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CRACK PATH PREDICTION BY THE MVCCI-METHOD AND EXPERIMENTAL VERIFICATION FOR SPECIMENS UNDER PROPORTIONAL BENDING AND SHEAR LOADING

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ABSTRACT

The Modified Virtual Crack Closure Integral (MVCCI)-method has proved to be a highly effective and versatile numerical procedure for the fracture analysis of various crack problems in linear elasticity. In the present paper it is shown that the MVCCI-Method also can readily be utilised for the computer aided prediction of curved crack growth paths under proportional loading conditions. Further it will be shown, that the common computer aided crack path prediction can be improved in accuracy in combination with the MVCCI-Method. The proposed numerical crack path prediction is still based on a step-by-step technique but using a new predictor-corrector procedure that results in a piece by piece curved approximation of the simulated crack path. By this powerful method both, the new locus of the crack tip and the slope of the crack path can be computed simultaneously by one virtual tangential crack extension with respect to the previous step.

In order to evaluate the validity and the efficiency of the proposed crack path simulation method in relation to the basic strategies, experiments of non-coplanar fatigue crack growth are carried out with two special specimens under combined proportional bending and shear loading. The specimens have been designed in order to produce non-homogeneous stress fields. Crack initiations from different positions along the edges of these two types of specimens are investigated for different interactions with the produced asymmetrical stress states.

In all cases considered the computationally predicted crack trajectories show an excellent agreement with the different types of curved cracks that are obtained in the experiments as a function of the position of crack initiation.

The present 2D investigation shows that the MVCCI-method in conjunction with the proposed predictor-corrector procedure provides a powerful numerical tool for a general computational approach to the fracture analysis of complex crack configurations under proportional loading conditions.

INTRODUCTION

Catastrophic failure of structures and components is often caused by cracks that frequently originate and extend in regions characterised by complicated geometrical shapes and asymmetrical loading conditions. Hence the developing crack paths are found to be curved and standard solutions for coplanar cracks do not apply. Therefore the prediction of such curved crack paths is essential for the accurate assessment of fatigue crack-propagation lives and of the final fracture modes of cracked components and structures. Several computational methods have been proposed for crack path predictions based on step-by-step analys***e***s by using finite elements or boundary elements (Bergquist and Gnex[1]; Theilig[2]; Sumi[3,4]; Linnig[5]; Theilig, Wiebe and Buchholz[6]; Portela and Aliabadi[7]; Richard, Schöllmann and May[8].

In the present paper, attention is focused on a new predictor-corrector procedure that results in an incremental curved approximation of the crack path on the basis of quantities which the straightforward Modified Virtual Crack Closure Integral Method can provide (Theilig, Döring and Buchholz[9]). In order to show the significance of the proposed simulation technique computational results are compared with findings from experimental investigations by the aid of specially designed specimens under proportional loading conditions, in particular under combined proportional bending and shear loading.

TWO-DIMENSIONAL CRACK PATH PREDICTION

Consider a crack in a two-dimensional linear elastic body under proportional mixed-mode loading conditions. The stresses ahead of the crack tip are given by

$$\sigma_{11}(x_1,0)=\frac{k_I}{\sqrt{2\pi x_1}}+T+b_I\sqrt{\tfrac{x_1}{2\pi}}+O(x_1), \tag{1}$$

$$\sigma_{22}(x_1,0)=\frac{k_I}{\sqrt{2\pi x_1}}+b_I\sqrt{\tfrac{x_1}{2\pi}}+O(x_1), \tag{2}$$

$$\sigma_{12}(x_1,0)=\frac{k_{II}}{\sqrt{2\pi x_1}}+b_{II}\sqrt{\tfrac{x_1}{2\pi}}+O(x_1), \tag{3}$$

where k_I and k_{II} are the stress intensity factors (SIFs). T, b_I and b_{II} are the included higher order stress field parameters. It is known that in such a situation the crack will propagate in a smoothly curved manner after kinking out of the original plane (Fig. 1). For several mixed-mode criteria the direction φ_0 of crack kinking depends only on the ratio k_{II}/k_I of the SIFs of the original straight crack, whereas for others a further dependence on Poisson's ratio ν is found. But for small ratios $0 < |k_{II}/k_I| < 0.1$ practically the same values $\varphi_0 = -2\, k_{II}/k_I$ are predicted by all criteria. This direction will result in the state of local symmetry at the branched crack tip ($K_{II} = 0$). The generalisation of the local symmetry criterion can be regarded as the natural basis for the evolution of the crack path in a homogeneous isotropic material (Goldstein and Salganik[10]). Thus, it can be stated that a continuously growing crack will form a smoothly curved path that experiences pure mode I at any crack tip position (Fig. 1). Therefore the state of stress ahead of the crack tip is given without K_{II} by

$$\sigma_{11}\left(x_1^*,0\right)=\frac{K_I}{\sqrt{2\pi x_1^*}}+T^*+b_I^*\sqrt{\tfrac{x_1^*}{2\pi}}+O\left(x_1^*\right) \tag{4}$$

$$\sigma_{22}\left(x_1^*,0\right)=\frac{K_I}{\sqrt{2\pi x_1^*}}+b_I^*\sqrt{\tfrac{x_1^*}{2\pi}}+O\left(x_1^*\right) \tag{5}$$

$$\sigma_{12}\left(x_1^*,0\right)=b_{II}^*\sqrt{\tfrac{x_1^*}{2\pi}}+O\left(x_1^*\right) \tag{6}$$

The coordinate system (x_1^*, x_2^*) is defined with the origin at the current crack tip with the x_1^*-axis along the tangential direction of the crack path at the tip. Therefore it can be stated that continuous crack deflections can only be caused by the existing non-singular stresses

According to Sumi[3,4] the crack path prediction can be performed by using the first order perturbation solution of a slightly kinked and curved crack. A virtually extended slightly kinked and smoothly curved crack path profile (Fig. 2) is assumed in the form

$$l(x_1)=\alpha x_1+\beta x_1^{3/2}+\gamma x_1^2+O\left(x_1^{5/2}\right) \tag{7}$$

where α, β and γ are the shape parameters.

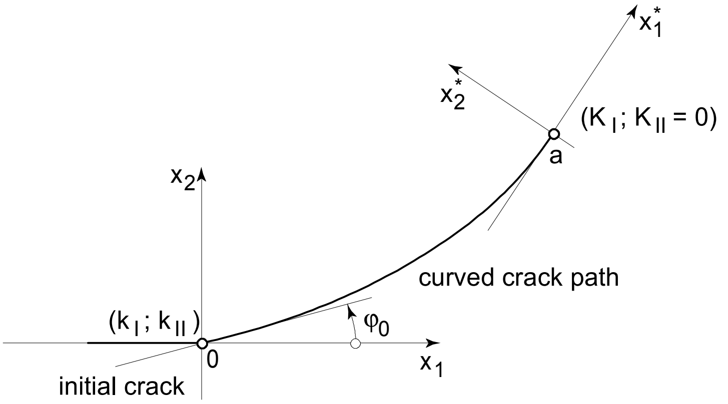


Fig. 1. A kinked and curved crack

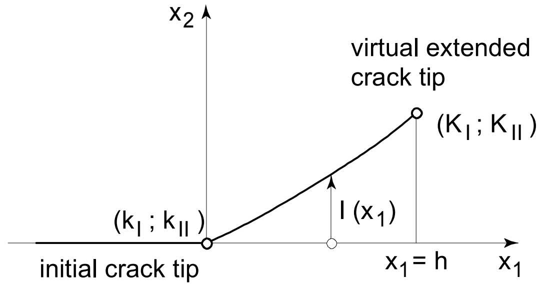


Fig. 2. A slightly kinked and curved virtual crack

The SIFs at the virtually extended crack tip are given by

$$\begin{aligned} K_I = {} & \left(k_I - \frac{3}{2}\alpha k_{II}\right) - \frac{9}{4}\beta k_{II} h^{1/2} \\ & + \left[\frac{b_I}{2} - \frac{5}{2}\alpha b_{II} - 3\gamma k_{II} + k_I \bar{k}_{11} - \alpha k_I \left(\bar{k}_{12} + \frac{3}{2}\bar{k}_{21}\right)\right. \\ & \left. + k_{II}\bar{k}_{12} - \alpha k_{II}\left(\bar{k}_{11} + \frac{3}{2}\bar{k}_{22}\right)\right] h + O\left(h^{3/2}\right), \end{aligned} \tag{8}$$

$$\begin{aligned} K_{II} = {} & \left(k_{II} - \frac{1}{2}\alpha k_I\right) - \left(\frac{3}{4}\beta k_I - 2\sqrt{\tfrac{2}{\pi}}\,\alpha T\right) h^{1/2} \\ & + \left[\frac{b_{II}}{2} - \frac{1}{4}\alpha b_I - \frac{3\sqrt{2\pi}}{4}\beta T + \gamma k_I + k_I \bar{k}_{21} + \alpha k_I \left(\frac{1}{2}\bar{k}_{11} - \bar{k}_{22}\right)\right. \\ & \left. + k_{II}\bar{k}_{22} + \alpha k_{II}\left(\frac{1}{2}\bar{k}_{12} - \bar{k}_{21}\right)\right] h + O\left(h^{3/2}\right), \end{aligned} \tag{9}$$

where the quantities $\bar{k}_{11}, \bar{k}_{12}, \bar{k}_{21}, \bar{k}_{22}$ represent the effects of stress redistribution due to the crack growth in a plane finite body under a load with unit stress intensity factors at the initial crack tip. If we follow the crack propagation criterion of local symmetry the SIF K_{II} vanishes along the crack path at any current crack tip position. Therefore the shape parameters of the natural crack geometry are obtained as

$$\alpha = -2\, k_{II} / k_I\,, \tag{10}$$

$$\beta = \frac{8}{3}\sqrt{\tfrac{2}{\pi}}\,\frac{T}{k_I}\,\alpha, \tag{11}$$

$$\gamma = -\left(k_{II}\bar{k}_{22} + k_I \bar{k}_{21} + \frac{b_{II}}{2}\right)\frac{1}{k_I} + \left\{\left[k_I\left(2\bar{k}_{22} - \bar{k}_{11}\right) + \frac{b_I}{2}\right]\frac{1}{2k_I} + 4\left(\frac{T}{k_I}\right)^2\right\}\alpha\,. \tag{12}$$

If we consider a straight crack under local symmetry at the initial crack tip, i.e. $k_{II} = 0$, we find $\alpha = \beta = 0$. Therefore the parabolic crack profile

$$l(x_1) = \gamma\, x_1^2 \tag{13}$$

is holding.

The shape parameter γ is given by

$$\gamma = -\left(\frac{b_{II}}{2} + k_I\,\bar{k}_{21}\right)\frac{1}{k_I} \tag{14}$$

and with eq. (8)

$$K_I = k_I + \left(\frac{b_I}{2} + k_I\,\bar{k}_{11}\right) h\,. \tag{15}$$

is found. In this case the crack will propagate without kinking with a continuous deflection. But in the case of a self-similar virtual crack extension of the postulated straight crack the SIF's

$$\bar{K}_I = k_I + \left(\frac{b_I}{2} + k_I\,\bar{k}_{11}\right) h\,, \qquad \bar{K}_{II} = \left(\frac{b_{II}}{2} + k_I\,\bar{k}_{21}\right) h \tag{16}$$

are obtained. From eqs. (15) and (16) $K_I(h) = \bar{K}_I(h)$ is found in consequence of the considered slightly curved crack extension. According to eqs. (13 - 16) one finally gets for a selected increment Δh the following information of the real crack path

$$\Delta\varphi = -2\,\frac{\Delta\bar{K}_{II}}{k_I}\,,\ \Delta l = -\frac{\Delta\bar{K}_{II}}{k_I}\Delta h\,,\ \Delta a \approx \Delta h\left[1 + \frac{2}{3}\left(\frac{\Delta\bar{K}_{II}}{k_I}\right)^2 - \frac{2}{5}\left(\frac{\Delta\bar{K}_{II}}{k_I}\right)^4\right]. \tag{17}$$

It is seen that under the local symmetry criterion $K_{II} = 0$ the change of the slope and the locus of the crack tip can be interpreted as the consequence of $\Delta\bar{K}_{II} \neq 0$ for a virtual tangential crack extension Δh (Fig. 3).

Therefore, in the case of proportional loading conditions the analysis of a smooth crack path can be carried out by a small virtual tangential crack extension as the predictor-step in combination with a finite change of the crack path as the corrector-step. Due to the predictor-step the calculation of $K_I = \bar{K}_I$ and $\Delta\bar{K}_{II}$ is necessary in conjunction with the related tangential crack extension Δh. This can be done by using the finite element method.

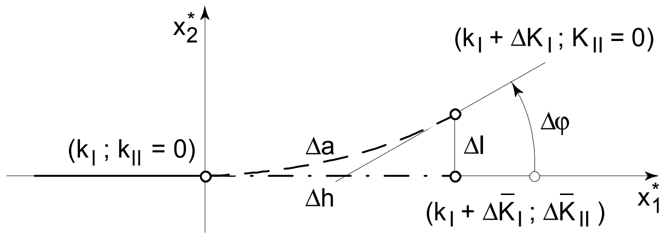


Fig. 3. Curved crack propagation

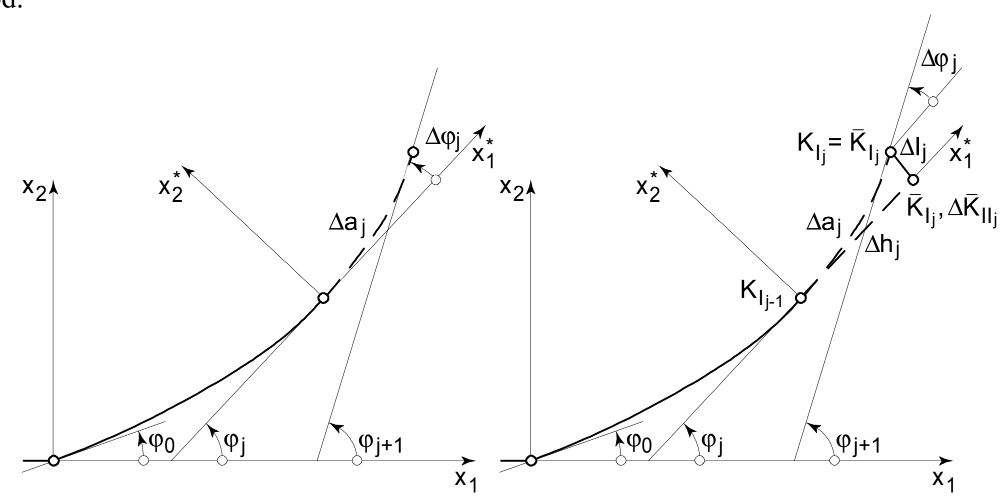


Fig. 4. Step by step crack path simulation with curved increments

According to Fig. 4 the proposed incremental crack extension procedure is given by

$$\varphi_{j+1} = \varphi_j - 2\left(\frac{\Delta \overline{K}_{II_j}}{K_{I_{j-1}}}\right), \Delta l_j = -\left(\frac{\Delta \overline{K}_{II_j}}{K_{I_{j-1}}}\right)\Delta h_i , a_j = a_{j-1} + \left[1 + \frac{2}{3}\left(\frac{\Delta \overline{K}_{II_j}}{K_{I_{j-1}}}\right)^2 \frac{2}{5}\left(\frac{\Delta \overline{K}_{II_j}}{K_{I_{j-1}}}\right)^4\right]\Delta h_j . \quad 18)$$

From eqs.(18) the need for an efficient numerical mode separation technique in conjunction with the step-by-step analysis can be seen. With respect to this requirement the MVCCI-method has proved to be highly advantageous, because it delivers the separated strain energy release rates of two modes simultaneously with excellent accuracy and without any additional effort. For 8-noded quadrilaterals at the crack tip (Fig.5), which are necessary to model the parabolic curved increments of the crack path, the following finite element representation of Irwin's crack closure integral relations can be given (Buchholz[10]; Krishnamurthy et al.[11]; Raju[12])

$$\overline{G}_I = \frac{1}{2\Delta ht}\left(F_{2,i}\ \Delta u_{2,i-1} + F_{2,i+½}\ \Delta u_{2,i-½}\right),\ \Delta\overline{G}_{II} = \frac{1}{2\Delta ht}\left(F_{1,i}\ \Delta u_{1,i-1} + F_{1,i+½}\ \Delta u_{1,i-½}\right). \quad (19)$$

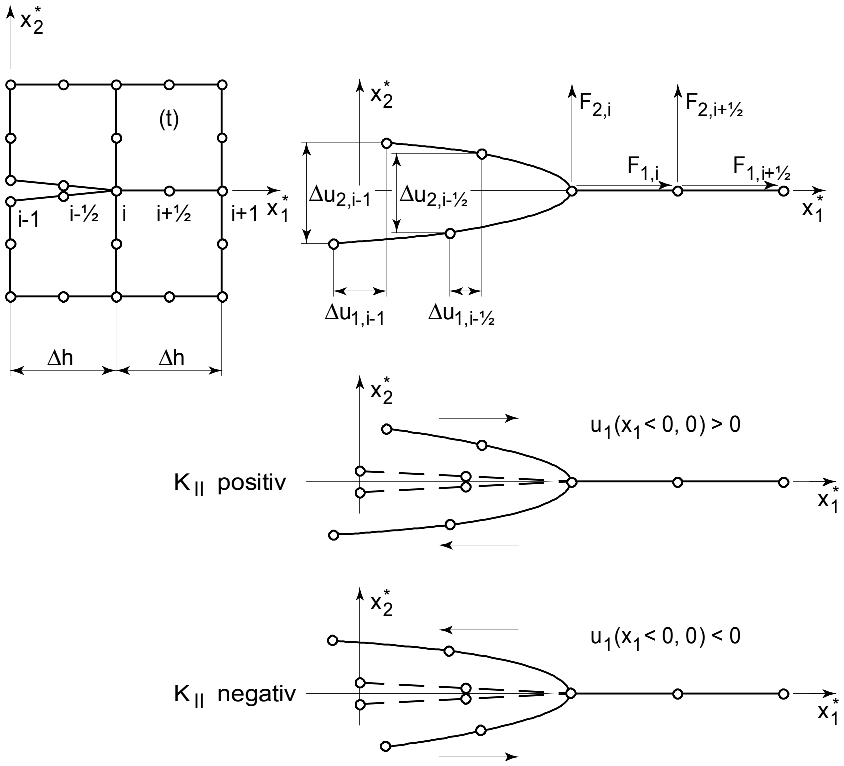


Fig. 5. Modified virtual crack closure integral method

CURVED FATIGUE CRACK GROWTH TESTS

In order to evaluate the validity and the efficiency of the proposed improved crack path simulation method in relation to the well known basic strategies, experiments of non-coplanar fatigue crack growth are carried out with two specially designed specimens under combined proportional bending and shear loading [2,13]. The specimens have been designed in order to produce non-homogeneous stress fields. The first specimen type to be investigated here is a non-symmetrical specimen with a circular shaped transition region for lateral force bending (LFBC-specimen). Along the circular shaped region of the specimen notches have been attached at different position α_N = 0,20,40 deg. from which cracks initiated and extended during fatigue tests (Fig. 6). Especially steel St34 u-2 has been selected as specimen material.

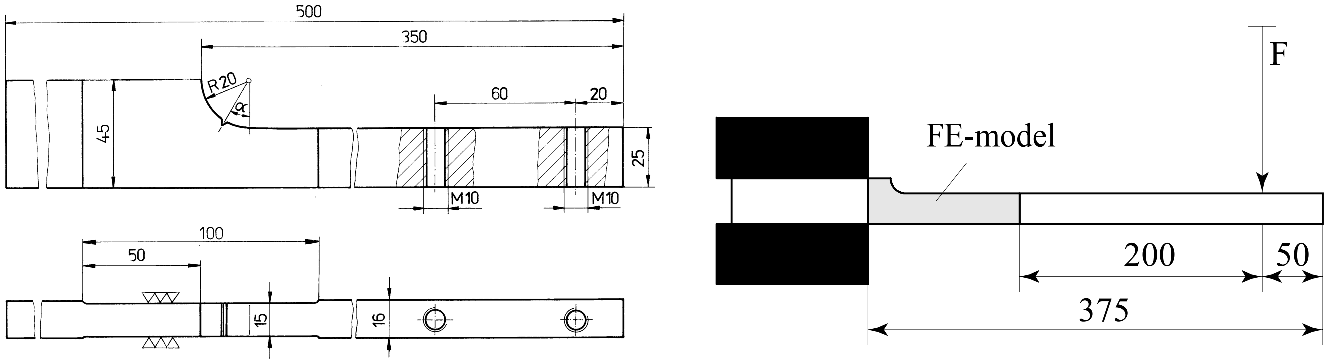


Fig.6. Lateral bend specimen with curved edge and loading conditions

The design of the second specimen type (LFBH-specimen) contains a hole in the centre of the specimen (Fig.7). The steel X20Cr13 has been selected as specimen material in this case. Crack initiations from notches at different positions l_K along the tensile loaded edge of these specimens are investigated to produce different crack interactions with the hole. In particular l_K = 65, 75 and 85 mm were selected. The notches have been manufactured with a width of 0.3mm and a maximum depth of 3mm.

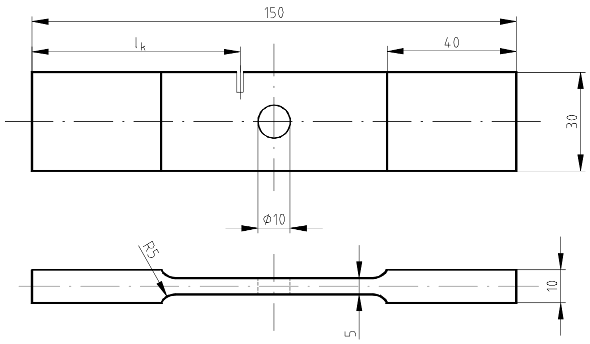


Fig.7. Dimensions and notch position of the LFBH-specimen

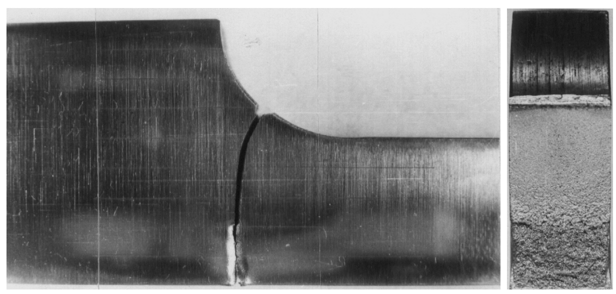


Fig.8. Broken LFBC-specimen with curved crack

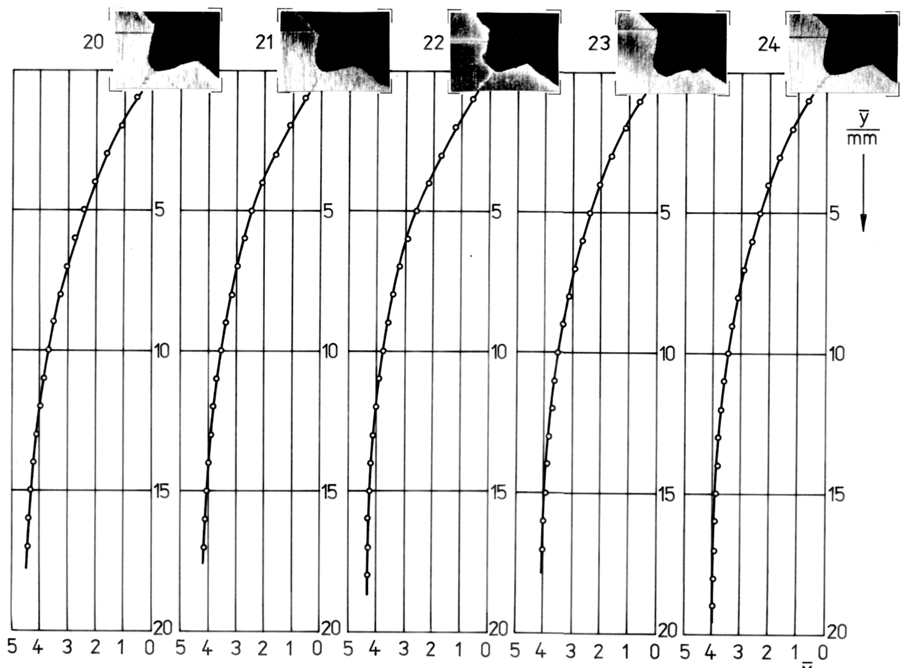
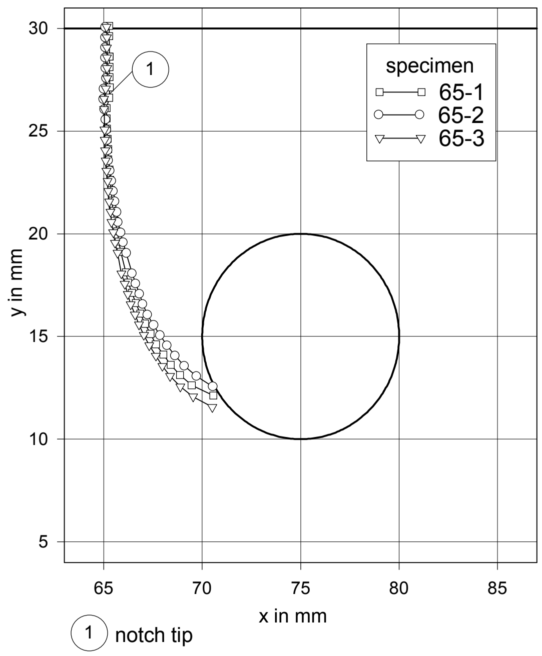


Fig.9. Fatigue crack paths of the LFBC-specimen with the notch position α_N = 40 deg.

For each notch position five LFBC-specimens have been investigated with a constant force amplitude ΔF under proportional loading conditions ($F_{max}/F_{min} > 0$). The specimens were broken by overloading after a significant reduction in cross section (Fig. 8). These crack paths were measured at the center line of the broken specimen. In Fig. 9 the experimentally obtained crack trajectories of the specimens with the notches at α_N = 40 deg. are shown and it is interesting to see that the experimental scatterband is with 0.6 mm rather narrow. It can be recognised that the local position of pre-cracks in the roots of the notches determine the scattering. These given experimental findings have formed the basis on which an early approach to computer aided crack path simulation has been evaluated [2].



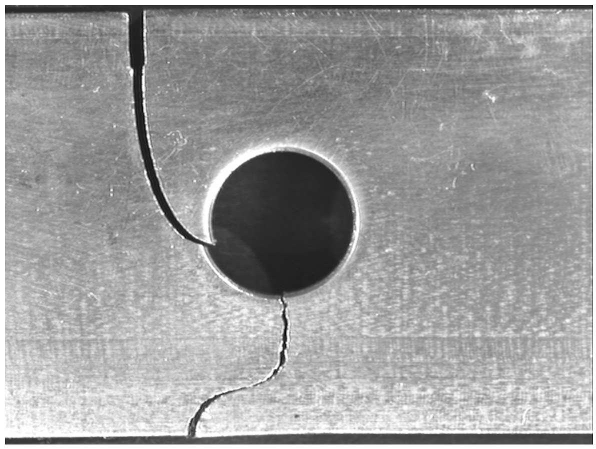
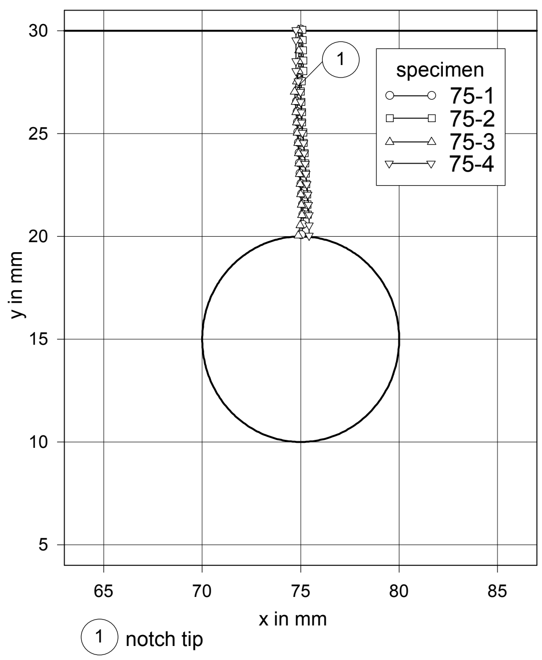


Fig.10a. Fatigue crack paths of the LFBH-specimen with the notch position l_K = 65mm



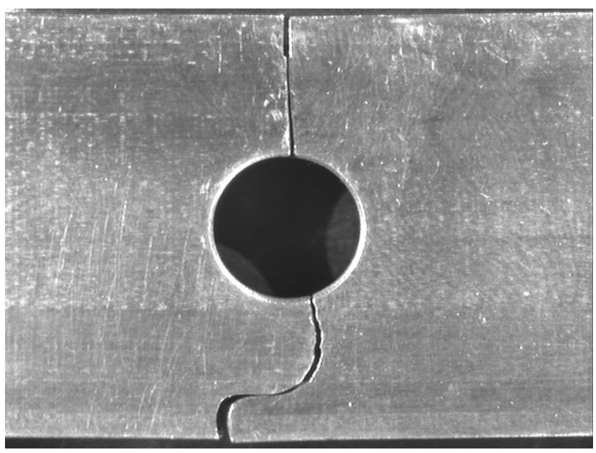
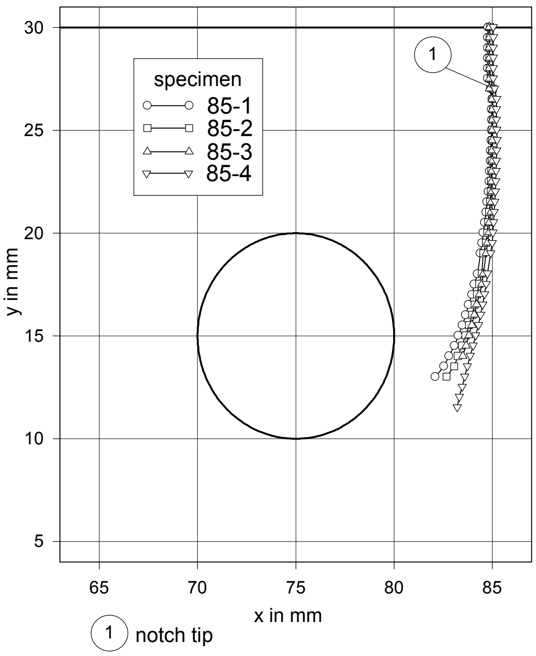


Fig.10b. Fatigue crack paths of the LFBH-specimen with the notch position l_K = 75mm



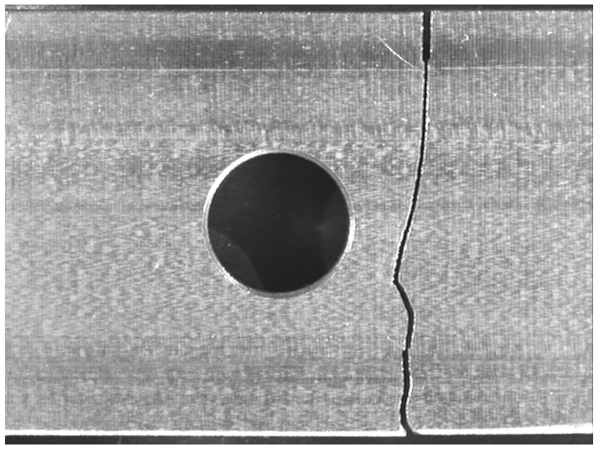


Fig.10c. Fatigue crack paths of the LFBH-specimen with the notch position l_K = 85mm

In Fig. 10 broken LFBH-specimens with curved fatigue crack path and the valid experimental results are given for the different tested notch positions. For the measurements the coordinate system x, y was defined with its origin at the centre of the bore. In these tests it can also be recognised that the intended small differences of the notch positions in relation to the bore and the local positions of the pre-cracks in the roots of the notches essentially determine the scattering.

NUMERICAL CRACK PATH PREDICTION

For the finite element calculations the models given in Fig. 11 were chosen in accordance with the design of the lateral bend specimens and the available test assembly. Especially the length l_1 of the LFBC-model was chosen to 125mm. The further dimensions are given with l_2 = 25mm, b_1= 25mm, b_2 = 45mm and t = 15mm (thickness). The lateral force was chosen to F = F_Q = 1kN with the consequence that M_b = 200Nm.

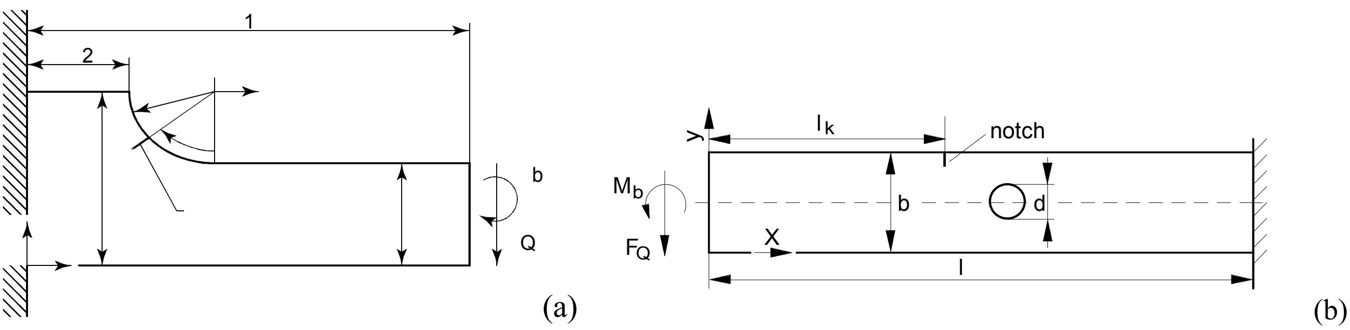
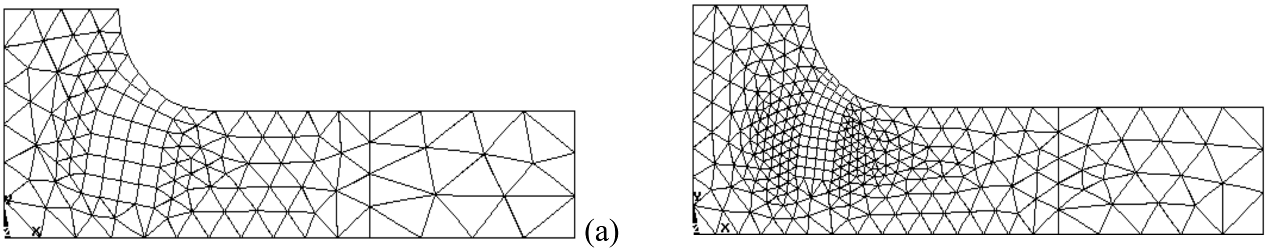


Fig.11. Geometrie and applied boundary conditions of the lateral force bending specimens

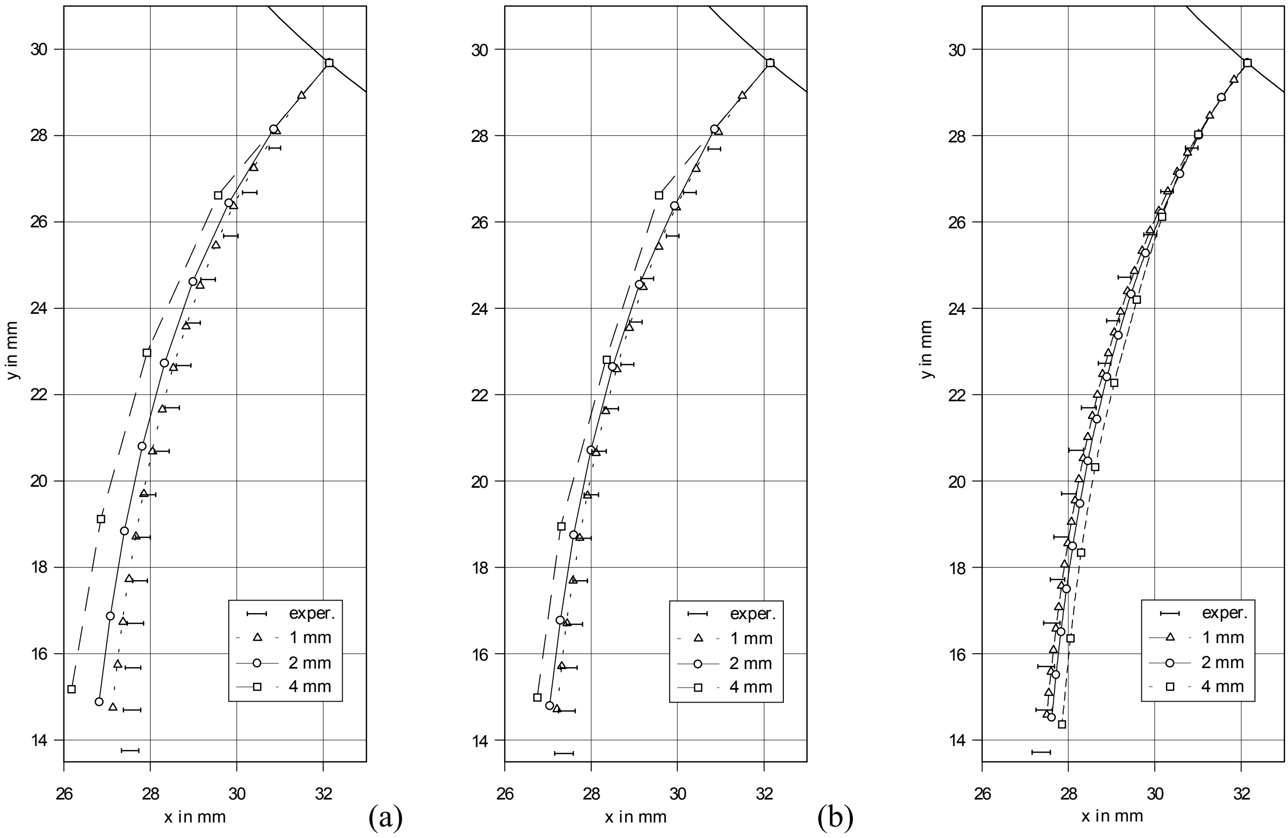
Considering linear elastic material behaviour Young's modules and Poisson's ratio were chosen to E = 2.1 10^5 N/mm^2 and ν = 0.3. In the case of the LFBH-specimen the applied loading was chosen to F_Q = 316N resulting in M_b = 21.5 Nm. Figure 12 shows two selected finite element discretisations of the LFBC-specimen, which were found to be suitable for the proposed simulation strategy.

After each predictor step of virtual tangential crack extension by Δh a remeshing is necessary in such a way, that the corrector step is realised in order to model the curved crack surfaces and that together with the following predictor-step new crack tip elements are generated providing additional nodes. All calculations were carried out with the FE-code ANSYS.



(a) (b)

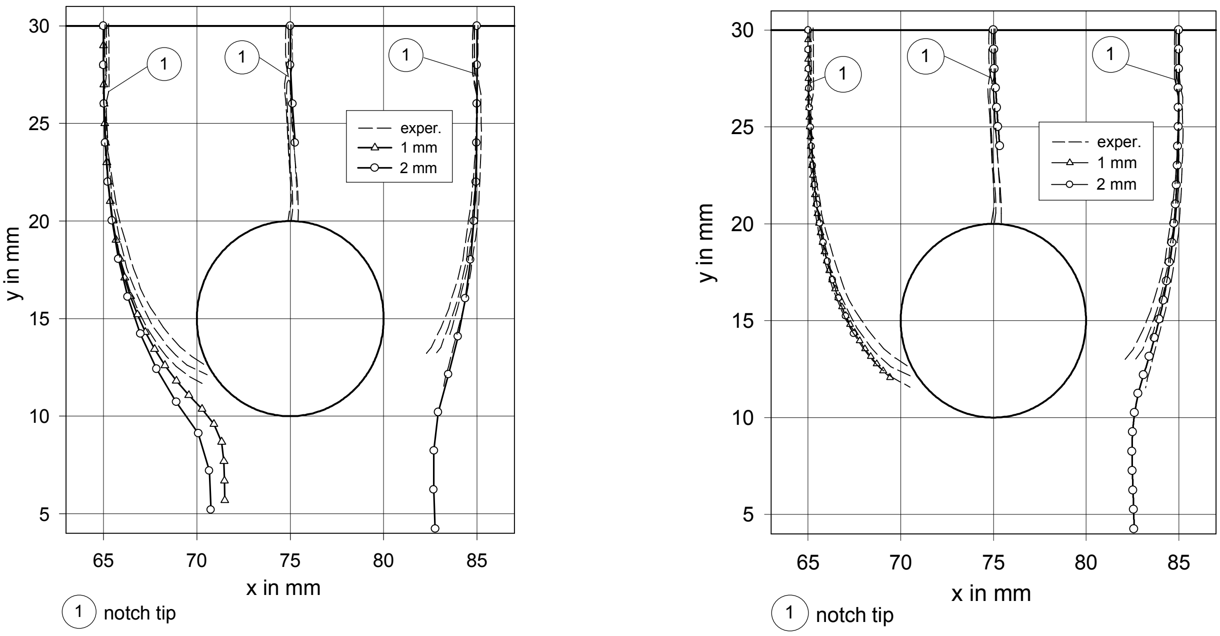
Fig. 12. FE-meshes of the LFBC-specimen with simulated extension steps α_N = 40deg. (a: Δh=4mm,b: Δh=2mm)



(a) (b) (c)

Fig.13 Simulated and experimentally obtained crack paths of the LFBC-specimen with the notch position α_N = 40deg.
(a: mixed mode interpretation; b: straight incremental steps; c: curved incremental steps with marked mid-side nodes)

In Fig. 13 the different computationally simulated crack trajectories for the notch position α_N = 40 deg. of the LFBC-specimen are shown together with the experimentally obtained scatterbands from the related fatigue crack growth experiments. In Fig. 14 the numerical results of the notch position l_K = 65, 75 and 85mm and the experimental findings of the LFBH-specimen are given. For all chosen increments Δh (4mm, 2mm, 1mm) an excellent agreement is found. Additional calculations were carried out without the proposed corrector step (straight increments) to asses the convergence of the new method. Especially in the case of the LFBH-specimen (l_K = 65 mm) the new method results in an accurate evaluation of the final fracture mode of the cracked specimen.



(a) (b)

Fig.14. Simulated and experimentally obtained crack paths of the LFBH-specimen
(a: straight incremental steps; b: curved incremental steps with marked mid-side nodes)

SUMMARY

This investigation has shown that the numerical tool of the MVCCI-method is delivering excellent crack path simulation results with 8-noded quadrilaterals and only moderately refined finite-element-meshes around the crack tip. The step-by-step simulation process with a piece by piece parabolic curved approximation of the crack path offers an efficient way for the numerical analysis of fatigue crack growth in complex two dimensional structures under proportional loading conditions. From the excellent agreement of the numerical and experimental results one can also conclude that the applied criterion of local symmetry works very well. The proposed new predictor-corrector-method in conjunction with the evaluation by the MVCCI-method provides the basis for a general computational approach to the fracture analysis of complex crack configurations and loading conditions.

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