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SENSITIVITY ANALYSIS OF METAL FORMING PROCESSES INVOLVING FRICTIONAL CONTACT IN STEADY STATE

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Resumen

Se presenta un elemento de contacto con fricción para procesos estacionarios de conformado de metales. Conjuntamente se desarrolla la formulación para el análisis de sensibilidad al coeficiente de fricción según la ley de Coulomb. El interés de semejante modelo resulta del análisis de procesos de laminación, corte, etc., donde debe determinarse la zona de contacto existiendo un estado estacionario durante la mayor parte del proceso. La formulación de flujo resulta un método adecuado para modelar eficientemente esta situación. Los elementos de contacto imponen una restricción en la componente de la velocidad normal a la superficie de contacto y una fuerza tangente opuesta a la velocidad. Las partes del contorno que no están en contacto son tratadas como superficies libres, las cuales deben cumplir la condición de ser líneas de corriente. Se presenta el análisis de sensibilidad con respecto al coeficiente de fricción mediante el método de diferenciación directa. Se discute el efecto de variar este parámetro para un caso de extrusión y otro de corte.

Abstract

A simple element to model frictional contact in steady state metal forming processes is presented together with the sensitivity analysis to the friction coefficient in a Coulomb friction law. The interest of such model arises from the analysis of rolling processes and a two dimensional approach to cutting problems, where the contact zone is to be determined, however a stationary state is present in most part of the operation. The flow approach proves to be an adequate method to handle efficiently this situation. The contact elements impose a restriction in the velocity component normal to the boundary and a tangential friction force opposite to the velocity. The parts of the boundary which are not in closed contact are treated as free surfaces, which must fulfill the condition of being streamlines. Sensitivity analysis with respect to the friction coefficient is performed by the Direct Differentiation Method (DDM). The effect of variations in this parameter is discussed for the simulation of an extrusion and a cutting problem.

1. Introduction

Numerical simulation of metal forming processes is a very clear example where the theoretical models can have important practical applications. In the last decades this field has been continuously developed by researchers encouraged by the interest shown by the industry. This development was made possible by the increasing available computing capabilities. For metal forming modeling two main approaches have become classical: the so-call solid, displacement based, approach and the flow

approach based on velocities, formulated in an Eulerian description. This one is best suited for processes involving large displacements but where the material particles follow the same paths, in a stationary state. In that case the whole problem is more simply described by giving the velocity field. Extrusion, rolling, and under certain assumptions, cutting are typical examples of such processes.
In another context, sensitivity analysis has become a widely accepted element for evaluating engineering problems. Applied most traditionally to structural mechanics, the advances and developments have mainly been done in this field. However other engineering problems have been enriched by the knowledge given by the sensitivity analysis and in some case been used successfully as a tool to optimal design. Metal forming analysis is one of the field where most recently these contributions have appeared. There where some first applications of sensitivity analysis to optimal design in metal forming [1] but without use of the concept of sensitivity, at least without calling it in that way. The gradients required in the optimization algorithm where calculated by the so called finite difference approach. Use of sensitivity procedures to evaluate the derivatives of the basic problem variables and of functionals built upon them was proposed for metal forming processes described in terms of the flow approach in [2] and [3], accounting for material parameters, and in [4] to shape parameters.
In this paper a simple element to model frictional contact in steady state metal forming processes is presented. The formulation aims to determine the contact zone and simulate friction between metal and tools in the steady state range of metal forming processes. The interest in such a model comes from the consideration of such processes as plane or tube rolling and a two dimensional approach to cutting. A Coulomb friction law is assumed, and sensitivity analysis to the friction coefficient is provided as a byproduct of the analysis problem.
Since in the quoted processes the stationary state covers a major part of the operation, it is worth taking advantage of this fact in order to avoid time integration, resulting in a simpler solution and computer implementation. The flow approach[5] proves to be an adequate method to handle efficiently this situation with a rigid-viscoplastic material model. A pressure stabilization procedure proposed by Hughes et al. [6] for Newtonian fluids in Stokes flow is extended to non-Newtonian fluids and applied to eliminate spurious pressure modes. Then the pressure can be used both to calculate friction force and to formulate a contact criterion. The contact elements are fixed to the solid boundary and connected to the material.
When contact takes place a restriction on the velocity normal to the solid boundary is imposed via the penalty method, while a friction force opposite to the velocity direction is calculated tangentially to the boundary. If the contact condition is not fulfilled, the contact element stiffness matrix is null. These elements are combined with the explicit treatment of free surfaces which proceeds by integration along a streamline. Analysis of sensitivity to the friction coefficient is carried out by the direct differentiation method. The procedure is illustrated by modeling an extrusion and cutting problems. This paper extends previous results of the authors in sensitivity analysis of metal forming processes [2, 3, 4]
First, the flow approach is briefly recalled and then the sensitivity analysis is developed following the direct differentiation method, and illustrated with an extrusion and cutting process simulations.

2. Flow approach

For a wide class of metal forming operations, it can be observed that a Eulerian approach is best suited to describe them. Besides, the metal under plastic work can be considered as a non-Newtonian fluid and studied by solving the corresponding equation of motion from fluid mechanics. As it results from an nondimensional analysis[7], the dynamic terms in the equation of motion can be neglected. Therefore we can write it in weak form

$$\int_{\Omega} \delta\dot{\boldsymbol{\varepsilon}} \cdot \boldsymbol{\sigma} \, d\Omega = \int_{\Omega} \delta\mathbf{v}\,\mathbf{f} \, d\Omega + \int_{\partial\Omega_t} \delta\mathbf{v}\,\mathbf{t} \, d(\partial\Omega) \tag{1}$$

where

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{2}$$

is the rate of deformation tensor, $\boldsymbol{\sigma}$, $\mathbf{f}$ and $\mathbf{t}$ are the stress tensor, body forces and boundary tractions acting on Ω_t, respectively. Stresses and strain rates are related by the constitutive equation

$$\sigma_{ij} = 2\mu\dot{\varepsilon}_{ij} + p\delta_{ij} = s_{ij}\dot{\varepsilon}_{ij} + p\delta_{ij} \tag{3}$$

where,

$$\mu = \frac{\sigma_0 + (\dot{\bar{\varepsilon}}/\gamma)^{\frac{1}{n}}}{3\dot{\bar{\varepsilon}}} \tag{4}$$

is the equivalent viscosity which is a function of the equivalent strain rate defined as

$$\dot{\bar{\varepsilon}} = \sqrt{\frac{2}{3}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}} \tag{5}$$

Together with equation (1) the incompressibility condition holds

$$\int_\Omega \delta p\, \dot{\varepsilon}_{ii} \mathrm{d}\Omega = 0 \tag{6}$$

After discretization by standard finite element techniques the system (1),(6) reads

$$\begin{bmatrix} \mathbf{K}_{(\mu)} & \mathbf{K}^T_{(p)} \\ \mathbf{K}_{(p)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}} \\ \bar{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{Q} \\ \mathbf{0} \end{bmatrix} \tag{7}$$

with

$$\begin{aligned} \mathbf{K}_{(\mu)} &= \int_\Omega \mu\, \mathbf{k}_0 \mathrm{d}\Omega = \int_\Omega 2\mu\, \mathbf{B}^T\mathbf{B}\mathrm{d}\Omega \\ \mathbf{K}^T_{(p)} &= -\int_\Omega \mathbf{B}^T\mathbf{m}\,\bar{\mathbf{B}}\mathrm{d}\Omega \\ \mathbf{Q} &= \int_\Omega \mathbf{N}\hat{\mathbf{f}}\mathrm{d}\Omega + \int_{\partial\Omega_t} \mathbf{N}\hat{\mathbf{t}}\,\mathrm{d}(\partial\Omega_t) + \mathbf{F} \end{aligned} \tag{8}$$

the vector $\mathbf{m} = [1\ 1\ 1\ 0\ 0\ 0]$ converting the total strain into the volumetric component, $\mathbf{B}$ denoting the strain-rate-velocity matrix $\dot{\boldsymbol{\varepsilon}} = \mathbf{B}\dot{\mathbf{q}}$ and $\bar{\mathbf{B}}$ the shape functions interpolating the pressure.
We can write (7) shorter, in the residual form

$$\mathbf{R} = \bar{\mathbf{K}}_{(\mu)}\bar{\mathbf{q}} - \bar{\mathbf{Q}} = \mathbf{0} \tag{9}$$

Equation (7) corresponds to the equations describing the nonlinear Stokes flow. By defining the residual at the i-th iteration as

$$\mathbf{R}^{(i)} = \bar{\mathbf{K}}^{(i)}_{(\mu)}\bar{\mathbf{q}}^{(i)} - \bar{\mathbf{Q}} \qquad , i = 1, 2, \ldots \tag{10}$$

we obtain the solution correction $\delta\bar{\mathbf{q}}$ from setting equal $\mathbf{0}$ the residual at the $i+1$-th iteration, which is approximated by the first order Taylor expansion

$$\mathbf{R}^{(i+1)} \cong \mathbf{R}^{(i)} + \frac{\partial \mathbf{R}^{(i)}}{\partial \bar{\mathbf{q}}}\delta\bar{\mathbf{q}} = \mathbf{0} \tag{11}$$

from which the solution correction is computed as

$$\begin{aligned} \delta\bar{\mathbf{q}}^{(i+1)} &= -\left(\frac{\partial \mathbf{R}^{(i)}}{\partial \bar{\mathbf{q}}}\right)^{-1}\mathbf{R}^{(i)} \\ \bar{\mathbf{q}}^{(i+1)} &= \bar{\mathbf{q}}^{(i)} + \delta\bar{\mathbf{q}}^{(i+1)} \end{aligned} \tag{12}$$

where

$$
\begin{aligned}
\frac{\partial \mathbf{R}^{(i)}}{\partial \bar{\mathbf{q}}} &= \bar{\mathbf{K}}_{(\mu)} + \int_\Omega \left(\bar{\mathbf{k}}_0 \bar{\mathbf{q}}\right) \frac{\partial \mu}{\partial \bar{\mathbf{q}}} d\Omega = \mathbf{K}^t_{(\mu)} \\
\bar{\mathbf{k}}_0 &= \begin{bmatrix} \mathbf{k}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}
\end{aligned}
\tag{13}
$$

is the (non–symmetric) tangent stiffness matrix while

$$
\frac{\partial \mu}{\partial \dot{q}_\alpha} = \frac{\partial \mu}{\partial \dot{\bar{\varepsilon}}} \frac{\partial \dot{\bar{\varepsilon}}}{\partial \dot{\varepsilon}_{ij}} \frac{\partial \dot{\varepsilon}_{ij}}{\partial \dot{q}_\alpha} = -\left[\sigma_0 + \left(1 - \frac{1}{n}\right)\left(\frac{\dot{\bar{\varepsilon}}}{\gamma\sqrt{3}}\right)^{\frac{1}{n}}\right] \frac{2\dot{\varepsilon}_{ij}}{\sqrt{3}\dot{\bar{\varepsilon}}^3} B_{ij\alpha}
\tag{14}
$$

$$
\frac{\partial \mu}{\partial \bar{p}} = 0
\tag{15}
$$

2.1 Frictional contact

The contact problem for steady state flow somewhat differs from the more traditional contact as considered for transient, displacement based situations. In fact, boundary conditions have to be adjusted in order to have no traction on the boundaries in contact with solid surfaces. This condition allows the definition of a criterion for determining the point where the material separates from (or enters in contact with) the solid boundary. Our analysis, with respect to the flow direction, deals rather with the separation point, but it can be directly extended to the point where the free surface meets the solid boundary, if such is the case.

Along the surface which may be either free or in contact with the solid boundary, normal velocities are constrained to zero if the flow tries to get through the solid boundary, and they are set free if a traction exerted on the flow by the wall is observed. That is, a fixed node of the contact surface is released if the pressure is negative, and a free node is fixed if it has penetrating normal velocity. The changes are performed one node at a time, on the neighbourhood of the present limit of the contact zone, keeping the rest in the previous state. In this way, the separation point is adjusted until the free boundary starting from it becomes a streamline.

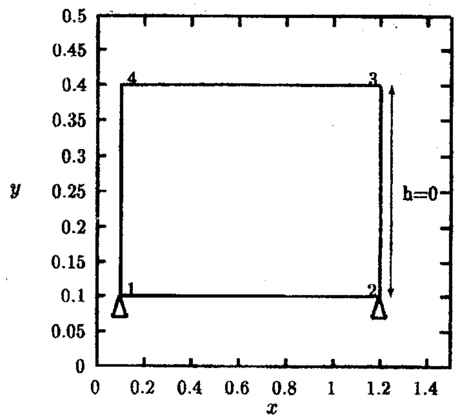
It is worth to point out that points on the free part of the surface may have a normal component of the velocity directed towards the solid boundary. However, this does not mean necessarily that the flow gets through the wall.

The imposition of optional restrictions on the velocity component normal to the wall is achieved by four noded contact elements which connect two nodes of the material with two auxiliary nodes fixed to the solid boundary (velocities and pressure are constrained to zero). At the beginning, both sides are coincident. If the element belongs to the free surface, the element matrix is null, and no restriction is imposed to the degrees of freedom of the fluid. But whenever the contact condition is satisfied, the normal velocity is constrained for the nodes in the fluid by equating them to their counterpart on the solid boundary (which is equal to zero) via a penalty parameter. According to the element numbering given in figure 1, the equations

$$
\alpha(v_1 - v_4) = 0 \qquad \alpha(v_2 - v_3) = 0
\tag{16}
$$

are added to the global system in the respective equations for v_3 and v_4 through the element matrix, being α the penalty parameter. Since v_1 and v_2 are set to zero, the fluid normal velocities v_3 and v_4 will also be zero. In figure 1 the nodal velocities are given in the local coordinate system. If the local axes are not coincident with the global ones, then the equations (16) are transformed and the penalty parameter will act in two equations of the global system for each node in order to assure zero velocity across the boundary.

Then, upon integration of the free surface, no change will result in the nodes defining the surface whenever the normal velocity is zero. This may happen either because the free surface is already a streamline, or because it is in contact with the solid boundary. On the other hand, a different updated configuration will be obtained if there is a non-zero normal velocity component.



$$k_{44}=k_{11}=-k_{41}=-k_{14}=\begin{cases}\alpha & \text{if (pressure in 4} > 0) \\ 0 & \text{otherwise}\end{cases}$$

$$k_{33}=k_{22}=-k_{32}=-k_{23}=\begin{cases}\alpha & \text{if (pressure in 3} > 0) \\ 0 & \text{otherwise}\end{cases}$$

Figure 1: Contact elements.

With this same contact elements friction conditions can be imposed by means of the force vector. The part of the boundary where the contact condition is active receives a distributed tangential friction force which is proportional to the pressure and opposite in direction to the velocity, according to the Coulomb friction law. (Other friction laws may also be assumed and easily implemented regarding all the information available at the element level). Then if p_3 and p_4 are, respectively, the pressures at nodes 3 and 4 (see figure 1), then the distributed friction force along the element side $\overline{34}$ will be

$$\mathbf{f} = -\frac{\mathbf{v}}{|\mathbf{v}|}(p_3\psi + p_4(1-\psi))\nu \tag{17}$$

where $\psi = (x - x_4)/(x_3 - x_4)$ and $x_4 \leq x \leq x_3$ is the local coordinate of side $\overline{34}$, $0 \leq \psi \leq 1$ and ν is the Coulomb friction coefficient. This gives the equivalent nodal friction forces

$$\begin{aligned} \mathbf{f}_3 &= \frac{\mathbf{v}}{|\mathbf{v}|}\nu\left(\frac{p_3}{3}+\frac{p_4}{6}\right) &= \frac{\mathbf{v}}{|\mathbf{v}|}\hat{f}_3 \\ \mathbf{f}_4 &= \frac{\mathbf{v}}{|\mathbf{v}|}\nu\left(\frac{p_3}{6}+\frac{p_4}{3}\right) &= \frac{\mathbf{v}}{|\mathbf{v}|}\hat{f}_4 \end{aligned} \tag{18}$$

However, the actual friction forces will be bounded by the no-compression case (where the friction force will be zero) and the sticking case, which gives the maximum (in absolute value) allowable friction force, $\mathbf{f}_i^s$. Then the actual module of the friction force will be

$$|\mathbf{f}_i| := \begin{cases} |\mathbf{f}_i^s| & \text{for } \hat{f}_i > |\mathbf{f}_i^s| \\ \hat{f}_i & \text{for } 0 < \hat{f}_i < |\mathbf{f}_i^s| \\ 0 & \text{for } \hat{f}_i < 0 \end{cases} \tag{19}$$

3. Sensitivity analysis

We consider now a general form of the sensitivity functional as

$$\phi = \phi[\mathbf{s}, \bar{\mathbf{q}}; h] \tag{20}$$

We wish to calculate the sensitivity of ϕ with respect to a material parameter h entering the theory. As h we may take any of the parameters of the constitutive equation (3)-(4) and in this case we take the friction coefficient ν.

Following the so-called direct differentiation method (DDM) we calculate the gradient

$$\frac{\mathrm{d}\phi}{\mathrm{d}h} = \frac{\partial\phi}{\partial h} + \frac{\partial\phi}{\partial \mathbf{s}}\frac{\mathrm{d}\mathbf{s}}{\mathrm{d}h} + \frac{\partial\phi}{\partial\bar{\mathbf{q}}}\frac{\mathrm{d}\bar{\mathbf{q}}}{\mathrm{d}h} \tag{21}$$

in which, given the solution of the equilibrium problem, $\partial\phi/\partial h$, $\partial\phi/\partial\bar{\mathbf{q}}$ and $\partial\phi/\partial\mathbf{s}$ are known, or can be routinely obtained, and $\mathrm{d}\mathbf{s}/\mathrm{d}h$ can be written in an easily computable way as

$$\frac{\mathrm{d}\mathbf{s}}{\mathrm{d}h} = \frac{\mathrm{d}\mathbf{s}}{\mathrm{d}\bar{\mathbf{q}}}\frac{\mathrm{d}\bar{\mathbf{q}}}{\mathrm{d}h} \tag{22}$$

where $\frac{\mathrm{d}\mathbf{s}}{\mathrm{d}\bar{\mathbf{q}}} = \left\{\frac{\mathrm{d}\mathbf{s}}{\mathrm{d}\dot{\mathbf{q}}}\frac{\mathrm{d}\mathbf{s}}{\mathrm{d}\bar{\mathbf{p}}}\right\}$, and, according to equations (3), (7) and (8)

$$\begin{aligned} \frac{\mathrm{d}\mathbf{s}}{\mathrm{d}\dot{\mathbf{q}}} &= 2\mu\mathbf{B} + 2\frac{\partial\mu}{\partial\dot{\mathbf{q}}}\mathbf{B}\dot{\mathbf{q}} \\ \frac{\mathrm{d}\mathbf{s}}{\mathrm{d}\bar{\mathbf{p}}} &= \mathbf{0} \end{aligned} \tag{23}$$

Therefore, in order to compute $\mathrm{d}\phi/\mathrm{d}h$ by equation (21) only $\mathrm{d}\mathbf{q}/\mathrm{d}h$ has to be obtained from additional calculations. By differentiating the equilibrium equation (1) with respect to h we have

$$\frac{\mathrm{d}}{\mathrm{d}h}\left(\bar{\mathbf{K}}_{(\mu)}\bar{\mathbf{q}} - \bar{\mathbf{Q}}\right) = \frac{\mathrm{d}\bar{\mathbf{K}}_{(\mu)}}{\mathrm{d}h}\bar{\mathbf{q}} + \bar{\mathbf{K}}_{(\mu)}\frac{\mathrm{d}\bar{\mathbf{q}}}{\mathrm{d}h} - \frac{\mathrm{d}\bar{\mathbf{Q}}}{\mathrm{d}h} = \mathbf{0} \tag{24}$$

where

$$\frac{\mathrm{d}\bar{\mathbf{K}}_{(\mu)}}{\mathrm{d}h} = \int_\Omega \frac{\partial\mu}{\partial h}\bar{\mathbf{k}}_0\mathrm{d}\Omega + \int_\Omega \left(\frac{\partial\mu}{\partial\bar{\mathbf{q}}}\frac{\mathrm{d}\bar{\mathbf{q}}}{\mathrm{d}h}\right)\bar{\mathbf{k}}_0\mathrm{d}\Omega \tag{25}$$

By combining equations (24) and (25) we arrive at

$$\left(\bar{\mathbf{K}}_{(\mu)} + \int_\Omega \left(\bar{\mathbf{k}}_0\bar{\mathbf{q}}\right)\frac{\partial\mu}{\partial\bar{\mathbf{q}}}\mathrm{d}\Omega\right)\frac{\mathrm{d}\bar{\mathbf{q}}}{\mathrm{d}h} = -\left(\int_\Omega \frac{\partial\mu}{\partial h}\bar{\mathbf{k}}_0\mathrm{d}\Omega\right)\bar{\mathbf{q}} \tag{26}$$

i.e., at

$$\mathbf{K}^t_{(\mu)}\frac{\mathrm{d}\bar{\mathbf{q}}}{\mathrm{d}h} = -\left(\int_\Omega \frac{\partial\mu}{\partial h}\bar{\mathbf{k}}_0\mathrm{d}\Omega\right)\bar{\mathbf{q}} \tag{27}$$

This equation can be used for finding the nodal velocity and pressure design gradients $\frac{\mathrm{d}\bar{\mathbf{q}}}{\mathrm{d}h}$ provided the nodal velocity and pressure vector $\bar{\mathbf{q}}$ has been solved for from the equilibrium problem.
For the case of friction modeled by the contact-friction elements, the sensitivity to the friction coefficient is now calculated by simply derivating the friction force, equation (19), with respect to ν

$$\frac{\partial\mathbf{f}_i}{\partial\nu} = \begin{cases} 0 & \text{for } \hat{f}_i > |\mathbf{f}_i^s| \\ \frac{\mathbf{v}}{|\mathbf{v}|}\hat{f}_i & \text{for } 0 < \hat{f}_i < |\mathbf{f}_i^s| \\ 0 & \text{for } \hat{f}_i < 0 \end{cases} \tag{28}$$

It is worth pointing out that for the sticking case these derivatives are zero, that is the solution is not sensitive to variations of the friction coefficient.

4. Numerical results

4.1 Direct extrusion

The sensitivity analysis with respect to the friction coefficient is shown for an extrusion process, where the horizontal part of the matrix has been covered by frictional contact elements. Figure 2 shows a

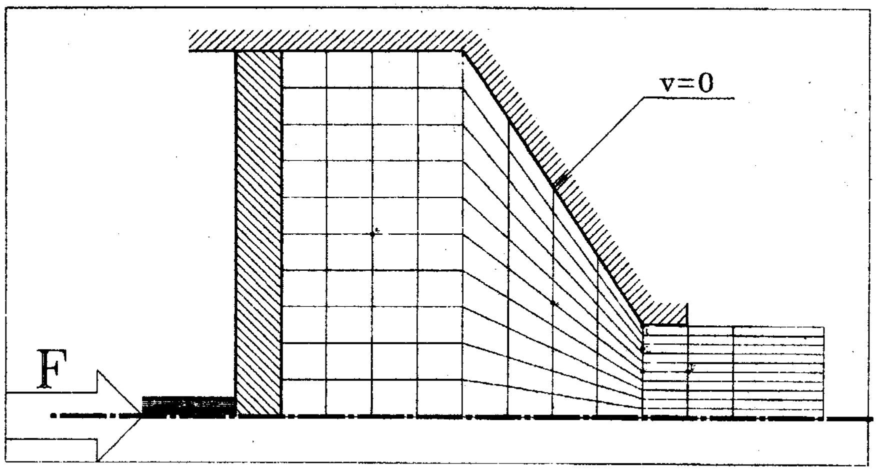
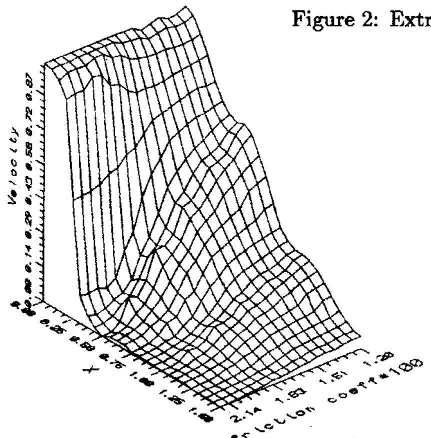


Figure 2: Extrusion process - layout



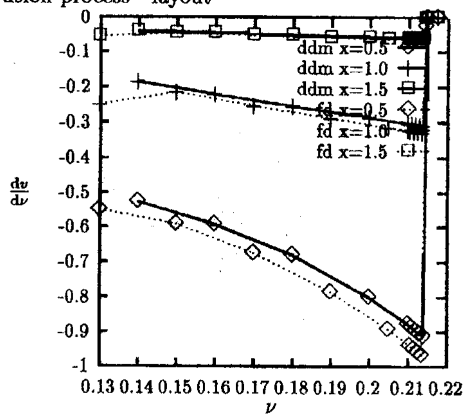


Figure 3: Velocity (left) and its sensitivity w.r.t. the friction coefficient ν (right) on the die boundary.

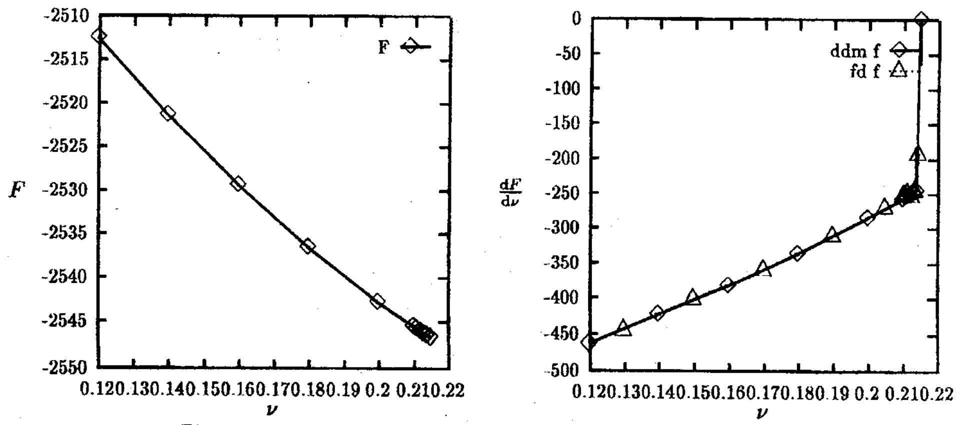


Figure 4: Extrusion force and sensitivity *vs* friction coefficient

scheme of the discretized model. Since during the process all the points inside the extrusion matrix are at compression, the contact condition will be active for all the contact elements. A constant ram velocity is imposed and the effect of changing the friction coefficient is studied. The boundary conditions obtained by variation of the friction coefficient range between the pure slip (no friction) to the sticking (zero velocity) condition. Figure 3*a* shows the velocity along the matrix boundary for different values of the friction coefficient. It can be seen that there is a transition zone between the velocity imposed by the ram and the dead zone. For increasing values of the friction coefficient, the velocity at the boundary decreases to zero in a shorter space interval. The sensitivities of the velocity at the boundary are plotted in figure 3*b*, where results obtained by DDM (full line) are compared to those of the central finite difference approach (dotted line). It can be seen that they match very well. From the 3-D plot of figure 3 we see that the variations in terms of the friction coefficient are quite irregular, therefore we can expect that relatively small perturbations will have to be taken in order to have a good approximation with finite differences. In fact, for the finite difference verification we have taken variations of the friction coefficient of 0.0002. Finally we show in figure 4 the plot of the extrusion force in terms of the friction coefficient and the sensitivities obtained by both methods and the same lines convention as in figure 3*b*.

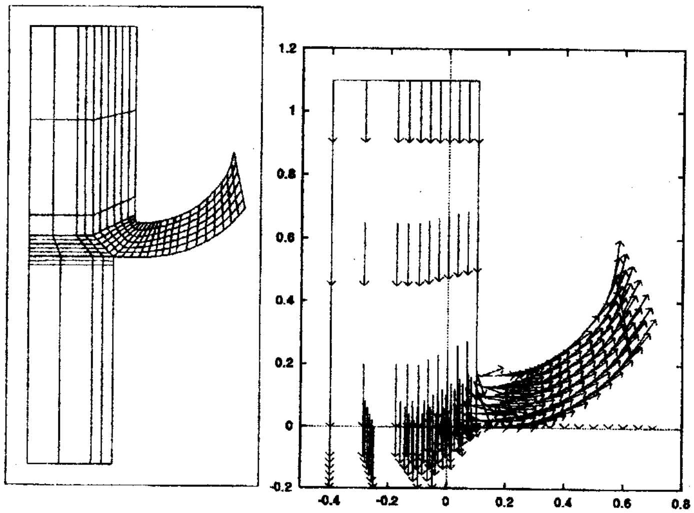


Figure 5: Cutting problem: final mesh (left) and velocity field (right).

4.2 2-D cutting

A cutting problem may be modeled by assuming a semi-infinite flow meeting a rigid tool wich causes the separation of a thin chip from the main flow. There is a small region where all the plastic work is concentrated. The flow is uniform in the rest of the domain. The most critical part of the problem is the chip, which has two free surfaces, one of them starts from the tool, from which it separates after some distance from the edge. The other free surface exhibits a sharp angle as a consequence of the shear deformation band runing from the tool edge to the corner of the surface, which in the ideally plastic case it is a discontinuity line. The chip shape is highly dependent on the friction between the metal flow and the tool. The final mesh is shown in figure 5a, where the sharp change of direction of one of the free surfaces can be seen, typical of perfectly (or almost) plastic materials, which is

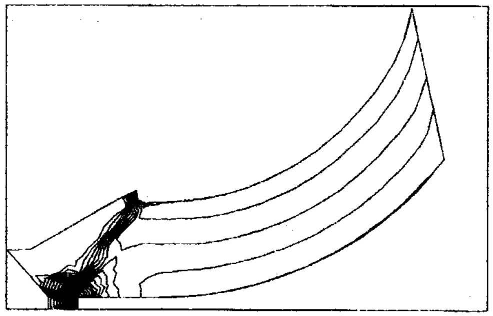


Figure 6: Cutting problem - velocity module

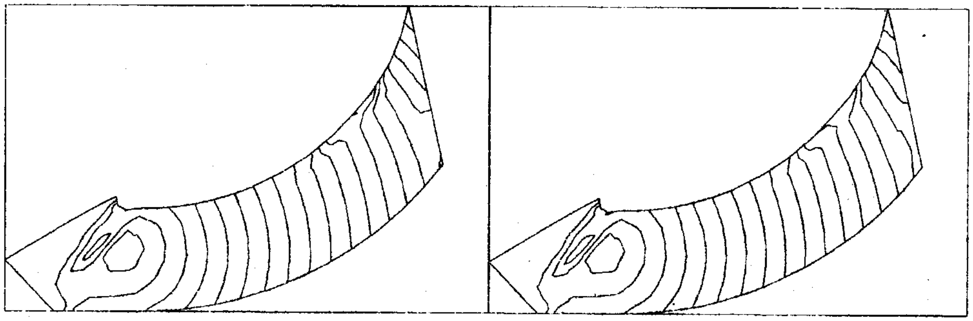


Figure 7: Cutting problem - module of the velocity sensitivity to the friction coefficient, $|dv/d\nu|$: DDM (right) and Finite differences (left)

the present case, with a inverse exponent of the visco-plastic law (4) of $n = 40$. This final mesh has been obtained after starting the calculation with a smooth shape for the corner. The velocity field is plotted in figure 5b, while figures 6 and 7 show the velocity module contours for the chip and the sensitivities calculated by DDM and finite differences, respectively. Apart from the agreement between both contour maps, we can point out that while the velocity gradient is directed across the chip, the gradient of the velocity sensitivity runs along the chip and it grows in the direction of the flow. This is a logical result since the chip tends to have a circular movement (so the velocity gradient is radial) but the longer the chip, the more unstable and sensible to any change it is.

5. Conclusions

The presented contact element allows the friction treatment with no additional degrees of freedom and the contact handling for steady state problems. As a byproduct, sensitivity analysis can be carried out and, for the analyzed problems, helps to indicate the effect of changing the frictional boundary conditions. From a practical point of view this can be a useful information when a decision is required about the lubricant to use in a given metal forming process.

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