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FINITE STRAIN LOCALIZATION IN DYNAMICS OF MULTIPHASE MATERIALS

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A computational analysis of dynamic strain localization in multiphase solids is presented in this paper. The theoretical framework is based on mixture theory, integrated by the volume fractions concept. The constitutive equations employed are developed within finite strain plasticity. The study of localization is based on continuum wave propagation. The directions of localization are obtained by means of an eigenvalue analysis of the acoustic tensor.

The investigation of the development of localized bands is carried out by means of a finite element code. The influence on localization of coupling between the constituents and of several material parameters is investigated. A strong influence of the spatial discretization it is also noted. With respect to this, alternative procedures are proposed.

1. INTRODUCTION

Finite strain localization in multiphase solids occurs with shear bands formation in zones of limited amplitude, where concentrated and amplified strains appear. In these regions, material behaviour is anelastic while the other zones are elastic and the strains are infinitesimal.

Physical examples of localization phenomena are those of strain concentrations in glass, metal cracking, concrete damaging, slope instability and soil fracture.

The analysis of finite strain localization has a noticeable interest in engineering science, since regions where such deformations appear play a dominant role with respect to the integrity of the affected solids or structures. Moreover, in these regions the onset of fracture phenomena or structural collapse is most probable.

The computational analysis of strain localization allows to collect a wide information spectrum about several aspects of the involved mechanics, like finite strain behaviour and material nonlinearity.

In this work we present the theoretical framework of localization in geomaterials and the results of the first developments of a larger computational investigation; the geomaterials are considered as multiphase materials in fully saturated or semi-saturated conditions.

The role of the fluids in localization is fundamental, since shear band formation preceeding cracking and fracture is mostly affected by the interaction between solids and fluids, in terms of time sequence of band formation and the way of their appearance.

Attention is also given to computational issues since highly strained elements are involved in localization, with consequences in terms of stability and convergence of these elements.

The topic of strain localization has been analysed in the last years by several authors. The problem of dynamic localization in one phase solids has been investigated, e.g., in Sluys [10] and some modelling generalizations were performed in Owen et al. [7]; the case of saturated porous materials was investigated, e.g., in Loret and Prevost [6]. The mechanics of porous materials has been studied, among others, by Schrefler et al. [9], Zienkiewicz and Shiomi [13], de Boer et al. [4] and Ehlers [5].

The foundation of this research can be found in classical continuum mechanics (Truesdell and Noll [11], Truesdell and Toupin [12]), with modifications to take into account the volume fraction concept, and extensions with respect to the constitutive model to incorporate finite plasticity.

Localization theory is here based on the dynamics of wave propagation (Chen [3]) and is described using the properties of the acoustic tensor (Briseghella and Majorana [2]). These are infact key features to formulate the problem in general form.

2. MECHANICS OF POROUS MATERIALS

The mass conservation for the i-th phase at macroscopic level can be written, in the spatial setting, as (de Boer et al. [4]):

$$\dot{n}^i + n^i div\, \dot{\mathbf{x}}^i = -n^i \frac{\dot{\rho}^{iR}}{\rho^{iR}} \tag{2.1}$$

and the momentum conservation equations are:

$$div\, \mathbf{T}^i + \rho^i\left(\mathbf{b}^i - \ddot{\mathbf{x}}^i\right) + \hat{\mathbf{s}}^i = \mathbf{0} \tag{2.2}$$

where $n^i = n^i(\mathbf{x},t) = \dfrac{dv^i}{dv}$ is the volume fractions, $\rho^{iR}(\mathbf{x},t)$ is the real density, $\rho^i = n^i \rho^{iR}$ is the apparent density, $\mathbf{T}^i$ is the Cauchy partial stress tensor of the i-th phase, $\mathbf{b}^i$ is the body force vector, $\ddot{\mathbf{x}}^i$ is the acceleration of the i-th phase and $\hat{\mathbf{s}}^i = \hat{\mathbf{p}}^i + \hat{\rho}^i \dot{\mathbf{x}}^i$ is an additional force, accounting for the effect of the i-1 phases over the examined i-th one (the symbol ^ indicates the exchange of the affected quantity with the other phases).

For non-polar continua, the momentum of momentum equations allow to find the simmetry of the partial stress tensor. The framework of the finite elasto-plasticity can be found in Ehlers [5].

3. DYNAMIC LOCALIZATION THEORY IN SATURATED POROUS SOLIDS

The finite strain localization theory is based on the analysis of wave propagation in continuous solids. The first fundamental investigations on this subject are due to Duhem and Hadamard (1903). A presentation of the theory in general form and systematic literature references can be found in Truesdell and Noll [11], Truesdell and Toupin [12] and in Chen [3].

Let Σ be the front wave (here considered as a Riemannian manifold in motion, through which the acceleration and the velocity gradient can be discontinous functions); **n** the normal directed outward from the above manifold. Indicating with

$$\eta^i = grad\, \dot{\mathbf{x}}^i$$

the velocity gradient of the i - phase along this manifold, Hadarmard's compatibility conditions can be written as

$$\left\| \dot{\mathbf{E}}^i \right\| = \frac{1}{2}\left(\eta^i \otimes \mathbf{n} + \mathbf{n} \otimes \eta^i\right) \tag{3.1}$$

$$\left\| \ddot{\mathbf{x}}^i \right\| = -c\,\eta^i \tag{3.2}$$

where $\mathbf{E}^i$ is the deformation tensor of the i - phase, $\|..\|$ indicates the jump of the quantity inside the symbol, c represents the velocity of the manifold Σ with respect to the material frame of the analysed solid and $\otimes$ indicates, as usual, the dyadic or tensor product. Consequently, momentum equilibrium equations impose:

$$\left\|\dot{\mathbf{T}}^i\right\| \cdot \mathbf{n} = \rho^i c^2 \eta^i \tag{3.3}$$

Expressing the constitutive relationship in incremental form, it can be found in Loret [6] that c^2, squares of the wave propagation velocities, is coincident with the eigenvalues of the acoustic tensor **B**, expressed by:

$$\mathbf{B} = \begin{pmatrix} \mathbf{B}^{SS} & \mathbf{B}^{SF} \\ \mathbf{B}^{SF} & B^{FF} \end{pmatrix} \tag{3.4}$$

where B^{FF} is a positive scalar quantity, $\mathbf{B}^{SF}$ is a vector and $\mathbf{B}^{SS}$ is a second order tensor containing the tensor of the solid moduli.

The hyperbolicity condition of the problem, implying an effective wave propagation, requires real values of the propagation velocity c. Consequently the eigenvalues of the tensor **B** must be real and positive, or in other words **B** must be a positive definite symmetric tensor; in the elastoplastic case it is sufficient for this to adopt associated plasticity for the solid skeleton. Moreover **B** is positive definite if $\det \mathbf{B} > 0$. This condition is assured if the plastic modulus h is positive. The transition from hyperbolicity to ellipticity is related to softening.

Développing the determinant of **B** matrix, it can be observed that the global result is related to the evolution of the constitutive parameters of the solid skeleton only.

4. NUMERICAL EXAMPLES

The two-dimensional domain of the porous material is discretized by means of isoparametric triangular or quadrilateral finite elements. Linear finite element have been chosen because of their computational efficiency in nonlinear analyses and their low distorsional characteristics.

The finite elements used for discretizing the problem have not been intentionally oriented along particular lines (unbiased mesh).

The performed analyses show in particular:

i) the necessity of softening in the constitutive law to have shear band formation,

ii) the influence of permeability on band growing,

iii) particular patterns of stresses, pressures and strains with respect to the corresponding ones in hardening plasticity or elasticity,

iv) a strong mesh dependence of the results in the absence of some regularisation.

4.1 Quadrilateral sample

The onset and growth of localized bands in a soil sample of rectangular shape made of saturated material with dimensions 25 x 35 m are analysed [Fig. 1]. The sample is subject to axial compression by means of uniformly distributed loads both on the upper and lower surfaces.

The solid and fluid domains are not subject to any initial stress state (hence gravitational effects or hydrostatic pressures are not accounted for). In the fluid discretization the top and botton surfaces are considered impermeable, in order to subject the fluid flux in principle to the same motion as the solid particles, and to evidence the degree of coupling between the solid skeleton and the fluid matrix.

In the considered model, homogeneous and isotropic solid and fluid phases are taken into account. The constitutive relationship of the solid skeleton is of Mohr-Coulomb type, with a linear displacement - strain relationship.

It can be noted that plastic strains are concentrated in narrow bands of finite amplitude where high strain gradients occur [Fig. 2]. In Fig. 3 localization directions are shown as found with the procedure based on the analysis of the acoustic tensor. Fig. 4 shows that in case of plasticity with hardening no band formation appears. The time transients of strains [Fig. 5] and pressures [Fig. 6] are characterized by a wave form, with a marked regularity up to the onset of the shear band formation ($t < 0.3$ sec.), since the plastic effect is yet limited. Plastic strain shows a different pattern, characterized by a plateau during the time between the onset of a loading wave and the subsequent one [Fig. 8].

Starting from the time ($t > 0.3$ sec.) in which a significant length of the band appears, corresponding to the first increment of plastic deformation, the loss of periodicity, typical of the linear elastic solution [fig. 5], can be noted, as well as a sudden increment of the pressures [Fig. 6]; this phenomenon is a direct consequence of the coupling between parameters of the solid and fluid fields in the equations governing the problem.

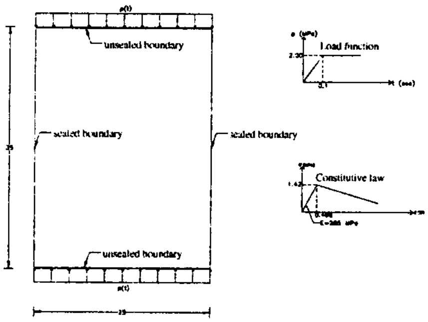

Fig. 1: Description of the geometrical and material characteristics of the reference example

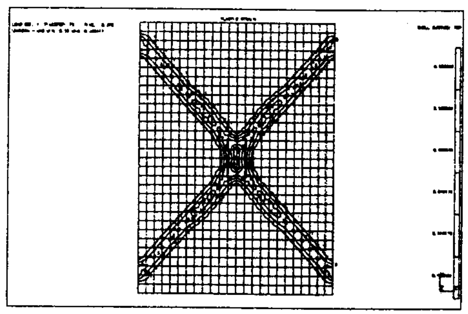

Fig. 2: Effective plastic strain field at t = 0.375 s

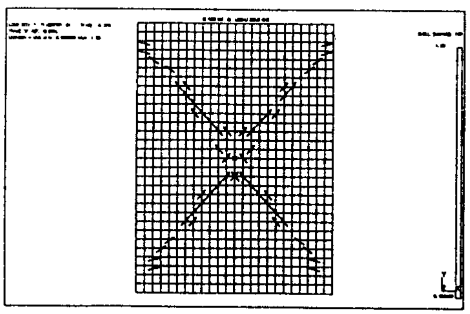

Fig. 3: Localization directions as following by the analysis of the acoustic tensor

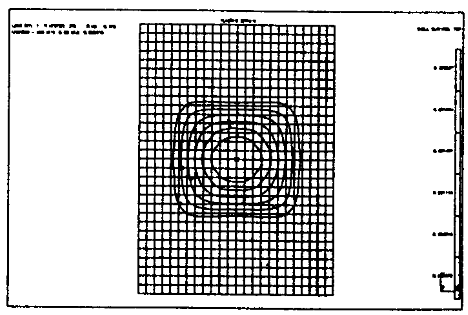

Fig. 4: Effective plastic strain at t = 0.375 s in plasticity with hardening

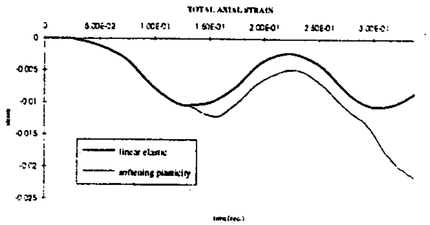

Fig. 5: Comparison between strains in a gauss point nearest the centre, in the linear elastic case and in plasticity with softening

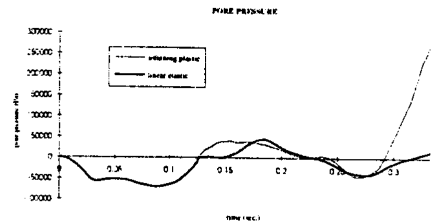

Fig. 6: Comparison between pressures in the central node in the linear elastic case and in plasticity with softening

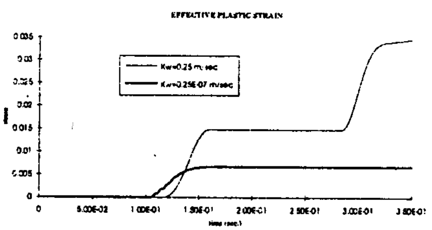

Fig. 7: Comparison between effective plastic strains vs. time, in a gauss point nearest the centre, for different permeabilities

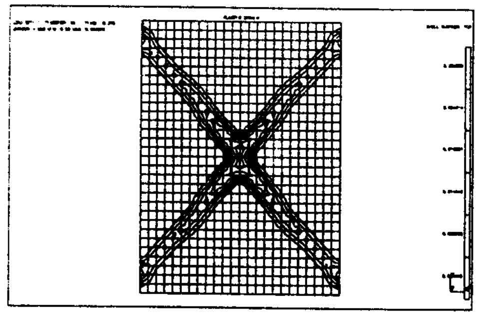

Fig. 8: Effective plastic strains at t = 0.375 s with a relative permeability of Kw=0.25 m/s

4.2 Influence of permeability

The permeability affects the degree of coupling between the two phases and presents a significant role in the development of localization. The lower is its value, the higher is the part of the load increment assumed by water and the slower is the transfer to the solid skeleton. Hence the coupling effects increase as the permeability decreases.

In the localization phenomenon this implies the variation of the plastic strain levels [Fig. 7], the change of band dimension [Figs. 8, 9] up to the disappeance of their formation [Fig. 10].

The coupling can be so strong that the fluid can be captured inside the bands and simultaneously the pressures may localize [Fig. 11].

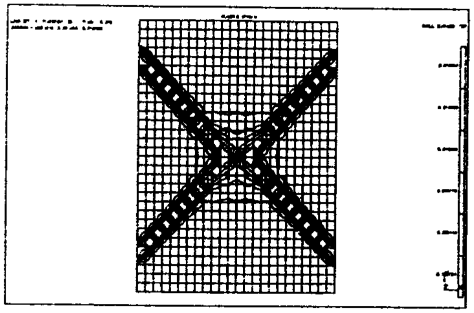

Fig. 9: Effective plastic strains at t = 0.375 s with a relative permeability of Kw=0.25E-03 m/s

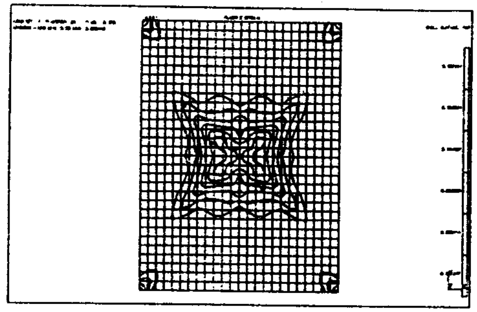

Fig. 10: Effective plastic strains at t = 0.375 s with a relative permeability of Kw=0.25E-10 m/s

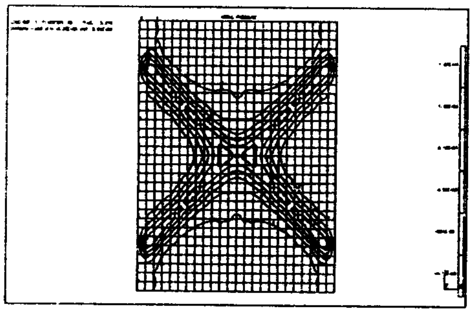

Fig. 11: Nodal pressures at t = 0.375 s with a relative permeability of Kw=0.25E-03 m/s

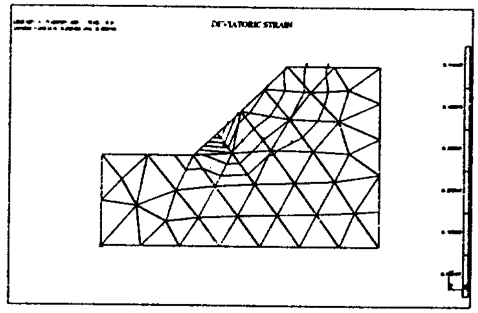

Fig. 12: Deviatoric strain (62 elements)

4.3 Dependence on spatial discretization

Using a rate-independent model the localization is subject to a strong dependence on the chosen discretization, and the numerical solution cannot have a physical meaning. This is connected with the presence of a softening branch in the constitutive relationship, responsible for the loss of hyperbolicity in the equations of motion (becoming elliptical). The wave propagation disappears because a wave with zero velocity appears or two waves with immaginary velocities (stationary jump). Such statement can be easily demonstrated in the one-dimensional case, where the wave velocity is equal to $\pm\sqrt{D_{ep}/\rho}$, being D_{ep} the elastoplastic modulus (negative in the softening branch). Hence the system of differential equations becomes ill posed, i.e. with a strong dependence on the initial and boundary conditions. The absence of a scale parameter in the constitutive law leads to the dependence of the band width on the element dimension.

4.4 Earth slope

The model of a hypothetical slope has been investigated. It is subject to a load in the upper edge, increasing in time monotonously.

Initially a discretization with 62 triangular elements has been set up, giving as result the formation of zones with concentrated strains along curves reproducing the typical instability circles of earth slopes [Fig. 12]. If a more accurate discretization with much more elements is used, (refining the mesh with a higher element density in the most significant zone) the phenomenon previously described manifests itself again: the strains are concentrated along aligned elements.

5. CONCLUSIONS AND FURTHER DEVELOPMENT

This paper shows the first results of a research in progress on localization in two or three phase geomaterials. One of the problem to be solved is the mesh dependence of the shear band width; to eliminate this drawback, a characteristic length parameter in the constitutive model must be introduced. This can be done by improving the continuum model and regularizing the solution. The first goal can be matched using e.g. a model with polar constituents, like micro-rotations and micro-moments (see Pastor et al. [8]), or introducing in the description of continuum mechanics a higher degree of strain, as in the second grade plasticity formulation due to Ehlers et al. [5] (both of non-easy experimental calibration). On the other hand, regularization can be obtained by a rate-dependent model, using in the constitutive law a non-physical viscosity (like the models by Perzyna and Duvait, Lions); working in this way, the system of differential equations retains its character (hyperbolic in dynamic anlysis, elliptic in the static one) remaining always well posed also in the softening range. Such regularization implies the effective independence of the discretization used only after a suitable calibration of the viscosity parameters.

A further problem concerning softening in the constitutive law is still open, i.e. instability of the material in the Drucker sense, with possible non uniqueness of the solution. The above limitation can be overcome using an enhanced plasticity model. Moreover, together with the above methods to better describe the shear bands, it is possible to improve the discretization in the localization regions. Two strategies appear suitable from this point of view: one is the mesh densification as emerging from the analysis of the acoustic tensor (remeshing) (Pastor et al. [8]) and the other one is to align the elements along the band directions again as indicated by the eigenvector solution of the acoustic tensor (biased mesh) (Bìcanic and Selman [1]).

It should be finally remarked that an enhanced numerical simulation of strain localization in porous materials would require a geometrically and material non linear dynamic analysis, followed by the transition to a contact problem between separating surfaces and a flow model through fractures.

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