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OPTIMAL DESIGN BY SENSITIVITY ANALYSIS IN METAL FORMING

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Resumen

Se presenta un algoritmo para determinar la forma óptima en procesos de conformado de metales con respecto a un determinado criterio, de modo de satisfacer distintas restricciones de diseño. Se estudian procesos de conformado de metales descriptos en términos de la formulación de flujo. Para ellos se desarrolla el análisis de sensibilidad a la forma. mediante el método de la configuración de referencia, en el cual se han tomado como variables de diseño las coordenadas de algunos nodos que sirven para definir la forma de la herramienta. El llamado método de diferenciación directa es aplicado para formular el problema de sensitividades. A partir de los resultados del ambos análisis (de equilibrio y de sensibilidad) se evalúa un funcional de costo y su gradiente en el espacio de diseño. Estos valores constituyen los datos de entrada para cada interación del algoritmo de optimización mediante el cual se obtienen el conjunto de valores de las variables que minimizan el funcional, de lo que resulta la forma óptima según el criterio adoptado. Se consideran el método de gradientes conjudados y el algoritmo de Schittkowski y se comparan ambos resultados. El análisis se realiza para procesos estacionarios. La técnica es ilustrada calculando el diseño óptimo de una matriz de extrusión con respecto a distintos criterios.

Abstract

An optimization algorithm is presented to meet various conditions during the design of a given metal forming operation. Metal forming processes described in terms of the flow approach are considered. Shape sensitivity analysis is obtained by the control volume approach, where the design variables are some selected nodal coordinates used to defined the tool shape. The so called direct differentiation method is followed to formulate the sensitivity problem. The results from the equilibrium and the sensitivity problems are used to evaluate a cost functional and its gradient in the design space. These values are the input for each iteration of the optimization algorithm, from which the new values of the designed variables are obtained, which minimize the functional, resulting in the optimal shape with respect to the adopted optimization criterion. The conjugate gradient method and Schittkowski's algorithm are used and compared. Steady state processes are considered. The procedure is illustrated by calculating the optimal design of an extrusion matrix with respect to different criteria.

1. Introduction

Industrial metal forming operations usually require expensive equipements and big amounts of energy. One of the criteria to evaluate the efficiency of a given process can be stated by its capability to

produce the desired piece regarding shape and mechanical properties with the least energy usage. Computer modelling of these problems has an obvious interest since it can lead to better designs and cost reductions. In addition, metal forming operations have attracted the attention of researchers because complex material behaviour and geometric configurations take place. More accurate constitutive models have arised and been implemented in numerical codes which are very useful tools for analysis and avoid the construction of prototypes. However, classical analysis does not tell the designer which are the key problem variables neither how design changes affect the process. Any improvement in this respect depends on the designer's experience. Nevertheless, in structural analysis, similar questions have been successfully answered by the design sensitivity analysis (DSA), which gives the gradient of state variables and/or response functionals in the design variables space. The main goal of DSA results when analytical (as opposed to finite difference) methods are applied for calculating the design derivatives. In this case sensitivities are obtained by solving a linear problem with the same system matrix as the tangent stiffness matrix already calculated and inverted for the equilibrium problem. Therefore the sensitivities result at a very low computational cost as compared with the equilibrium problem. On the other hand, by the so-called finite difference approach for sensitivity analysis at least twice the original computational effort must be employed to obtain the numerical derivatives for each design variable, and additionally it is always an unsolved problem which should be the design increment size in order to have low truncation and approximation errors. It is commonly stated that optimal design is the natural following step after DSA. In fact, although the information produced by DSA is already very important in itself, it is clear that further advantage can be taken from it. An optimization problem can then be defined in which a functional subject to design constraints is to be minimized. In another context, such an optimization has been attempted in [2] but the approach used there does not make use of sensitivity techniques. Moreover, it used a non-gradient optimization algorithm.
The value of the functional and its gradient at each design configuration is used to find the optimum design. Recently DSA has been extended to metal forming processes both to parameter and to shape problems[1, 3]. In this paper these results are used to find the optimum design of a given process according to an adopted optimization criterion.

2. Analysis problem

We consider metal forming processes modeled by the finite element method in terms of the flow approach [4, 5, 6]. Velocities and pressures are obtained by solving the equilibrium equation in its weak form

$$\int_{\Omega} \boldsymbol{\sigma}^{T} \delta\dot{\boldsymbol{\varepsilon}}\ d\Omega = \int_{\Omega} \mathbf{f}^{T}\, \delta\mathbf{v}\ d\Omega + \int_{\partial\Omega_t} \mathbf{t}^{T}\, \delta\mathbf{v}\ d(\partial\Omega) \tag{1}$$

together with the incompressibility condition in which the nodal pressures act as Lagrange multipliers. The metal is modeled as a viscoplastic (rate hardening) material described by the constitutive equation

$$\boldsymbol{\sigma} = 2\mu\dot{\boldsymbol{\varepsilon}} + p\mathbf{I}_{(p)} \quad ; \quad \mu = \frac{\sigma_0 + (\dot{\bar{\varepsilon}}/\gamma)^{\frac{1}{n}}}{3\dot{\bar{\varepsilon}}} \tag{2}$$

where $\boldsymbol{\sigma}$ is the stress tensor, $\dot{\boldsymbol{\varepsilon}}$ the strain rate tensor, p the pressure, $\mathbf{I}_{(p)}$ the unit tensor and μ the equivalent viscosity.
By standard finite element patterns the equations (1) and the incompressibility condition are discretized and a non-linear system of equations is obtained. The velocity and pressure field is obtained after solving the system by iterations: either by so-called back substitution, in which case the (secant) stiffness matrix is used, or by the Newton Raphson method, where the tangent stiffness matrix is needed. The specific choice may be given by the viscous character of the material being modeled and by the way the loads are applied: constant or by imposition of constant velocities.
The flow approach in its "classical" version has been extended with the treatment of free surfaces, friction with contact and particular techniques are also included to eliminate some inherent numerical

problems, e.g. matrix scaling and spurious pressure mode elimination. Details about these features can be found in the already quoted papers.

3. Shape sensitivity analysis and optimization algorithm

Given a generic response functional

$$\Psi = \int_{\Omega} G(\boldsymbol{\sigma}, \dot{\boldsymbol{\varepsilon}}, \mathbf{v}, b)\ \mathrm{d}\Omega + \int_{\partial\Omega_v} g(\mathbf{v}_F, \mathbf{t}, b)\ \mathrm{d}(\partial\Omega_v) + \int_{\partial\Omega_t} h(\mathbf{v}, \mathbf{t}_F, b)\ \mathrm{d}(\partial\Omega_t) \tag{3}$$

we are interested in finding its sensitivity to a given set of design variables, which in our case will define the problem geometry. In fact, this means to find the response functional gradient in the design space. The two more popular methods for sensitivity analysis are the Adjoint system method (ASM) and the Direct differentiation method (DDM). Both have been applied to parameter sensitivity analysis of metal forming processes in a flow approach context in [1]. The former method builds an equivalent fictitious structure where the all the "adjoint" quantities have the meaning of sensitivities. The latter finds the design derivative of (3) by intermediate calculation of the equilibrium problem variables. In both cases the tangent stiffness matrix of the equilibrium problem is obtained as the matrix (or its transposed, in ASM) for the sensitivity problem. Therefore the sensitivity solution results as a byproduct of the equilibrium solution, usually as a linear problem.

Further, shape sensitivity analysis involves the definition of how all the problem quantities depend on the design variables, which in this case are coordinates or in general geometric entities defining the problem shape. A survey and comparison of different methods for shape sensitivity analysis for linear structural analysis can be found in [7]. The extension to metal forming processes described in terms of the flow formulation is presented in [3] by application of the direct differentiation method to the continuous expresion of the equilibrium equation (1). Derivatives with respect to geometric variables result by considering, according to the Domain parametrization method, a fixed reference configuration and design-dependent mappings to every actual configuration. By taking design variations from (1) we get

$$\begin{aligned}\int_{\Omega} \tilde{\delta}\boldsymbol{\sigma}^T \delta\dot{\boldsymbol{\varepsilon}}\, J\ \mathrm{d}\Omega_0 = &\int_{\Omega} \partial(\mathbf{f}^T J)\delta\mathbf{v}\ \mathrm{d}\Omega_0 + \int_{\partial\Omega} \partial(\mathbf{t}^T \partial J)\delta\mathbf{v}\ \mathrm{d}(\partial\Omega_0)\\ &- \int_{\Omega} [\partial\boldsymbol{\sigma}^T \delta\dot{\boldsymbol{\varepsilon}}\, J + \boldsymbol{\sigma}^T \partial(\delta\dot{\boldsymbol{\varepsilon}}) J + \boldsymbol{\sigma}^T \delta\dot{\boldsymbol{\varepsilon}}\, \partial J]\ \mathrm{d}\Omega_0\end{aligned} \tag{4}$$

and a similarly obtained expression for the incompressibility condition. Both equations give, after discretization, the same tangent matrix as the analysis problem. The quantities $\tilde{\delta}x$ and ∂x denote, respectively, implicit and explicit variations of x with respect to the design variables. From the state variable sensitivities the sensitivity of the response functional (3) is obtained.

The preceding problem is solved at each design iteration of an optimization algorithm which minimizes (3) with respect to the design variables. Two optimization algorithms have been employed: the first one is based on the conjugate gradient method[8] and the second is the Schittkowski's algorithm[9].

The optimization problem can be formulated as follows

Optimization problem:

Minimize the functional Ψ

subject to

$$\Upsilon_i = x_{i+1} - x_i - f(y_i, y_{i+1}, y_{0i}) \geq 0$$

for $i = 1, NC$ where NC is the number of constraints.

4. Numerical illustration

The procedure is illustrated by finding the optimum design of an extrusion matrix, where the extrusion ratio is fixed and the design variables are some nodal coordinates defining the matrix shape.

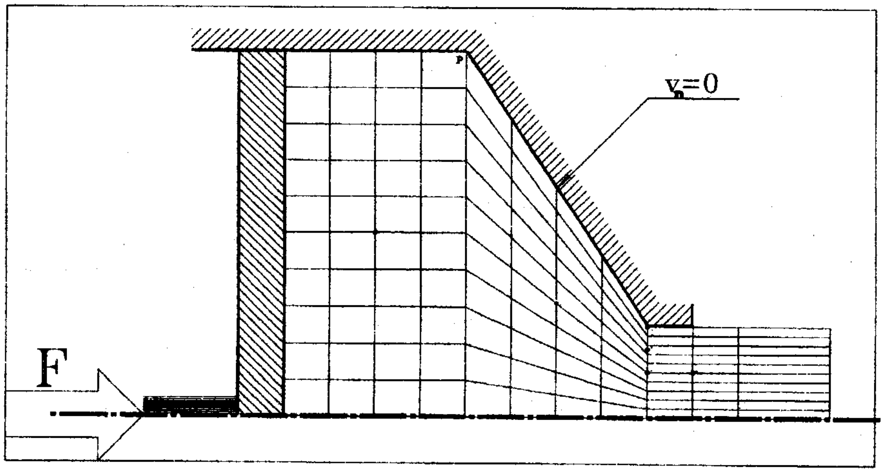

Figure 1: Layout of the extrusion process

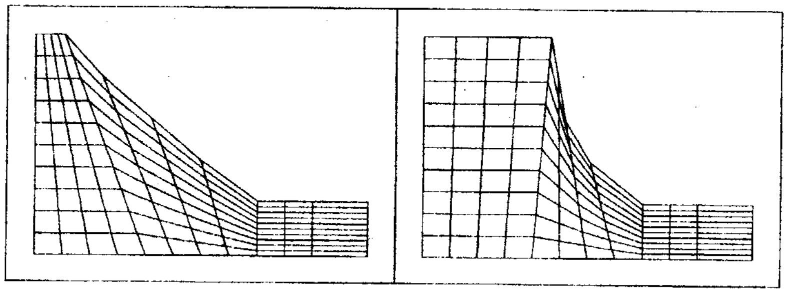

Figure 2: Optimal shape – Criteria: energy rate (left) and maximum local energy rate (right)

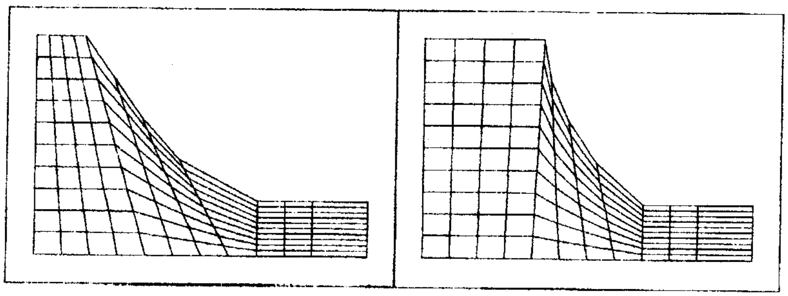

Figure 3: Optimal shape – Criteria: averaged effective strain rate deviation (left) and maximum local effective strain rate (right)

Optimization criteria of minimizing the deformation energy, the maximum strain rate and uniform rate of deformation are investigated.
Fig. 1 shows the scheme of an extrusion problem with its discretization into finite elements. A constant velocity is imposed on the left boundary (so the extrusion force is design variable) and no friction boundary conditions have been assumed. By the flow approach we obtain the velocity, strain and stress solution. Further, for any given configuration the shape sensitivities may be obtained by application of the procedure outlined in Section 3 for appropriately defined shape parameters. Shape sensitivity analysis of such a problem with respect to the die angle has been presented in [3]. On that occasion the die profile was supposed to be a straight line so that only one design parameter (the horizontal coordinate of the upper right die corner) was enough to define any geometry variation. Therefore the discretized domain was divided into only three macroelements (two rectangular and one trapezoidal) in order to calculate the dX_i/dh derivatives through the mapping $X_i = X_i(\xi_j, h)$, where ξ_j are the local coordinates at the (fixed) reference configuration.
Now we want to find the optimal die profile with respect to certain response functionals which shall be defined later on. In order to allow a maximum of degrees of freedom for the die contour we take this time –instead of one trapezoidal design macro-element for the conical zone– as many as segments the die profile will have. This number is limited by the number of nodes in that part of the boundary. Now we consider the response functionals we want to optimize. In each case we will get as the optimization result the set of coordinates defining the "optimal" die. The following functionals have been investigated.

4.1 Proposed Functionals

Energy rate

$$\Psi_E = \int_\Omega \sigma_{ij}\dot{\varepsilon}_{ij}\, d\Omega$$

Maximum local energy rate

$$\Psi_e = \max_\Omega\left(\frac{1}{2}\sigma_{ij}\dot{\varepsilon}_{ij}\right)$$

Averaged effective strain rate deviation

$$\Psi_{D\dot{\bar{\varepsilon}}} = \frac{1}{\Omega}\int_\Omega (\dot{\bar{\varepsilon}} - \dot{\bar{\varepsilon}}^*)^2\, d\Omega \quad \text{where} \quad \dot{\bar{\varepsilon}}^* = \frac{1}{\Omega}\int_\Omega \dot{\bar{\varepsilon}}\, d\Omega$$

Maximum effective strain rate

$$\Psi_{\dot{\bar{\varepsilon}}_l} = \max_\Omega(\dot{\bar{\varepsilon}})$$

Overall distorsion rate

$$\Psi_D = \frac{1}{2}\int_\Omega \dot{\varepsilon}_{12}\, d\Omega$$

Maximum distorsion rate

$$\Psi_{\dot{\varepsilon}_{12}} = \max_\Omega(\dot{\varepsilon}_{12})$$

which are optimized by the corresponding die profiles shown in figures 2 to 4.
Table 1 shows the reduction in all of the considered functionals, achieved after optimization with respect to the x coordinate of the four nodes laying on the conical part of the die. We can see two different families of optimal shapes obeying to the functional character: local or global. For the specific case of the overall deformation energy (the same conclusion can be drawn about others) the global functional presents a practically straight optimal die profile while the corresponding local criterion (i.e. to minimize the maximum local plastic deformation energy rate) exhibits a strongly concave profile. It is interesting to notice, however, that both give approximately the same angle between the die and the material outlet, which is the most critical zone, where the strain rate and stresses are concentrated.

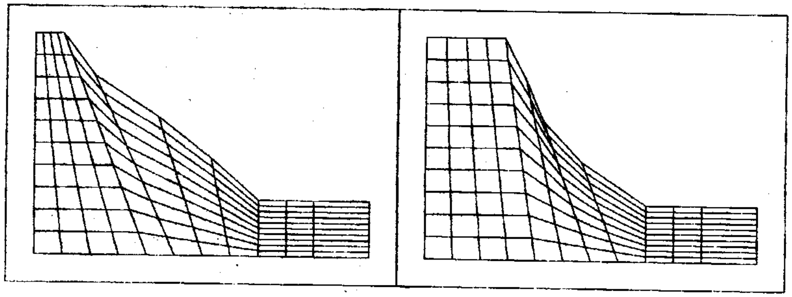

Figure 4: Optimal shape – Criteria: overall distorsion rate (left) and maximum local distorsion rate (right)

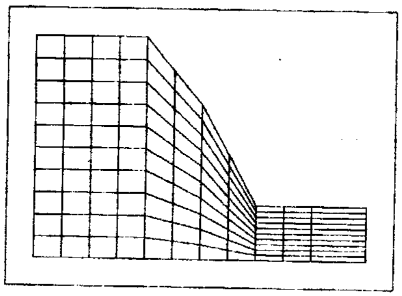

Figure 5: Optimal shape with sticking friction – Criterion: energy rate

Functional	Ψ_E	Ψ_e	$\Psi_{D\dot{\varepsilon}}$	$\Psi_{\dot{\bar{\varepsilon}}_l}$	Ψ_D	$\Psi_{\dot{\varepsilon}_{12}}$
Initial value	0.8658E+5	0.3684E+6	4.802	6.8844	7.119	5.068
Optimal value	0.7922E+5	0.2686E+6	2.913	5.3424	6.358	3.997

Table 1: Initial and optimal values of the design functionals

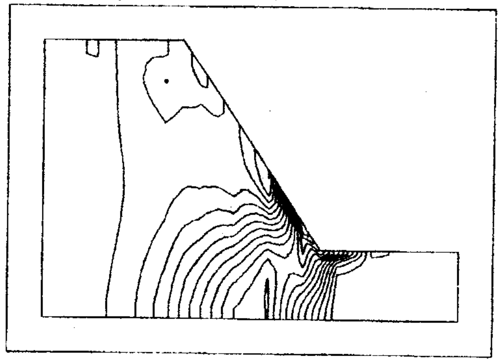

Figure 6: Energy rate isocurves for the initial die

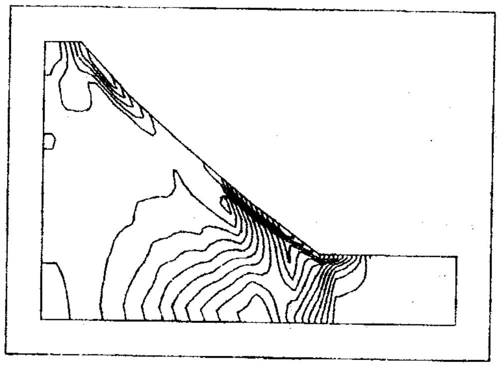

Figure 7: Energy rate isocurves for the global-energy rate optimized die

The optimal solution also strongly depends on the bounary conditions. In contrast with the already shown solutions, where, as stated, no friction conditions have been assumed, Fig. 5 shows the optimal solution with respect to the Ψ_E deformation energy criterion for a problem modeled with sticking friction condition. We see that, besides from the different shape, a much less significant reduction is achived: from $\Psi_{E0} = 0.1552E + 6$ to $\Psi_{Ef} = 0.1530E + 6$. This fact is explained by the dead zone appearing in the neighbourhood of the die corner, which is larger for such boundary conditions.

Finally, comparative plots of the energy rate are shown for the initial (Fig. 6), optimized (Fig. 7) configurations and for the one optimized with respect to the local criterion (Fig. 8). It can be seen that the last plot shows visible differences with respect to the original and the optimized ones, which suggests that the minimum for both criteria (local and global) are very different. On the other hand the isocurves for the energy rate have a similar pattern in both the initial and the global-optimized configurations, in the latter we see smoother energy gradients and the deformation zone more spreaded throughout the domain.

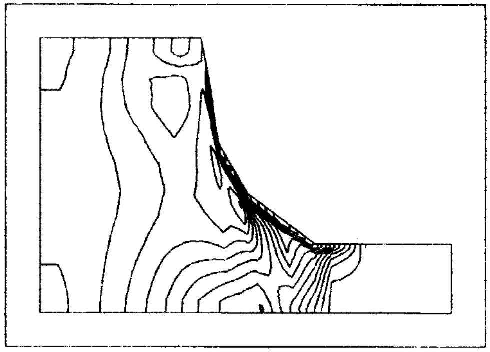

Figure 8: Energy rate isocurves for the local-energy rate optimized die

5. Final remarks

In this work a poligonal line for the die profile has been adopted in such a way that every nodal coordinate on the die profile is an independent design variable. With no additional difficulty other solutions can be adopted as well, for example to define the die profile in terms of spline lines which decide about the nodal coordinates. In that case the design parameters will be given by the spline coefficients, with the additional advantage that no further restrictions are necessary in order to avoid non-realistic situations.

Nevertheless, an amazing repeatibility has been found for a same functional when results from different sets of design parameters as compared. Accepting for the analysed case that the crucial magnitude is the die angle near the material outlet, this value is virtually the same either taking from one to four design parameter, provided they give enough degrees of freedom so that the system may reach this configuration. Moreover, this angle is very much similar comparing the global and local version of a same criterion and even among different optimization criteria.

The optimizations yielded up to 40% reduction of the cost functional, although in many cases this value was less significant. On the other hand, the "optimal" shape is strongly dependent on the functional choice and on the boundary conditions. Further research is necessary in order to evaluate the functionals behaviour at the other functionals minima.

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