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ISSUES OF THE LOCAL RADIAL BASIS COLLOCATION METHOD IMPLEMENTATION FOR SOLVING SECOND ORDER PARTIAL DIFFERENTIAL EQUATION

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Abstract. In this work, it is presented the performance of the RBF collocation method to predict the unknown field variable in non-homogeneous and variable coefficient boundary value problems. The local RBF collocation approach differs of the classical global RBF collocation approach in the way that a radial basis function (RBF) interpolation function is defined. The former chooses to represent the meshless approximation by an expansion around a few supporting points (it constitutes a computational molecule). Any Lagragian or Hermitian RBF Hardy's interpolation can be used to construct the meshless locally supported shape functions which can reconstruct the field variable in each point into the molecule. In this way, several strategies have been proposed to possibly improve the imposing of the derivative boundary conditions in a strong-form approach. Three representative linear examples are solved by means different RBF collocation approaches and its results compared. It is found that all local RBF approaches performed very well. In addition, the RBF shape parameter affects the computed solution differently for each method.

1 INTRODUCTION

In many problems of engineering interest governed by partial differential equations (PDEs), variable coefficients and non-homogeneities are presents. For example, in treating of nonlinearities or the time derivative term, the transformed original equations can generate those problem types. Hence any intended numerical method should be able to approximate such more general problems.

In this paper, we focus on the use of the radial basis function (RBF) method to solve linear second order boundary value problems. We consider both the global RBF approach (known as symmetric RBF method), Fasshauer (2007); La Rocca et al. (2005); La Rocca and Power (2008) and the recently proposed local RBF approach seen as an overlapping multi-domain approach, Zuppa and Cardona (2003); Lee et al. (2003); Wright and Fornberg (2006). It be should point out that pioneered Zuppa and Cardona (2003)'s work uses an Optimal Point Interpolant instead classical Hardy's interpolant. The RBF method is a meshfree scheme which has become very popular for the solution of PDEs because it is a very accurate and powerful technique which was initiated in 1990 by Kansa (1990) using a global supported Hardy's interpolant. Here the RBF method combines a meshless RBF approximation with a point collocation discretization of the governing PDEs (known as a strong-form approach). The main drawback of the global RBF collocation method is that, as the number of nodes increases, the resulting system of equations becomes ill-conditioned and computationally expensive. Several procedures have been proposed to overcome the difficulties mentioned. A very promising one is the local approach proposed. Although the collocation technique give satisfactory results for a large class of problems, some instability issues are observed at the boundary specially when it involves singularities (Bernal et al., 2008) or derivative boundary conditions (Zuppa and Cardona, 2003; Kee et al., 2007).

Several techniques have been intended to improve the robustness of the RBF method. At a global RBF framework, Fedoseyev et al. (2002) proposed an improved Kansa–MQ method by using an additional set of collocation points beyond of the boundary (at the exterior) and corresponding additional set of equations obtained via collocation of the governing equation on the boundary, e.i., two equations are imposed, one equation from the boundary condition and one from PDE. Recently, following similar idea of Fedoseyev, La Rocca and Power (2008) suggested an improved symmetric RBF method by using a double boundary collocation approach but without additional fictitious points beyond of the boundary, e.i., not additional meshing work is needed.

At a local framework, loss of accuracy is observed when the construction of the meshless local RBF interpolation is involved without any special treatment specially when it is concerned a derivative boundary condition. An improvement was observed by Zuppa and Cardona (2003), when a double collocation approach is performed at the boundary points. To achieve they followed and compared two procedures, one based on troublesome fictitious point beyond of the boundary and other on additional degrees of freedom corresponding to normal derivative of the solution to the derivative boundary. In Liu et al. (2005), a similar local interpolation (using Hermite RBF interpolation) has been used for the construction of local RBF shape functions which include the known gradient information at the boundary. This Hermite interpolation can deal with one or more interpolation conditions per data point. In addition, they close the system of equation by using PDE collocation on the boundary so the accuracy and stability on the Neumann boundary were improved. Recently, Wright and Fornberg (2006) proposed a local Hermite-type interpolation to built local RBF shape functions but including the known linear

PDE information. This approach is referred as RBF-HFD. They observed a remark improvement of the order of accuracy when solving constant-coefficient Poisson-type problems along with Dirichlet boundary conditions. Unlike Liu et al. (2005)'s procedure, this approach does not generate additional degrees of freedom. In a previous work (Gutierrez and Florez, 2008), we can extend successfully the ideas from Wright and Fornberg (2006) to linear constant coefficient convective-dominated problems subject to derivative boundary conditions. Besides we compare local vs global RBF collocation approaches and it found that the former outperforms the second one.

In present work, an attempt has been made to extend the approach by Wright and Fornberg (2006) to non-homogeneous and variable coefficient boundary value problem subject to mixed boundary conditions. This approach will be referred as the local RBF PDE collocation approach (known as RBF-HFD approach in Wright and Fornberg (2006)). Besides we compare its performance with other RBF approaches in which are included the symmetric RBF collocation method (Fasshauer, 2007), the local RBF single collocation approach (known as the RBF-FD approach in Wright and Fornberg (2006)) (Lee et al., 2003; Shu et al., 2003) and the local RBF double boundary collocation approach (Liu et al., 2005). All of them based on the early Hardy's interpolation. An excellent review is provided by Fasshauer (2007) which concern with the simplest form of RBF interpolation and present alternatives to improve.

Like the governing differential equation, the derivative boundary condition can be also discretized by cited RBF approaches. However, Shu et al. (2003) also confirmed that the local RBF pure Hardy's collocation approach yields to a less accurate approximation when involves Neumann boundary condition then they followed a different approach where using locally orthogonal point layers adjacent to the boundary and combining the RBF method with one sided finite difference method (it is only used to implement the Neumann boundary condition). This approach belong with the local RBF double boundary collocation approach have been specially suited to treat the derivative boundary conditions. Maybe this is the first demonstration of the numerical features for the different RBF approaches for solving more general boundary value problems. We have chosen three representative problems in 2-D involving non-homogeneities, variable coefficient and mixed boundary conditions. The exact solutions are available and used to check computed results.

2 IMPLEMENTATION OF THE RADIAL BASIS FUNCTION METHOD

Solving PDEs by collocation of radial basis functions is already a common practice. We briefly introduce the procedures here. Consider the PDE belong with boundary conditions: for given $g(x)$, $f(x)$ find $u(x)$ (the field variable) from

$$\mathcal{L}u(x) = f(x) \quad \text{in } \Omega \tag{1}$$

$$\mathcal{B}u(x) = g(x) \quad \text{on } \partial\Omega \tag{2}$$

where Ω is a bounded domain with the boundary $\partial\Omega$, $\mathcal{L}$ is a linear variable coefficient second order partial differential operator and $\mathcal{B}$ is a linear boundary operator.

2.1 The symmetric RBF approach

Here the following global supported meshless RBF expansion for approximate the unknown function u is proposed (Fasshauer, 2007, pag. 348):

$$\hat{u}(x) = \sum_{i=1}^{NB} \lambda_i \mathcal{B}^{\boldsymbol{\xi}} \phi\left(\|x - \xi_i\|\right) + \sum_{i=NB+1}^{N} \lambda_i \mathcal{L}^{\boldsymbol{\xi}} \phi\left(\|x - \xi_i\|\right) \tag{3}$$

where the notation $\xi_1, \ldots, \xi_N$ is used for the centers of radial basis function ϕ. These centers will usually be selected to coincide with the collocation point set $\theta_h = \{x_i\}_{i=1}^N$. However the following equations are clearer if there is a formal distinction between centers ξ_j and collocation points x_i. Moreover NB represents the number of collocation points on the boundary of Ω, $N - NB$ the number of the internal collocation points, $\mathcal{L}^{\boldsymbol{\xi}}$ and $\mathcal{B}^{\boldsymbol{\xi}}$ are the differential operators of governing equation but acting on ϕ viewed as a function of the second argument (center). The coefficients $\lambda_i,\ i = 1, \ldots, N$ are going to be determined by solving a linear collocation system of equations.

After enforcing the conditions relating to governing equations (point collocation technique):

$$\mathcal{L}\hat{u}(x_i) = f(x_i), \quad x_i \in \Omega \setminus \partial\Omega \tag{4}$$

$$\mathcal{B}\hat{u}(x_i) = g(x_i), \quad x_i \in \partial\Omega \tag{5}$$

We end up with a symmetric collocation matrix of size $N \times N$ that is of the same type as the generalized Hermite interpolation matrices and therefore it is non-singular as long as ϕ is chosen appropriately. We would like to point out that the discretization of the boundary condition at boundary points precludes discretization of the PDE at corresponding points. Once the coefficients λ_i are found, the approximate solution $\hat{u}$ at any $x \in \Omega$ is given by the interpolation formula Eq. (3).

In Fasshauer (2007), it is stated that a symmetric RBF collocation implementation for the variable coefficient problem solution, while theoretically possible, is very cumbersome and is not clear how to deal with non-linear problems. Any more information about this approach can find in references (Fasshauer, 2007; La Rocca et al., 2005; La Rocca and Power, 2008).

2.2 Local RBF approaches

The local RBF approaches can be seen as the implementation of the classical overlapping domain decomposition from the RBF formulation in the limiting case of a very large number of overlapping sub-domains but free of iterations, e.i., no information transmission between sub-domains is required. Even, it could be argued that the local method is not truly meshless since for each node it is necessary to know its $n - 1$ neighboring points, which implies some *minor* type of connectivity (Bernal et al., 2008). In the following, we outline the steps used for the construction of the local meshless RBF shape functions (see Shu et al. (2003); Liu et al. (2005); Wright and Fornberg (2006); Gutierrez and Florez (2008) for more details). A locally supported shape function is required to produce a sparse discretized system of equations that can be solved efficiently which is a key feature for solving large systems.

We assume that, where is needed each node $x_j \in \theta_h$; we have selected in some way (Liszka et al., 1996; Wright and Fornberg, 2006) a subset $S_j \subset \theta_h$, called the computational molecule of x_j such that S_j is the set of surrounding points (centers) of x_j (the star point) which includes $n << N$ distinct grid-points of which $m \leq n$ could be used in a double collocation way. It assumes that $x_j \in S_j$, and it is the first element. At every molecule S_j, a locally approximate solution $\hat{u}_j$ is generate by a Hermitian RBF interpolation function augmented with a constant polinomial term:

$$\hat{u}_j(x) = \sum_{i=1}^{n} \lambda_i \phi\left(\|x - \xi_i\|\right) + \sum_{k=1}^{m} \alpha_k \gamma^{\boldsymbol{\xi}} \phi\left(\|x - \xi_k\|\right) + \chi \tag{6}$$

or in a vector-matrix notation:

$$\hat{u}_j(x) = \left|\boldsymbol{\phi}\left(\|x - \xi_i\|\right) \quad \gamma^{\boldsymbol{\xi}}\boldsymbol{\phi}\left(\|x - \xi_k\|\right) \quad 1\right| \left|\boldsymbol{\lambda} \quad \boldsymbol{\alpha} \quad \chi\right|^T \tag{7}$$

which interpolates both value function $u(x)$ on all points and derivative information $\gamma u(x)$ at the double collocation points. Note that this interpolant guarantees reproduction of constant functions. If $m = 0$ (a Lagrange interpolation) it interpolates functional values only, and if $m \neq 0$ then a RBF double collocation is possible to exploit, e.i. in these locations, two interpolation conditions are simultaneously satisfied. The latter can be a way to increase the accuracy, without increasing the size of the molecule, whether there is information about the derivatives of the unknown function at some data points. The interpolation coefficients λ_i, α_k and χ are going to determined.

By evaluating the different interpolation conditions on Eq.(6) at the corresponding supporting points (n) from the molecule, this leads to $n + m + 1$ linear equations, n equations by value function, m equations by derivative value plus one homogeneous constraint by the polinomial term. The block matrix form of these equations is:

$$\underbrace{\begin{vmatrix} \boldsymbol{\phi}\left(\|x-\xi\|\right) & \gamma^{\xi}\boldsymbol{\phi}\left(\|x-\xi\|\right) & \boldsymbol{e} \\ \gamma\boldsymbol{\phi}\left(\|x-\xi\|\right) & \gamma\gamma^{\xi}\boldsymbol{\phi}\left(\|x-\xi\|\right) & \gamma\boldsymbol{e} \\ \boldsymbol{e}^T & \gamma\boldsymbol{e}^T & 0 \end{vmatrix}}_{A^H} \begin{vmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\alpha} \\ \chi \end{vmatrix} = \begin{vmatrix} \boldsymbol{u} \\ \boldsymbol{\gamma u} \\ 0 \end{vmatrix} \tag{8}$$

where $\boldsymbol{e}_i = 1$, the coefficient matrix A^H is symmetric and $\boldsymbol{u}^T$ as the values of the unknown field variable u at all n supporting points. Hence the interpolation coefficients depend on the unknown field variable only. The latter is important bear in mind.

Using Eq.(6) belong with the linear system Eq.(8), the meshless approximate function can be expressed in term of shape functions as:

$$\hat{u}_j(x) = \sum_{i=1}^{n} \Phi_i(x) u_i + \sum_{j=1}^{m} \Phi_j(x) \gamma u_j \tag{9}$$

and any linear partial differential operator can be easily discretized applying it to shape functions, i.e.

$$\mathcal{L}\hat{u}_j(x) = \sum_{i=1}^{n} \mathcal{L}\Phi_i(x) u_i + \sum_{j=1}^{m} \mathcal{L}\Phi_j(x) \gamma u_j \tag{10}$$

The derivative term $\gamma u(x)$ in Eq. (9) can be assumed in different ways looking for better approximations. Three options have been proposed

1. When $m = 0$, e.i. not derivative information is assumed known, e.i. there is not a special treatment for the derivative boundary condition. This formulation is named as the local RBF single collocation approach. It is very good for well-posed Dirichlet problem only.

2. When $m \neq 0$ in molecules at and near to the derivative boundary since the derivative information is assumed $\gamma u(x) = \frac{\partial u}{\partial \boldsymbol{n}} = q(x)$, e.i, if there are m points belong to the derivative boundary within a molecule, their normal derivatives $q(x)$ are considered as the additional unknowns. At derivative star point, the PDE and the normal derivative boundary condition are satisfied simultaneously; it increases the number of collocation equations. This formulation is named as the local RBF double boundary collocation approach.

3. When $m \neq 0$ in all molecules since the derivative information is assumed to come from the right hand term of the PDE, e.i., $\gamma u(x) = \mathcal{L}u(x) = f(x)$. This formulation is named

as the local RBF PDE collocation approach. In our tests, $m < n$ in interior molecules and $m \leq n$ in molecules with the star point along the derivative boundary were assumed. The first condition rejects explicitly the derivative information at the star point (Wright and Fornberg, 2006). Beside, the m nearest points to the star point were chosen. Note that this formulation can be well applied to Dirichlet and Dirichlet plus Neumann problems.

Finally, choosing the point x to be located at the star point of each molecule and substituting Eq. (9) in the governing equations Eq. (1,2), we end up with a formulation that couple the field variable at the star point with the corresponding field value of neighbouring points (point collocation technique). By an assembling process on all the molecules in the domain a global closed system of the equations for u is obtained. To impose the Dirichlet boundary condition, the Kronecker delta property is always used, but additional equations relating to the given field variable value are not seen as further rows in the global system of equations. It is considered in the other equations on the corresponding right hand terms.

Once the molecules have been established in a preprocessing stage, the overall cost of the different above approaches can be summarized as follows:

1. The cost of the computations of the shape functions at each star points which has an order of $n+m$. We use a direct solver in order to the inversion of the small symmetric equation system.

2. The cost of the assembly process of a large global sparse matrix. For example, this cost could be critical when the global system has to be assembled at each step from a nonlinear iteration loop when solving a strongly nonlinear problem. Here an efficient data structure is needed.

3. The cost of the linear solver in order to the global system inversion. Here we could use a preconditioned conjugate gradient algorithm or employ a direct sparse solver.

Our code is still in an earlier stage of development so it could be quite improved. However we felt that it is much faster than the global approach.

3 NUMERICAL RESULTS

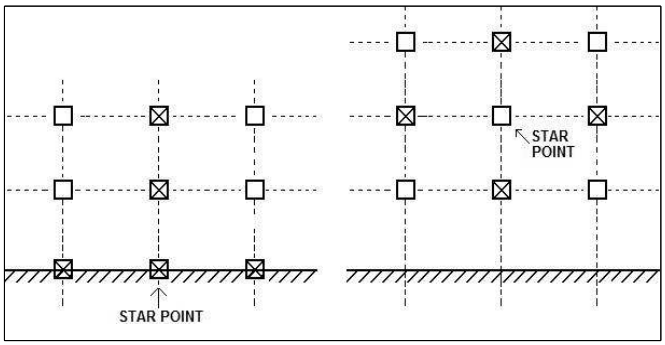

Figure 1: Computational molecules assumed on derivative boundary and internal star points, respectively including nine supporting points.

To gain some insight into the numerical characteristics of different radial basis function collocation approaches and to assess their stability using variations of RBF shape parameter, global and local frameworks and various strategies for handling derivative boundary conditions; numerical experiments have been performed for three representative boundary value problems with analytical solutions: A non-homogeneous mixed boundary condition problem, the forced heat equation with an exponential decay of the heat source intensity from an incident boundary and a convection-diffusion equation with variable velocity field. In all computations, the multiquadric RBF (MQ) $\sqrt{r_j^2 + c_j^2}$ plus a constant polinomial term have been used.

In this earlier stage of our code development, the molecule selection around each star point is easily accomplished by a distance criterion. This procedure is considered in a preprocessing stage. Fig. 1 show computational molecules for internal and boundary points when nine supporting points are chosen. The marked squares are assumed as m double collocation points in the local RBF PDE collocation approach. Uniform distributions of points and two sizes of molecules $n = 5$ (only marked squares in Fig. 1) and $n = 9$ have been used.

3.1 Example 1

$$\nabla^2 u + u = f(x, y) \tag{11}$$

and the boundary conditions given:

$$\begin{array}{ll} u = g(x, y) & \text{on } y = 0, 1 \text{ and } 0 \leq x \leq 1 \\ \frac{\partial u}{\partial x} = h(x, y) & \text{on } x = 0, 1 \text{ and } 0 < y < 1 \end{array} \tag{12}$$

with analytical solution given:

$$u(x, y) = x \exp(xy) \tag{13}$$

The non-homogeneous parts have been taken from the given analytical solution. In computations, two uniform distributions of points 11×11 and 41×41 discretise the unitary square domain.

Fig. 2 shows the r.m.s. relative errors of the field variable along the center line of the domain using four RBF approaches: The results based on the global symmetric RBF single collocation method (La Rocca and Power, 2008) are refereed as *meth 1*, the ones using the local RBF single collocation method as *meth 2*, the ones using local RBF double boundary collocation method as *meth 3* and the ones using local RBF PDE collocation method as *meth 4*. Moreover, the errors are studied by changing the MQ shape parameter, the density of domain points and the size of the computational molecule. In Fig. 2, it is exhibited evidences of the effects of the parameter c on the error in all RBF collocation approaches. The smoothest variations of the errors with the shape parameter are observed when using the coarser distribution of points in both numbers of supporting points. On the finer distribution of points, the error curves in the global symmetric and local PDE collocation methods, e.i., *meth 1* and *meth 4*, exhibit a critical point (a minimum), after which instabilities of the error are noted. The natural trend is that local RBF method yields to less accurate solutions than the global counterpart which is a known behavior when solving a problem that has a smooth solution (see Bernal et al. (2008)). However, in some cases the method refereed as *meth 4* is very competitive in comparison with the global method presented. Clearly, changing the density of the domain points, the solutions are more accurate on a wide range of c values. Now changing the molecule size of five a nine points do not report apparently more accurate solutions in all local RBF approahces.

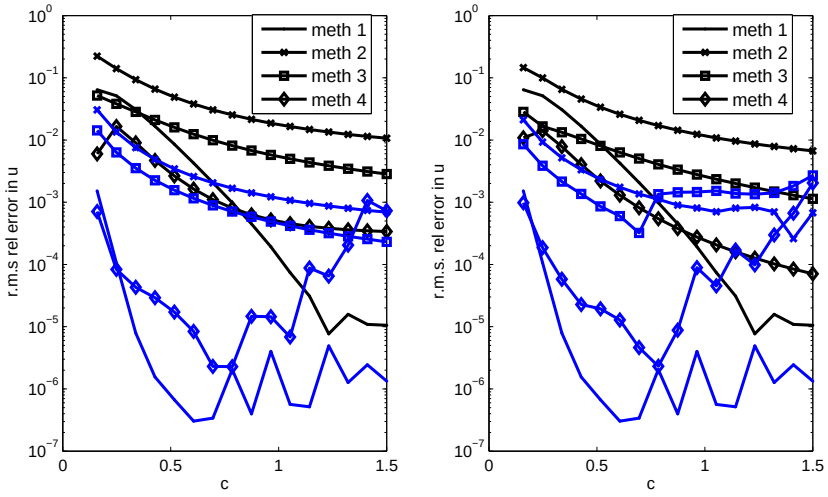

Figure 2: Comparison between the r.m.s. relative errors of the field variable regarding to shape parameter for the example 1. Five and nine supporting points form the molecules respectively on the left and on the right. The black and blue lines are for distributions of points 11×11 and 41×41, respectively.

It is worth noting that the local RBF double boundary collocation method (*meth 3*) can yield to any more accurate solution than the simplest local RBF collocation method (*meth 2*). Hence, the former can be viewed a feasible strategy for imposing the derivative boundary condition.

3.2 Example 2

$$\nabla^2 u + \beta \exp(-\gamma x) u = 0 \tag{14}$$

This simplified model equation governs the microwave heating process of an isotropic body (Sladek et al., 2004). It is assumed that the thermal absorptivity of material obey a lineal model in u and an exponential decay of the electric field is the x-direction only where γ is a decay constant. A thermal runaway is possible if a certain material, denoted by a critical β_o value, is exposed to a corresponding energy level (γ).

On all boundaries of the square plate, an uniform unity temperature is prescribed to analyze the thermal runaway. In reference Sladek et al. (2004), the β_o value was obtained analytically for $\gamma = 4$ which is 86.1. In our numerical calculations, an unity side length of the plate and an uniform distribution of points 11×11 are used.

In Fig. 3, we show our way of predict the thermal runaway. We ran the two local RBF single and PDE collocation approaches in the vicinity of the β_o known value and clearly it is seen a tremendous peak of the field variable in the β_o values predicted by our schemes. This procedure was used to observe the accuracy of our schemes with varying MQ shape parameter c and the number of neighboring points. It can be observed in Table 1 a good agreement between our results and analytical one for the critical β_o value on a wide range of MQ shape parameters. However, the local RBF PDE collocation method (*Meth 2*) show less sensitive with variations of MQ shape parameter. We notice any accuracy improvement in *Meth 2* when pass of five to nine supporting points. In this example is not apply the local RBF double boundary collocation method due to the boundary conditions are Dirichlet type only.

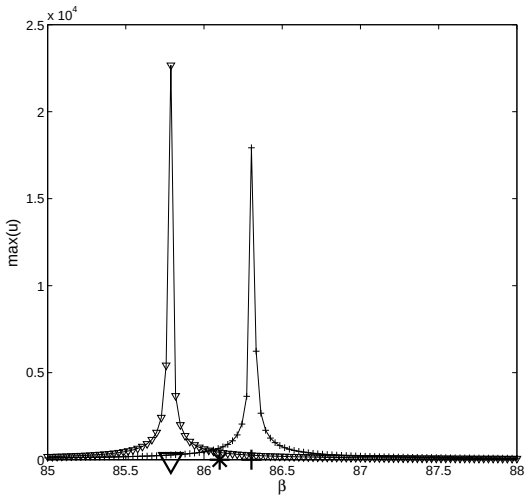

Figure 3: The field variable maximum as function of β values in the vicinity of the critical β_o value. The triangle and plus marked curves correspond to Meth 1 and Meth 2, respectively.

	Meth 1		Meth 2	
c	5-supp	9-supp	5-supp	9-supp
0.25	–	–	4.88E-03	6.27E-03
0.5	2.65E-02	2.18E-02	6.04E-03	1.28E-03
1	4.65E-03	2.44E-03	5.23E-03	2.67E-03
1.5	1.07E-02	1.51E-03	4.99E-03	2.44E-03
2.5	1.28E-02	3.72E-03	4.76E-03	2.44E-03

Table 1: Relative errors respect to 86.1 in numerical solutions obtained with different c and neighboring points. A distribution of point 11×11 has been used.

3.3 Example 3

$$\nabla^2 u - (A + kx)\frac{\partial u}{\partial x} - ku = 0 \tag{15}$$

where $A = \ln\left(\frac{u_1}{u_o}\right) - k/2$ and the boundary conditions given:

$$\begin{array}{ll} u = u_o = 300 & x = 0 \text{ and } 0 \leq y \leq 0.2 \\ u = u_1 = 200 & x = 1 \text{ and } 0 \leq y \leq 0.2 \\ \frac{\partial u}{\partial y} = 0 & y = 0, 0.2 \text{ and } 0 < x < 1 \end{array} \tag{16}$$

with an analytical solution given as:

$$u(x, y) = u_o \exp\left(\frac{k}{2}x^2 + Ax\right) \tag{17}$$

This convection diffusion equation with a variable velocity field has been chosen by different researchers due to strong range of variation of the unknown variable in computational domain which make its numerical solution a non-trivial task. In our computations, a parameter $k = 40$ and the domain into the rectangle $[1 \times 0.2]$ have been used which were also used

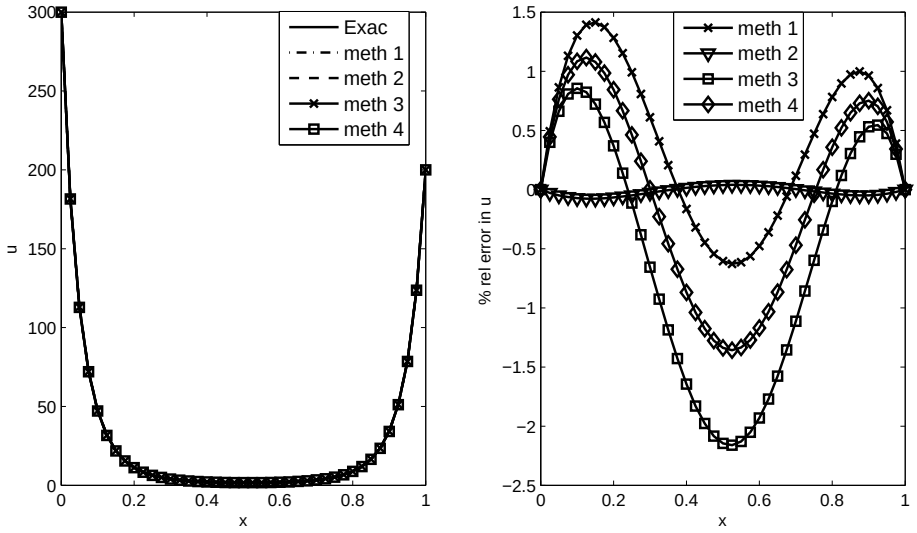

Figure 4: Comparison between the analytical solution and various local RBF collocation method solutions (on the left) for example 3. A graph of the corresponding relative percentage errors (on the right).

in La Rocca and Power (2008), but they compared global symmetric RBF single and double boundary collocation methods.

This example is analyzed using two uniform distributions of points 41×9 and 71×15 totalling 369 and 1065 nodes respectively and two molecule sizes including five and nine points. Four different ways of constructing the local meshless RBF approximations are adopted on a local framework. They are several strategies that can be used to treat the derivative boundary conditions and these approaches will be tested in this numerical example. They are: local RBF single collocation method (*meth 1*), local RBF PDE collocation method (*meth 2*), local RBF double boundary collocation method (*meth 3*) and local RBF single collocation method combining with one sided finite difference formula to discretise the homogeneous Neumann boundary condition (*meth 4*), see Shu et al. (2003) for more details. A comparison between the analytical solution and different RBF approaches and the corresponding curves of relative percentage errors along the longitudinal line of the domain are shown in Fig. 4. This figure has been obtained with the coarser distribution, a parameter $c = 0.2$ and molecules including nine points. It can be seen from Fig. 4 an excellent agreement between the exact solution and several computed solutions and free of visible spurious oscillations. On the right side, it is clearly shown that *meth 2* is the more accurate of the approaches. We compare present results with the ones of La Rocca and Power (2008) and notice that local methods are very more efficient than the global counterparts. A previous analysis already indicated this behavior when solving convective-dominated problems (Gutierrez and Florez, 2008).

In addition, we study in above local methods the effects of variations of shape parameter and two molecule sizes using the finer point distribution. From Table 2, it is clearly seen that almost all methods have an error minimum in $c = 0.2$. Changing of five to nine points in molecule produces the most appreciate improvement of the accuracy in named *meth 2*. Notice that other approaches do not suffer important variations respect to molecule sizes. In the cases of *meth 3* and *meth 4* which are especially suited for treating derivative boundary conditions, we notice similar performances although they are dissimilar conceptually. The named *meth 1* is seen as slightly less accurate of all methods tested.

	5-supp				9-supp			
c	meth 1	meth 2	meth 3	meth 4	meth 1	meth 2	meth 3	meth 4
0.05	0.3285	0.0163	0.3974	0.3807	0.1425	0.0361	0.2005	0.1669
0.10	0.0666	0.009	0.0849	0.0815	0.0243	0.0026	0.0378	0.0308
0.20	0.0078	0.0123	0.0065	0.0065	0.0168	0.0004	0.014	0.0154
0.50	0.0265	0.0192	0.0258	0.0259	0.0281	0.0014	0.0272	0.0279
0.80	0.0289	0.0231	0.0286	0.0286	0.0291	0.0018	0.0286	0.029
1.00	0.0294	0.0255	0.0292	0.0293	0.0332	0.0444	0.032	0.0331

Table 2: r.m.s relative percentage errors in the obtained field variable with a distribution of points 71×15 for different c and supporting points.

4 CONCLUSIONS

The RBF collocation method based on the Hardy's interpolation is used to solve three representative linear boundary value problems which include variable coefficient and nonhomogeneous mixed boundary conditions. Five different RBF strategies were tested. Among them, one with global character and four with local character. Two local strategies are especially suited for treating of derivative boundary conditions, one based on a boundary collocation of the PDE without fictitious points lying beyond of the derivative boundary and other one based on an one-sided second order finite difference formula. We observe that local methods tested can dealt with more general second order partial differential equations with accuracy and efficiency. In some cases the local methods are more accurate than global counterparts and when solving convection-dominated problems, the former are generally more accurate and stable than the second ones. It has been found that in the global RBF symmetric collocation and local RBF PDE collocation methods, the accuracy of the solution suffers the amplest variations respect to MQ shape parameter. Furthermore, it is found that pass of five to nine supporting points in the molecules to improve the accuracy it was only appreciated in the local RBF PDE collocation method. It can be seen as the best strategy among all approaches tested (based on RBF strong-form) for solving general linear boundary value problem. However, our results are based on uniform distributions of points, further work in non-uniform distributions is needed. Here the issue aims in developing a suitable search algorithm for the molecules.

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