$

$$uu_x + vu_y = -\varphi p_x + \frac{\varphi}{\mathrm{Re}} \left\{ \left[2\mu/3 \left(2u_x - v_y \right) \right]_x + \left[\mu \left(u_y + v_x \right) \right]_y \right\}, \tag{2}$$

$$uv_x + vv_y = -\varphi p_y + \frac{\varphi}{\mathrm{Re}} \left\{ \left[2\mu/3 \left(2v_y - u_x \right) \right]_y + \left[\mu \left(u_y + v_x \right) \right]_x \right\}, \tag{3}$$

$$u\varphi_x + v\varphi_y = \frac{\varphi}{\mathrm{Re}\,\mathrm{Pr}} \left[(k\varphi_x)_x + (k\varphi_y)_y \right], \tag{4}$$

$$uY_x + vY_y = \frac{\varphi}{\mathrm{Re}\,\mathrm{Pr}\,L_F} \left[(D_F/\varphi Y_x)_x + (D_F/\varphi Y_y)_y \right], \tag{5}$$

where the index denote derivatives, that is $\varphi_x = \partial\varphi/\partial x$. Here Re corresponds to the Reynolds number, $\mathrm{Re} = \rho_0^* u_0^* H^*/\mu_0^*$, L_F is the Lewis number of the reactant, $L_F = \rho_0^* D_{F0}^* C_{p0}^*/k_0^*$ and Pr is the Prandtl number, $\mathrm{Pr} = \mu_0^* C_{p0}^*/k_0^*$. Here, C_{p0}^* is the heat capacity measured at the inlet conditions. The associated boundary conditions are:

$$u = v = Y_y - \frac{L_F \varphi_w}{D\gamma}\omega = \varphi - \varphi_w = 0 \text{ at } y = 0, 1 \text{ and } 0 \le x \le L \tag{6}$$

$$u - 1 = v = \varphi - 1 \text{ at } x = -L_i \tag{7}$$

$$u_x = v = \varphi_x = 0 \text{ at } x = L + L_f \tag{8}$$

$$v = u_y = \varphi_y = Y_y = 0 \text{ at } y = 0, 1 \text{ and } 0 > x > L, \tag{9}$$

where ω is the nondimendional heat generation rate, $\omega = \omega^* H^*/(k_0^* T_0^*)$ and γ is the heat release parameter defined by $\gamma = Q^* Y_{F0}^*/C_{p0}^* W_F T_0^* = (T_e^* - T_0^*)/T_0^*$. Here Q^* is the heat release per unit mol of fuel consumed, T_e^* corresponds to the adiabatic flame temperature and φ_w is to the nondimensional temperature at the plate surface, to be found after solving the nondimensional energy equation of the plate given by

$$\lambda \varphi_{wxx} + k\varphi_y\big|_{y=0} + \omega(x) = 0. \tag{10}$$

In this equation, the thermally thin approximation has been employed for the plate, which means that the transverse variation of the temperature is assumed to be very small compared with overall temperature difference and is true if the aspect ratio of the plate h^*/L^* is very small compared with unity. λ is a nondimensional parameter which measures the ability of the plate to transfer heat in the longitudinal direction,

$$\lambda = \frac{k_w^*}{k_0^*}\frac{h^*}{H^*} \tag{11}$$

The associated boundary conditions to Eq. (10), for very thin plates are assumed to be given by adiabatic edges, $\varphi_x = 0$ at $x = 0$ and $x = L$. Introducing the stream function ψ, such as $u = \varphi\psi_y$ and $v = -\varphi\psi_x$, and the vorticity Ω, $\Omega = v_x - u_y$, the nondimensional governing equations (1) to (4) transform to

$$-\Omega = \varphi\left(\psi_{xx} + \psi_{yy}\right) + \varphi_x\psi_x + \varphi_y\psi_y \tag{12}$$

$$\psi_y\Omega_x - \psi_x\Omega_y - \varphi_x\left(\psi_y\psi_{yy} + \psi_x\psi_{xy}\right) + \varphi_y\left(\psi_y\psi_{xy} + \psi_x\psi_{xx}\right) =$$
$$\frac{1}{\mathrm{Re}}\left[\mu\left(\Omega_{xx} + \Omega_{yy}\right) + 2\mu_x\left(v_{xx} + v_{yy}\right) - 2\mu_y\left(u_{xx} + u_{yy}\right) + 2\mu_{xy}\left(v_y - u_x\right) + \right.$$
$$\left.\left(u_y + v_x\right)\left(\mu_{xx} - \mu_{yy}\right)\right], \tag{13}$$

$$\psi_y\varphi_x - \psi_x\varphi_y = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left[k\left(\varphi_{xx} + \varphi_{yy}\right) + k_x\varphi_x + k_y\varphi_y\right]. \tag{14}$$

$$\psi_y Y_x - \psi_x Y_y = \frac{1}{L_F\,\mathrm{Re}\,\mathrm{Pr}}\left[\frac{D}{\varphi}(Y_{xx} + Y_{yy}) + \frac{1}{\varphi}(D_x Y_x + D_y Y_y) - \frac{D}{\varphi^2}(Y_x\varphi_x + Y_y\varphi_y)\right]. \tag{15}$$

For not very rich mixtures of methane and air, the heterogeneous heating rate can be given by Treviño (1999a)

$$\omega^* = \frac{Q^*\Gamma r_{Fa} r_{Od}}{r_{Oa}}, \tag{16}$$

where Γ is the surface molar concentration in mol/cm^2, and corresponds to the surface site density ($\sim 10^{15}$ sites/cm^2) divided by the Avogadro number, $6.0221367 \times 10^{23}\,\mathrm{mol}^{-1}$, then the assumed value for $\Gamma \simeq 1.6603 \cdot 10^{-9}$ mol/cm^2. r_{Fa} and r_{Oa} are the adsorption rates for the fuel

Nr	Reaction	S	A	E
Fa	$CH_4 + 2Pt(s) \rightarrow CH_3(s) + H(s)$	0.01	–	–
Oa	$O_2 + 2Pt \rightarrow 2O(s)$	$.07(300/T)$	–	–
Od	$2O(s) \rightarrow O_2 + 2Pt(s)$	–	$3.7 \cdot 10^{21}$	190

Table 1: Heterogeneous reaction model. Units: **A** (mol,cm,s), **E** (KJ/mol)

and the molecular oxygen, respectively and r_{Od} represents the rate of desorption for molecular oxygen. All rates are in s^{-1} units. Therefore, the nondimensional heating rate takes the form

$$\omega = \frac{HQ^*\Gamma^2 A_{Oa}}{k_0^* T_0^*} \frac{S_F}{S_O} \left(\frac{W_O}{W_F}\right)^{3/2} \frac{Y_{F0}^*}{Y_{O0}^*} Y(x,0) \exp\left(-\frac{E_{Od}}{RT_0^* \varphi_w}\right). \tag{17}$$

Here S_F and S_O are the sticking probabilities for the fuel and molecular oxygen, respectively, W_F and W_O are the molecular weights of the fuel and molecular oxygen, respectively, A_{Od} and E_{Od} are the pre-exponential and the activation energy of the desorption reaction for molecular oxygen, respectively. R represents the universal gas constant. The kinetic parameter are shown in table 1.

3 ASYMPTOTIC LIMIT $\lambda \rightarrow \infty$

For large values of λ compared with unity, the temperature of the plate tends to be uniform and an asymptotic solution for $\lambda \rightarrow \infty$, can be obtained by assuming the following asymptotic expansion, with $1/\lambda$ as the small parameter of expansion,

$$\left\{\begin{array}{c} \varphi \\ \varphi_w \\ \psi \\ \Omega \\ Y \end{array}\right\} = \left\{\begin{array}{c} \varphi_0(x,y) \\ \varphi_{w0} \\ \psi_0(x,y) \\ \Omega_0(x,y) \\ Y_0(x,y) \end{array}\right\} + \sum_{j=1}^{\infty} \frac{1}{\lambda^j} \left\{\begin{array}{c} \varphi_j(x,y) \\ \varphi_{wj}(x) \\ \psi_j(x,y) \\ \Omega_j(x,y) \\ Y_j(x,y) \end{array}\right\}, \tag{18}$$

resulting the value of φ_{w0}, which is a uniform along the plate. The resulting nondimensional energy equations for the plates, given by Eq. (10), are

$$\varphi_{w1xx} = -\left.\varphi_{0y}\right|_{y=0} - \omega, \tag{19}$$

$$\varphi_{wjxx} = -\left.\varphi_{(j-1)y}\right|_{y=1}, \text{ for } j > 1, \tag{20}$$

to be solved with the adiabatic boundary conditions

$$\varphi_{wjx}(0) = \varphi_{wjx}(L) = 0, \text{ for all } j. \tag{21}$$

The adiabatic boundary condition at the leading order then requires that

$$\int_0^L \left[\left.\varphi_{0y}\right|_{y=0} + \omega\right] dx = 0. \tag{22}$$

To study the ignition for high values of parameter λ compared with unity, we retain only the leading order in λ. Due to the fact that in this limit the temperature of the plate is uniform, the resulting nondimensional heating rate can be written as

$$\omega = \Delta Y_0(x,0) \exp\left[\frac{E_{Od}}{RT_0^*} \frac{\varphi_{w0} - 1}{\varphi_{w0}}\right], \tag{23}$$

where Δ is the Damköhler number defined by

$$\Delta = \frac{HQ^*\Gamma^2 A_{Oa}}{k_0^* T_0^*} \frac{S_F}{S_O} \left(\frac{W_O}{W_F}\right)^{3/2} \frac{Y_{F0}^*}{Y_{O0}^*} \exp\left(-\frac{E_{Od}}{RT_0^*}\right). \tag{24}$$

Due to the large value of the activation energy of the desorption reaction for oxygen is large ($E_{od} = 190$ Kcal/mol), the departure of φ_{w0} from unity of the order RT_0^*/E_{Od} increases the reaction rate e times, and then a new scale of the nondimensional temperature is needed in order to study the ignition process

$$\begin{Bmatrix} \varphi(x,y) \\ \varphi_w(x) \\ \psi \\ \Omega \\ Y \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ \psi_0(x,y) \\ \Omega_0(x,y) \\ 1 \end{Bmatrix} + \frac{RT_0^*}{E_{Od}} \varphi_{w1} \begin{Bmatrix} h(x,y) \\ 1 \\ \psi_1(x,y) \\ \Omega_1(x,y) \\ g(x,y) \end{Bmatrix} + ..., \tag{25}$$

where $\psi_0(x,y)$ and $\Omega_0(x,y)$ are obtained after solving the following set of equations

$$-\Omega_0 = \psi_{0xx} + \psi_{0yy} \tag{26}$$

$$\psi_{0y}\Omega_{0x} - \psi_{0x}\Omega_{0y} = \frac{1}{\mathrm{Re}} \left[\Omega_{0xx} + \Omega_{0yy}\right]. \tag{27}$$

Once $\psi_0(x,y)$ and $\Omega_0(x,y)$ are known, the first order corrections can be obtained from a linearized version of Eqs. (12) to (15) given by

$$-\Omega_1 = \left(\psi_{1xx} + \psi_{1yy}\right) + h(\psi_{0xx} + \psi_{0yy}) + h_x\psi_{0x} + h_y\psi_{0y} \tag{28}$$

$$\psi_{0y}\Omega_{1x} + \psi_{1y}\Omega_{0x} - \psi_{0x}\Omega_{1y} - \psi_{1x}\Omega_{0y} - h_x\left(\psi_{0y}\psi_{0yy} + \psi_{0x}\psi_{0xy}\right)$$

$$+h_y\left(\psi_{0y}\psi_{0xy} + \psi_{0x}\psi_{0xx}\right) = \frac{1}{\mathrm{Re}}\left(\Omega_{1xx} + \Omega_{1yy}\right), \tag{29}$$

$$\psi_{0y}h_x - \psi_{0x}h_y = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(h_{xx} + h_{yy}\right). \tag{30}$$

$$\psi_{0y}g_x - \psi_{0x}g_y = \frac{1}{L_F\,\mathrm{Re}\,\mathrm{Pr}}\left(g_{xx} + g_{yy}\right). \tag{31}$$

These equations have to be solved with the boundary conditions

$$\psi_0 - y = \psi_1 = g_y - \frac{L_F\,\mathrm{Ze}}{\gamma}\Delta\exp\left(\varphi_{w1}\right) = h - 1 = 0 \text{ at } y = 0, 1 \text{ and } 0 \le x \le L \tag{32}$$

$$\psi_{0y} = \psi_{1y} = \Omega_0 = \Omega_1 = h_y = g_y = 0 \text{ at } y = 0, 1 \text{ and } 0 > x > L, \tag{33}$$

$$\psi_0 - y = \psi_1 = g = h = 0 \text{ at } x = -L_i. \tag{34}$$

Here Ze is the Zeldovich number defined by $\mathrm{Ze} = E_{Od}/RT_0^*$, which is in general very large compared with unity. The energy equation of the plate, Eq. (22) then can be written as

$$\frac{\Delta\,\mathrm{Ze}\exp\left(\varphi_{w1}\right)}{\varphi_{w1}} = I(L_j,\mathrm{Re},\mathrm{Pr}) = -\frac{1}{L}\int_0^L h_y|_{y=0}\,dx, \tag{35}$$

where L_j denotes the geometrical influence including L_i, L and L_f. Clearly, ignition ($d\Delta/d\varphi_{w1} = 0$) occurs as φ_{w1} reaches the value of unity and thus, the critical Damköhler number for ignition is

$$\Delta_I = \frac{I(L_j,\mathrm{Re},\mathrm{Pr},L_F)}{\mathrm{Ze}\exp(1)}. \tag{36}$$

Fig. 2 shows the value of integral I, which is directly related to the ignition Damköhler number, as a function of the Reynolds number for three different values of the channel length. Using the kinetic parameters given in Table 1, the ignition temperature can be obtained from the definition of the Damköhler number and is shown in Fig. 3.

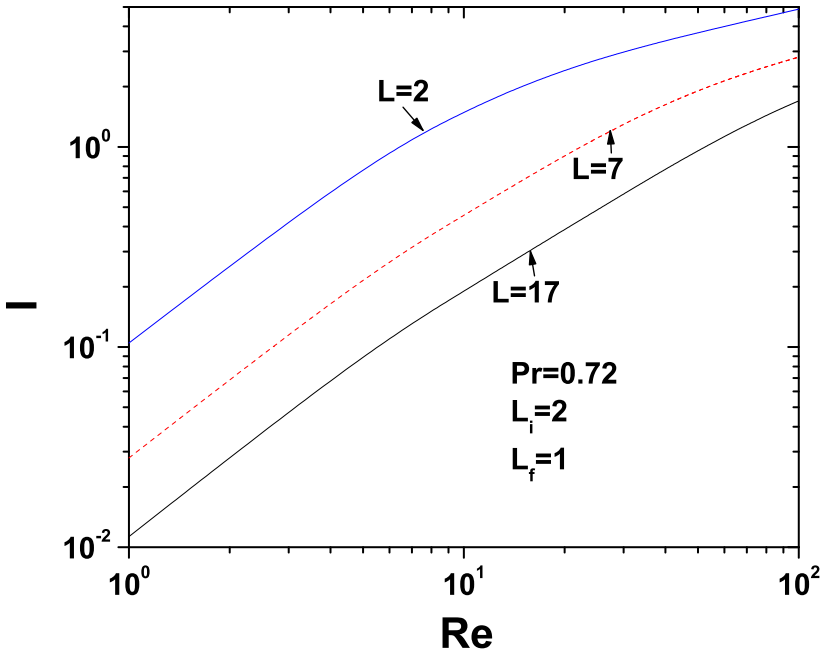

Figure 2: Integral I for three different values of the nondimensional length L, in the asymptotic limit $\lambda \to \infty$.

4 ASYMPTOTIC LIMIT $\lambda \to 0$

On the other hand, for small values of λ compared with unity, the heat conduction along the plate can be neglected. In this case the boundary condition at the plate surface is given by

$$\varphi_y\big|_{y=0} = -\frac{\gamma}{L_F\varphi_w}\,Y_y|_{y=0} = -\omega(x) = -\Delta Y(x,0)\exp\left[\frac{E_{Od}}{RT_0^*}\frac{\varphi_w-1}{\varphi_w}\right]. \tag{37}$$

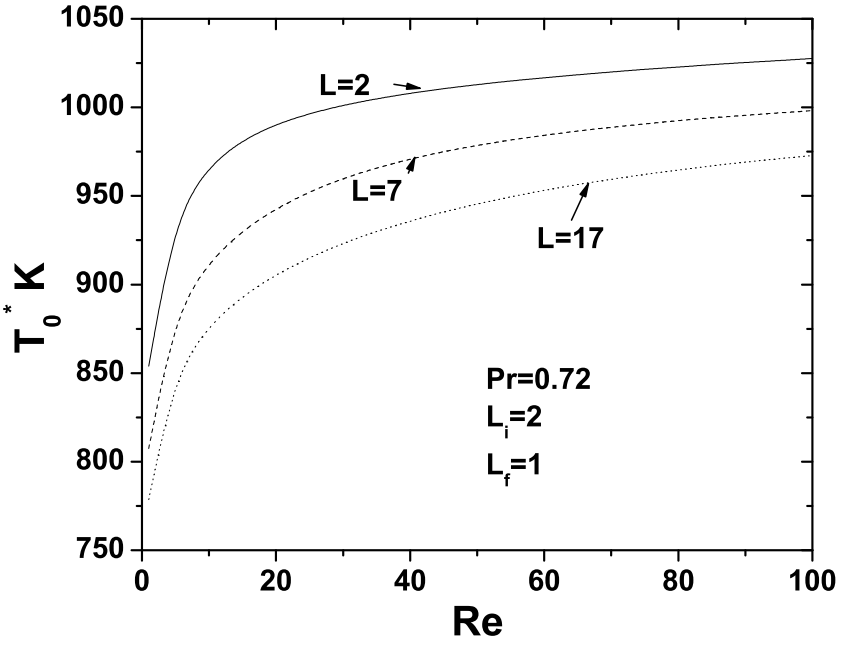

Figure 3: Ignition temperature for three different values of the nondimensional length L, in the asymptotic limit $\lambda \to \infty$.

In this limit the solution leading to ignition can be obtained by assuming the following expansions

$$\left\{ \begin{array}{c} \varphi(x,y) \\ \varphi_w(x) \\ \psi \\ \Omega \\ Y \end{array} \right\} = \left\{ \begin{array}{c} 1 \\ 1 \\ \psi_0(x,y) \\ \Omega_0(x,y) \\ 1 \end{array} \right\} + \frac{1}{\mathrm{Ze}} \left\{ \begin{array}{c} h(x,y) \\ h_w \\ \psi_1(x,y) \\ \Omega_1(x,y) \\ g(x,y) \end{array} \right\}, \tag{38}$$

where the solution to the leading oder equations are obtained after solving the same equations (26) to (27). h can be obtained after solving Eq. (30) with the boundary conditions at the plate given by $h_y|_{y=0} = -\Delta\,\mathrm{Ze}\exp(h_w)$. For fully developed flow, Eq. (30) can be integrated along the transverse coordinate to give

$$h_\zeta = h_{\zeta\zeta} + \overline{\Delta}\exp(h), \tag{39}$$

where $\zeta = \mathrm{Pe}\,x$, $\overline{\Delta} = 2\Delta\,\mathrm{Ze}\,/\,\mathrm{Pe}^2$. The number 2 arises because the integration is performed from 0 to $1/2$. For simplicity the subindex w has been removed. Eq. (39) has to be solved with the boundary conditions $h_\zeta = h_0$ at $\zeta = 0$ and $h_\zeta = 0$ at $\zeta = \zeta_f$. The boundary condition at $\zeta = 0$ comes from patching with the solution for $\zeta < 0$. For a given value of $\overline{\Delta}$ there are two possible values of h_0 resulting in a single value of $\zeta_f < \zeta_I(\overline{\Delta})$. There is not any solution for $\zeta_f > \zeta_I$, indicating this length as the ignition length. Fig. 4 shows the Damköhler number for ignition as a function of the length of the plate, after solving numerically Eq. (39).

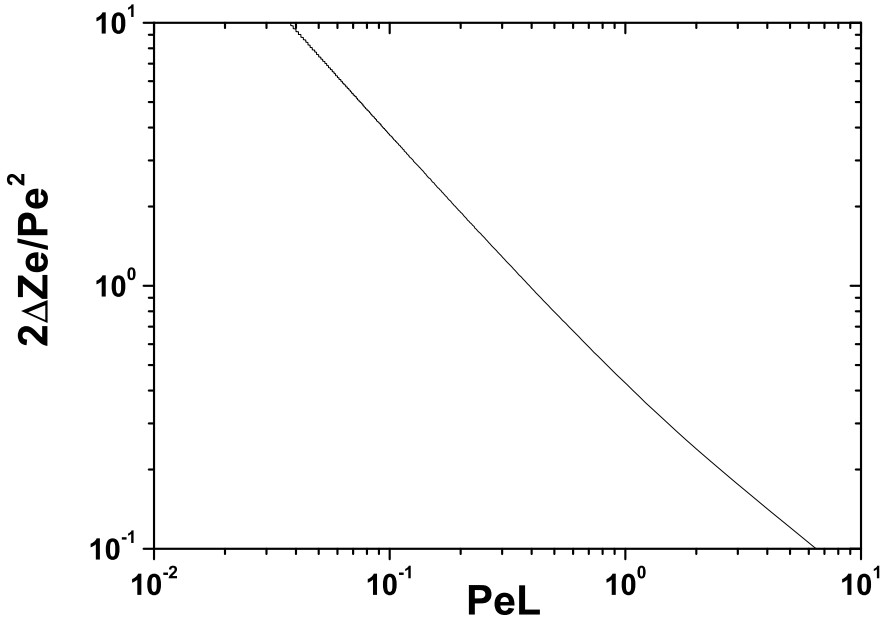

Figure 4: Damköhler number for ignition in the limit of $\lambda \to 0$.

5 NUMERICAL SOLUTION PROCEDURE

The governing equations with the associated boundary conditions are solved numerically using finite differences employing a pseudo-time as a tool to obtain convergence. The integration procedure is the following: An initial guess of the plate nondimensional temperature is employed and the stream function, vorticity and temperature of the gas flow are obtained by solving the discretized versions of Eqs. (12) to (14). A new nondimensional plate temperature profile is obtained after using the transient version of Eq. (10) with a nondimensional time step (time scaled with H^*/u_0^*) of 10^{-2}. This procedure is repeated until full convergence is achieved. For numerical stability, positive and negative values of the convective terms in the energy and the vorticity transport equations are discretized with upwind and rearward differencing, respectively, while the buoyancy and diffusion terms are discretized with a central difference formulation. All calculations were performed using air ($\mathrm{Pr} = .72$) as the gas, and wall vorticities were evaluated using the classical Thom's first-order formula Tom (1933)

$$\Omega(x,0) = \frac{2\left[\psi(x,\Delta y) - \psi(x,0)\right]}{\Delta y^2} \tag{40}$$

where Δy is the grid space normal to the wall. Higher order evaluations are not always known to lead to stable solutions Anderson et al. (1984). In order to get better accuracy at lower cost, a strongly non-uniform staggered grid system with a denser clustering near the heated plates was necessary. The computational domain was discretized using coordinate transformation functions to generate a variable grid size system that is closely spaced near the plates, using the boundary layer concepts Bender and Orszag (1978). For the transverse direction a coordinate transformation function $\eta(y)$ has been employed, to be obtained from the solution of the fol-

lowing differential equation

$$\varepsilon^2 \frac{d^2\varphi}{ds^2} = \varphi - m_1 s \tag{41}$$

with the boundary conditions $\varphi\,(-1/2) = -1/2$ and $\varphi\,(1/2) = 1/2$. This differential equation is known to have two boundary layers at $s = -1/2$ and $s = 1/2$. Here ε is the resulting boundary layer thickness and is assumed very small compared with unity and m_1 is the slope outside the boundary layer and is therefore positive and very small compared with unity. The solution to Eq. (41) is given by

$$\varphi = m_1 s + \frac{(1-m_1)}{2} \frac{\sinh(s/\varepsilon)}{\sinh(1/2\varepsilon)}. \tag{42}$$

Substitution of φ by $2\eta - 1$ and s by $2y - 1$, gives the desired function as

$$\eta(y) = \frac{1}{2} + m_1 \left(y - \frac{1}{2}\right) + \frac{(1-m_1)}{2} \frac{\sinh\left[k_1\,(y - 1/2)\right]}{\sinh\left[k_1/2\right]}, \tag{43}$$

where $k_1$must be large compared with unity. The packing parameters of the mesh m_1 and k_1 chosen were $m_1 = 0.4$ and $k_1 = 10$. For the solution of the system of equations, even grid spacing was used in x and η, generating the desired mesh in the original coordinates x and y. The following geometrical values were chosen: $L_i = 2$, $L_f = 1$ with $L = 17$. Different mesh sizes have been used depending on the Reynolds number. However a grid size of 221 x 84 was found to be excellent up to Reynolds of 100. For simplicity, in order to reduce the complexity of the problem, the thermal conductivity and the viscosity of the gas has been assumed to be constants, $\mu = k = 1$. The assumed value for the Prandtl number corresponds to that of air, $\mathrm{Pr} = 0.72$.

6 RESULTS

Numerical results were obtained for two different values of the longitudinal heat conduction parameter λ, one very large compared with unity $\lambda = 10^3$ and the other very small compared with unity, $\lambda = .05$ for two very different values of the Reynolds number, $\mathrm{Re} = 100$ and $\mathrm{Re} = 1$. Fig. 5 shows the final temperature distribution at the plate surface as a function of the streamwise coordinate for an initial temperature of $T_0^* = 850$ K, for a Reynolds number of $\mathrm{Re} = 1$ and the two different values of parameter λ. In both cases we got the ignition condition. The temperature distribution is almost flat for large values of λ and is rather flat at the downstream region for small values of λ. For very large value of the Reynolds number and a higher value of the initial temperature of $T_0^* = 1000$ K, the same is plotted in Fig. 6. For very small values of λ, there is an abrupt change in the temperature at a well distance from the channel entrance. From these two figures we can conclude that ignition occurs earlier for larger values of the heat conduction parameter λ.

7 CONCLUSIONS

In this paper, the ignition and combustion of a methane and air gas mixture flowing along an infinite array of parallel catalytic plates has been studied by inclusion of the gas expansion effects and the finite heat conduction on the plates. The system of equations consider the full Navier-Stokes equations coupled with the energy equations of the plates. It was assumed that the inlet Mach number is very small compared with unity, thus the pressure variations along the

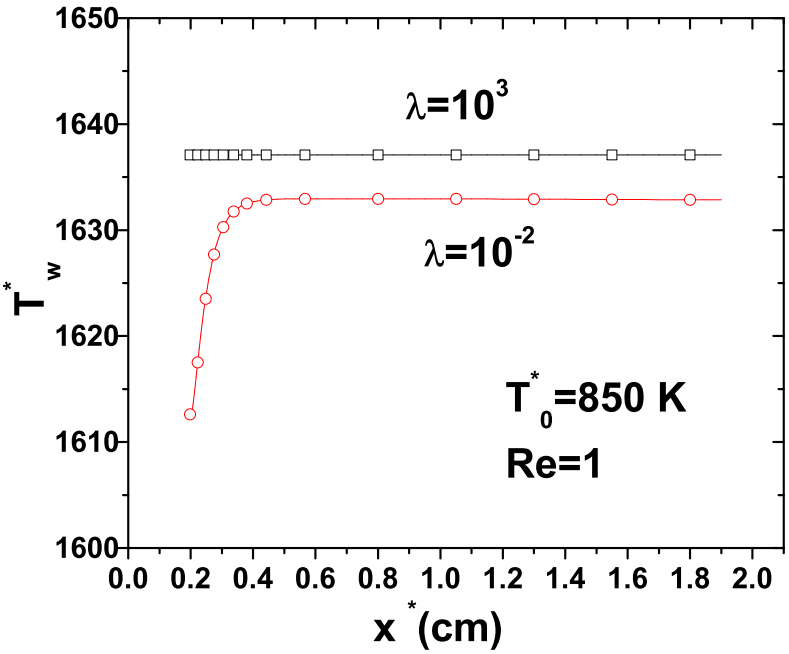

Figure 5: Final temperature distribution for an inlet temperature of $T_0^* = 850$ K for a Reynolds number flow of $\mathrm{Re} = 1$.

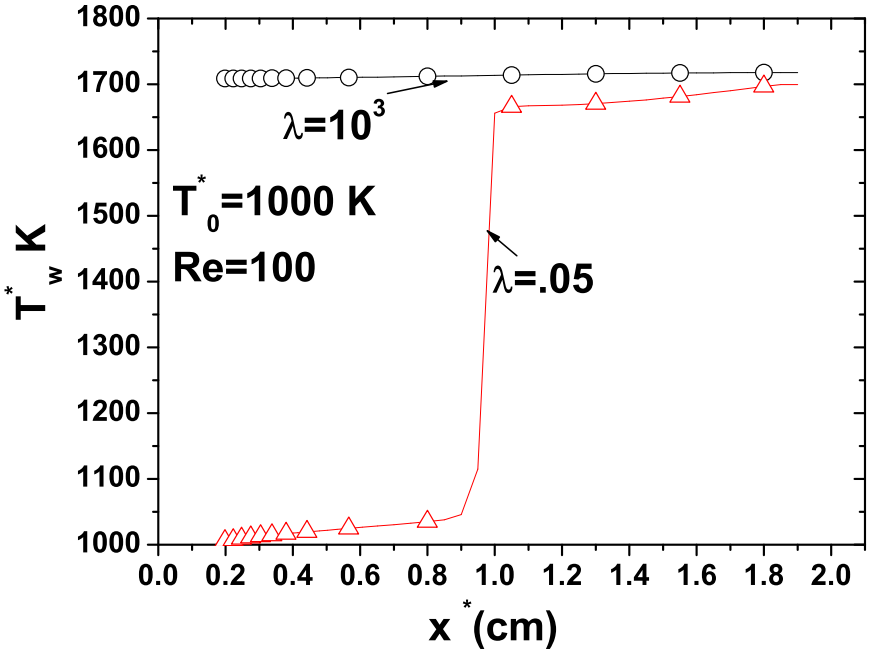

Figure 6: Final temperature distribution for an inlet temperature of $T_0^* = 1000$ K for a Reynolds number flow of $\mathrm{Re} = 100$.

system are very small compared with the pressure itself. Gas expansion then arises only from temperature changes. The system of equations are solved numerically using finite differences employing a pseudo-time as a tool to achieve convergence. The thickness of the plates are assumed to be very small compared with the plates separation. Due to the large number of parameters involved (L, L_i, L_f, Re, Pr, ω, λ), the computations were restricted to the case of $L = 17$, $L_i = 2$, $L_f = 1$, $\mathrm{Pr} = 0.72$, for a given value of the Reynolds number. For large values of parameter λ compared with unity, the longitudinal heat conduction along the plate produces an homogenization of the temperature and the resulting temperature gradients in the longitudinal direction are then of order $1/\lambda$. In both limits of very large and very small values compared with unity of parameter λ, the ignition conditions are obtained, in closed form, in the asymptotic limit of very large values of the Zeldovich numbers for the desorption reaction of atomic oxygen. Preliminar results indicate that ignition occurs earlier as parameter λ increases. However, more work is to be done in order strength this conclusion.

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