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STATISTICAL ANALYSIS OF HETEROGENEITIES AND THEIR EFFECT ON WELL TEST PRESSURE RESPONSE

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RESUMEN

Nuestro objetivo es estudiar la influencia de las variaciones espaciales de permeabilidad y porosidad en la respuesta de presión obtenida durante un ensayo de pozo. Se utilizan datos de campo, provenientes de tres pozos, y datos sintéticos. Los datos de campo consisten en mediciones de permeabilidad y porosidad en función de la profundidad y datos de presión medidos en ensayos de los mismos pozos. A fin de llevar a cabo este análisis se aplican dos herramientas diferentes: caracterización estadística de heterogeneidades y método de interpretación de ensayos de pozo. Para los datos estudiados se puede concluir que: (1) los valores de permeabilidad resultan mejor representados por distribuciones de tipo exponencial, (2) el valor constante de porosidad que mejor ajusta las presiones medidas coincide casi exactamente con la media aritmética, mientras que las estimaciones constantes de permeabilidad son valores situados entre la mediana y la media aritmética, (3) pequeñas variaciones de permeabilidad causan importantes cambios en la respuesta de presión.

ABSTRACT

Our aim is to study the effect of permeability and porosity spatial variations on well test pressure response. Data from three wells and synthetic data are used. Field data consist of permeability and porosity as functions of depth and pressure transient test measurements from the same wells. To achieve our aim two different tools are applied: statistical characterization of heterogeneities and a well test interpretation method. For our data: (1) permeabilities are best represented by exponential distribution functions; (2) constant porosity estimates that best fit measured transient pressures are almost equal to the statistical arithmetic mean, while constant permeability estimates lie between the median and the arithmetic mean, (3) minor permeability variations cause important changes in pressure response.

INTRODUCTION

Statistical tools, used to characterize permeability and porosity data measured as function of depth, are described. These tools are: *correlation between both variables* and *measures of central tendency* which involve the estimation of the corresponding *probability or frequency distribution function*. Besides, an inverse method for obtaining permeability, porosity and skin factor from well test pressure-time data is described (the skin factor quantifies the permeability change near the wellbore, as a result of drilling and completion practices). During a well test, the pressure response to changing production or injection conditions is obtained. The reservoir properties characterize that response, therefore, we try to infer that properties analyzing the pressure behavior. Flow in the reservoir is simulated by the diffusivity equation which represents a single-phase, slightly compressible, one dimensional model. Radial porosity and permeability variations are considered and the equation is solved by numerical means. Discretization of the differential equation is performed by applying an implicit finite difference technique. Parameters are found with a nonlinear regression method: the Quasi-Newton approximation for the least squares problem [1,2]. In this way, permeability, porosity and skin factor are found by minimizing the sum of squares of the differences between observed and numerically calculated pressure data.

Those methods, statistical characterization and well test interpretation technique, are applied to analyze data from Argentine fields and synthetic data. Field data consists, on one hand, of permeability and porosity as functions of depth, which have been measured through core tests from three wells; and, on the other hand, of pressure transient measurements from the same wells. Synthetic pressure data are obtained feeding our numerical simulator with adequate radial distributions of permeability and porosity.

In a first step, permeability and porosity data are statistically characterized [3,4]. Correlations, distribution functions and averages are found. Power transformations (Jensen et al [5]) are applied to permeability data to achieve acceptable distribution functions. In a second step, the well test interpretation method is applied in order to obtain constant estimates of permeability and porosity that best fit pressure data. These estimates are then compared with the statistical measures of central tendency. Finally, the influence of permeability and porosity variations on pressure transient test response is analyzed and discussed.

THEORY - STATISTICAL CHARACTERIZATION

All permeable media properties are heterogeneous. For example, in a certain petroleum field, the permeability and porosity values vary among the different points of the reservoir rock. Nevertheless, constant approximations of these variables are generally used to perform different computations. Those constant values must be carefully chosen to properly represent the physics of the flow.

In order to analyze vertical permeability and porosity measurements, a statistical study is performed, which consists of:

1) Finding a *relationship between permeability and porosity values*, measured from core laboratory tests. Although porosity (ϕ) affects permeability (k) values, there is no universal equation that relates both variables (the well known Carman-Kozeny equation involves additional parameters, such as tortuosity and specific surface area). In practice, plotting $\log(k)$ *vs.* ϕ for a specific reservoir, a certain functional dependency between these two variables is often observed . Straight lines or curves are then interpolated from this empirical relationship and their validity is checked by means of correlation coefficients. If the $\log(k)-\phi$ plot exhibits a large scatter, predictions based on the correlation obtained may be meaningless.
2) Computing statistical measures that extract the essential information contained in the data under consideration. In particular, *Measures of Central Tendency* provide "average values". These measures are: *mean* (arithmetic, geometric, harmonic, which are the most commonly found in the petroleum literature, mainly as permeability estimates [6]), *median* and *mode*. There are two ways of obtaining measures of central tendency: by computing straightforward approximations from the measured data or by applying the theoretical definition which involves the corresponding *Frequency or Probability Distribution Function*. This function is also estimated using the measured data.

Let us now consider *frequency distributions*. The spatial variation of a variable is not as arbitrary as it may seem. In fact, although permeability and porosity may take values over a certain range, each value has a specific probability of occurrence. That probability is determined by a frequency or probability distribution function, $pdf(x)$. Therefore, permeability and porosity are considered random variables associated with a distribution function that can be determined. There are more than one hundred probability distribution functions observed in nature. Only the two distribution functions used in this work are mentioned here. One of them is the *normal distribution* $(N(u,\sigma))$. Two parameters, the mean u and the standard deviation σ, determine this function, as follows,

$$pdf(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad , \qquad -\infty < x < \infty \quad . \tag{1}$$

The other one is the *exponential distribution* $(\mathcal{E}(\lambda))$ which depends only on one parameter, λ,

$$pdf(x)=\lambda e^{-\lambda x} \qquad x \geq 0 \tag{2}$$

$$\ln k \quad , \quad \alpha = 0 \quad .$$

An optimum α-value is estimated that best fits transformed data into a known pdf. Then, statistical tests, like Lilliefors Test for Exponential Distribution or Shapiro Wilk Test for Normal Distribution [4,7] are applied in order to validate the proposed frequency function.

THEORY - WELL TEST INTERPRETATION METHOD

This method was developed at our laboratory. It consists of

- a mathematical model of flow which includes radial permeability and porosity variations;
- a nonlinear regression technique used to fit well test pressure response.

radial diffusivity equation [6]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk(r)\frac{\partial p(r,t)}{\partial r}\right) = \phi(r)\mu c\frac{\partial p(r,t)}{\partial t} \quad . \tag{4}$$

In this equation, radial distance, r, and time, t are the independent variables and pressure $p(r,t)$ is the unknown. The parameters are viscosity, μ, compressibility, c, absolute permeability, k, and porosity, ϕ. The assumptions needed to obtain this equations are:

* One-dimensional and radial flow
* The reservoir is isotropic and has uniform thickness;
* Oil is a fluid of small and constant compressibility and constant viscosity;
* Gravity forces are negligibly small.

Besides, radial variations of permeability and porosity values are included in this model.

Eq. 4 is a parabolic partial differential equation. The initial condition is the known reservoir pressure $p_o(r)$ at the beginning of the well test,

$$p(t=0,r) = p_o(r). \tag{5}$$

Let us consider a bounded, cylindrical reservoir, with a well located in its axis. The boundary condition at the well is

$$k(r)r\frac{\partial p(r,t)}{\partial r}\bigg|_{r=r_w} = \frac{q(t)\mu}{2\pi h}, \qquad t>0; \tag{6}$$

the flow rate q may vary with time, so that different types of well tests can be simulated; r_w is the well radius. The outer boundary condition is

$$r\frac{\partial p(r,t)}{\partial r}\bigg|_{r=r_e} = 0, \qquad t>0; \tag{7}$$

it assumes the absence of flow; r_e is the external radius.

To take into account the skin effect, an additional pressure drop at the well is considered.

$$\Delta p_{skin} = s\left(\frac{q\mu}{2\pi kh}\right). \tag{8}$$

If permeability and porosity are constant values, an analytical solution for equation (4) with initial and boundary conditions (5), (6) and (7) exists [8]. This solution is given by a series expansion whose terms include Bessel Functions.

If permeability and porosity are not constant, an analytical solution exists only for special cases; for instance, when permeability is an arbitrary function of position with small variations from a mean value and porosity is constant [9]. However, in many cases the problem cannot be solved analytically and a numerical solution is required. In this work, that numerical solution is approximated using a *finite difference scheme*.

But, before applying finite differences, a new variable is introduced by means of the following transformation

$$x = ln(r/r_w). \tag{9}$$

The aim of Eq. 9 is to obtain a spatial grid with more points near the wellbore (where a detailed study is necessary). A constant increment in the x variable is used. With this transformation, Eq. 4 becomes

$$e^{-2x}\frac{\partial}{\partial x}\left(k(x)\frac{\partial p(x,t)}{\partial x}\right) = \phi(x)\mu c\frac{\partial p(x,t)}{\partial t} \tag{10}$$

with initial and boundary conditions,

$$p(t=0,x) = p_o, \qquad x > 0, \tag{11}$$

$$\left.\frac{\partial p(x,t)}{\partial x}\right|_{x=0} = \frac{q(t)\mu}{2\pi k(r_w)h}, \qquad t > 0, \tag{12}$$

$$\left.\frac{\partial p(x,t)}{\partial x}\right|_{x=ln(r_e/r_w)} = 0, \qquad t > 0. \tag{13}$$

Discretization is performed applying an implicit finite difference method. Let us denote the value of pressure at the representative mesh point $(x_i,t^n) = (i\Delta x, n\Delta t)$ by p_i^n, where Δx and Δt are the spatial and time discretization steps. Therefore, the following scheme represents Eq.10,

$$e^{-2x_i}\left(\frac{k_{i+\frac{1}{2}}\dfrac{p_{i+1}^{n+1}-p_i^{n+1}}{\Delta x} - k_{i-\frac{1}{2}}\dfrac{p_i^{n+1}-p_{i-1}^{n+1}}{\Delta x}}{\Delta x}\right) = \phi_i\mu c r_w^2\left(\frac{p_i^{n+1}-p_i^n}{\Delta t}\right). \tag{14}$$

Besides, central differences are applied to the derivative boundary conditions. The stability of the resulting scheme is proved by means of a matrix analysis.

Nonlinear Regression Technique - Parameter estimates are obtained minimizing an objective function. This objective function is determined applying the Maximum Likelihood Method [10] - in which normally distributed measure errors with diagonal covariance matrix are assumed. Therefore, the objective function becomes the weighted sum of the squares of the differences between measured well test pressure data and the pressure values calculated with the numerical model already described,

$$F(\phi,k,s) = \sum_{j=1}^{N} w_j(p_j^e - p_j^c(\phi,k,s))^2. \tag{15}$$

Superscripts e and c stand for "experimental" and "computed" respectively. N is the number of data points and s is the skin factor. The weighting factors, w_j, are estimated by

$$w_j = \frac{1}{\delta_j^2} \tag{16}$$

where δ_j^2 is the variance of absolute error in the j^th measurement. In this work, δ_j is considered proportional to p_j^e, the j^th pressure measurement.

Eq. 15 is minimized by means of a Quasi-Newton type optimization technique, specifically designed to solve the least-squares problem (Quasi-Newton Approximation for the least-squares problem - QNA). This method has shown the best behavior to solve different reservoir characterization problems [1,2,11] and, as far as we know, it has not been previously applied to well test analysis.

DATA

The theory already described is applied to field data and synthetic data.

Field Data - The field data have been obtained from wells located in different Argentine fields. Results corresponding to three wells, Well A , Well B and Well C, are presented. The data set consist of:

- permeability and porosity as functions of depth, measured through core laboratory tests;
- well test pressure measurements from the same wells.

Reservoir dimensions and fluid properties are shown in Table I. With these data, only the influence of vertical heterogeneities can be analyzed. There are no field measurements of permeability and porosity as functions of radius.

TABLE I -	RESERVOIR DIMENSIONS AND PROPERTIES			
	WELL A	WELL B	WELL C	SYNTHETIC
r_w (m)	0.111	0.122	0.111	0.122
h (m)	14	21.6	7	7
μ (Pa.s)	7.25×10^{-4}	2.7×10^{-4}	6.3×10^{-4}	9.2×10^{-4}
c (Pa^{-1})	7.96×10^{-9}	1.32×10^{-8}	7.90×10^{-9}	1.27×10^{-9}
qB (m3/s)	1.60×10^{-3}	3.75×10^{-3}	1.73×10^{-3}	5.57×10^{-3}
p_i (MPa)	15.82	23.84	24.20	41.42

Synthetic Data - Three examples of drawdown well test pressure response are simulated in order to analyze the influence of radial heterogeneities. The simulation is performed using the numerical model (Eq. 14), data from Table I [12] and random values of permeability and porosity for each grid point. Normal distribution functions are used to generate those random values,

Example 1: $\ln k \sim N(4,1)$; $\phi = 20\%$

Example 2: $k = 55\text{mD}$; $\phi \sim N(20,5)$

Example 3: $\ln k \sim N(4,1)$; $\phi \sim N(20,5)$

Definition of Average Relative Error

In order to quantify the goodness of fit to the measured pressure values, an average relative error per measurement is defined as follows,

$$\overline{E}_{rel}\% = \sqrt{\frac{F}{N}}x100 \qquad (17)$$

where F is the objective function defined in Eq.15 and N is the number of data points.

RESULTS AND DISCUSSION

We start analyzing field data. With permeability and porosity as functions of depth, the statistical characterization of properties is performed. Then, the well test interpretation method is applied to analyze the pressure response.

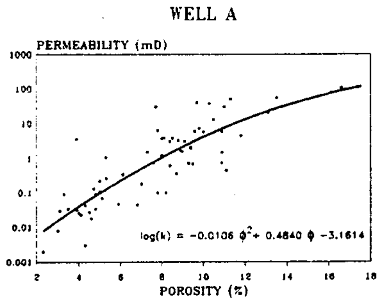
Statistical Characterization

The data consist of 64 permeability and porosity values as functions of depth for Well A, 109 for Well B and 65 for Well C, all of them measured on cores.

First of all, *a functional relationship between permeability and porosity values is searched.* With that aim, logarithm of permeability values are plotted in the vertical axis and porosity values are plotted in the horizontal axis for each set of data. Only in one well, Well A, a good correlation is found, as it is shown in Fig. 1. The functional relationship may be represented by a quadratic function, which improves the correlation given by a straight line. The quadratic function is obtained using a linear multiple regression algorithm:

$$\log(k) = -0.0106\phi^2 + 0.4840\phi - 3.1614 \qquad (18)$$

The correlation coefficient between variables is 0.85. In the other two cases (Wells B and C) the correlation coefficients are lower (0.40 and 0.16 respectively). Therefore, only the Well A-correlation is useful for prediction purposes.



Correlation Coefficient

WELL A
0.85

WELL B
0.40

WELL C
0.16

Fig. 1: Correlation between permeability and porosity for Well A. Correlation coefficients for Well A, Well B and Well C

Finally, *the permeability and porosity measures of central tendency* are also calculated. Then, *the frequency distribution functions of porosity and permeability* are estimated for each well.
On one hand, using porosity values without transformations, almost symmetric histograms are obtained in the three cases. So, a normal distribution is proposed and the Shapiro-Wilk Test accepts this hypothesis. The porosity arithmetic mean is 10% for Well A, 13% or Well B and 11% for Well C. Similar values are obtained for the mean, median and mode. This fact is in agreement with the behavior corresponding to a normal distribution. Therefore, the arithmetic mean is used as a constant estimate of porosity.

On the other hand, power transformations (Eq. 3) have to be applied to permeability values to fit a known distribution function. Transformed data with different α-values do not follow a normal behavior. In fact, the Shapiro-Wilk Test rejects the normality hypothesis at 1% level of significance for the examined cases. Nevertheless, for each well, an optimum α-value is found that fits transformed data to an exponential type distribution, hypothesis accepted by the Lilliefors Test in every case. The optimum α-value and the exponential function parameter λ estimated corresponding to each well are shown in Table II.

TABLE II	OPTIMUM α-VALUE AND EXPONENTIAL DISTRIBUTION PARAMETER		
	WELL A	WELL B	WELL C
α - value	0.4	0.5	0.56
λ	0.27	0.23	0.17

Therefore, the common assumption of α-normality must be carefully checked before being accepted. It should be noted that Lambert [13] also obtained exponential distribution functions for 102 wells over the 689 she studied.

With the exponential function thus found, the corresponding frequency distributions of permeability are calculated (i.e. pdf for Well B is plotted in Fig. 2).

Table III shows the permeability measures of central tendency computed straightforward from permeability data and computed using the theoretical definition through the corresponding frequency distribution functions. Comparing both tables, It may be seen that central tendency measures computed using both ways are very similar.

TABLE III	PERMEABILITY MEASURES OF CENTRAL TENDENCY -					
	Straightforward from Data			Using Frequency Distribution Function		
MEASURES(mD)	WELL A	WELL B	WELL C	WELL A	WELL B	WELL C
Arithmetic mean	10.27	13.60	13.07	9.37	14.80	14.25
Geometric mean	0.78	7.07	3.36	0.93	6.94	3.57
Harmonic mean	0.08	3.93	0.13	0.08	3.92	0.39
Median	0.92	6.40	5.40	0.79	6.28	4.63

In Fig. 2 the corresponding measures of central tendency computed for Well B are also included. Let us notice that, for exponential type distributions, the mode is always the minimum value, so it is not a useful estimator.

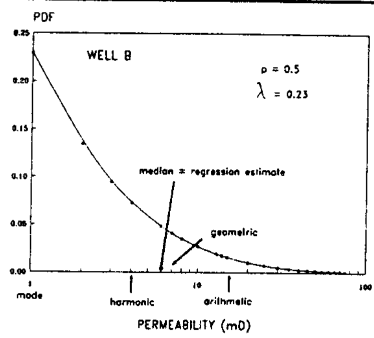


Fig. 2: Permeability Frequency Distribution Function of Well B obtained from transformed data

Analysis of Field Well Test Pressure Data
Pressure data corresponding to Well A and Well C were measured during an initial build-up test of each well while pressure data from Well B were measured during a drawdown test. The well test interpretation method described above is applied to those data and afterwards a sensitivity analysis is carried out in order to study how each property affects pressure response.

Table IV shows the results obtained by applying the nonlinear regression technique to the pressure response of the three wells. Constant estimates of permeability, porosity and skin factor are obtained. In Well A, the permeability-porosity correlation is included, so that only two independent parameters have to be determined.

TABLE IV	RESULTS OBTAINED APPLYING *NON-LINEAR REGRESSION TECHNIQUE*		
	WELL A (build-up)	WELL B (draw-down)	WELL C (build-up)
k	7.5 mD	6 mD	14 mD
ϕ	11%	11.5%	11%
s	-2.4	0.27	7
E_{rel}	0.29%	0.38%	0.33%

Measured pressures and pressures simulated with the above optimum parameters are drawn in Fig.3. A better goodness of fit is found for Well A.

Let us compare the statistical measures of central tendency (Table III) with the nonlinear regression estimates (Table IV). Porosity estimates are similar to the arithmetic mean values. For Wells A and C, permeability estimates are closer to the arithmetic mean, while, for Well B, it is almost equal to the median.
In order to carry out the *sensitivity analysis*, different permeability and porosity values, selected among the vertical measurements, are used to feed the numerical simulator. The purpose is to study the effect of each property independently, so that only variations of one of them are considered, while the other remains fixed at the nonlinear regression estimate.
For Well A, Fig. 4-a shows the porosity influence and Fig. 4-b the permeability influence. Both properties affect pressure response. As any one of them increases, lower pressure values are obtained. But pressure response is much more affected by permeability changes. In fact, a significant decrease in pressure is produced by a slight increase in permeability.

Therefore, on one hand, porosity may be approximated by a constant value; in fact, the arithmetic mean computed from vertical measurements is a good estimator. On the other hand, the choice of a suitable permeability cannot be established as a general rule because this is the parameter of major impact on well response.

So far we have analyzed the influence of different constant permeability and porosity values selected among vertical measurements. Let us now study the radial variations by means of synthetic examples.

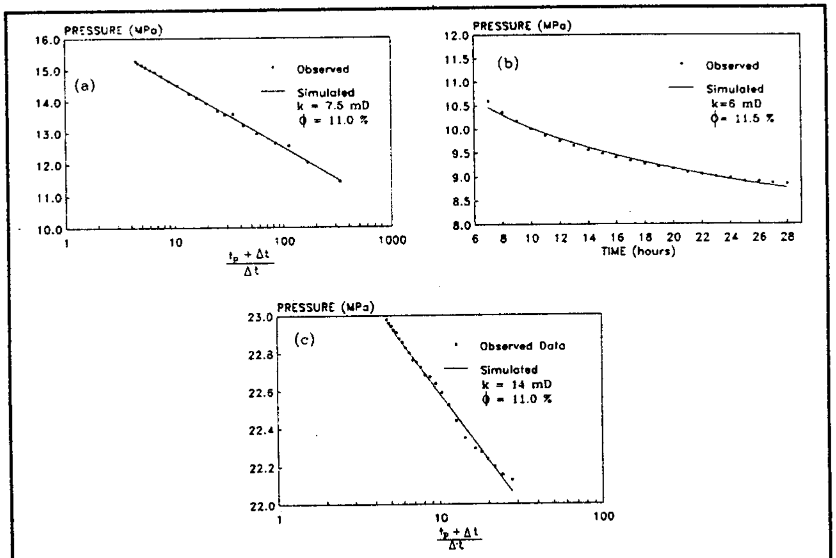


Fig. 3 : Computer match of observed pressures. (a) Well A (Horner plot), (b) Well B , (c) Well C (Horner plot)

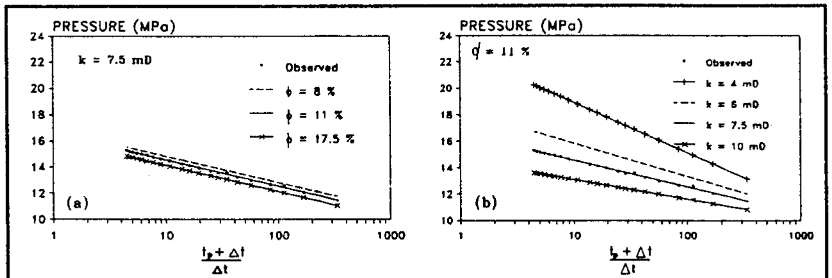


Fig. 4 : Sensitivity Analysis - Well A, (a) porosity and (b) permeability influence

Analysis of Synthetic Well Test Pressure Data

Three draw-down test examples are generated using the mathematical model and radial variations of permeability and porosity. In these examples the skin factor is not taken into account. In Example 1 pressure drawdown data are simulated with radial permeability variation ($\ln k \sim N(4,1)$) and constant porosity (ϕ =20%). They are shown in Fig. 5a. Let us notice that the log-normal has a mean of 4, so the mean permeability value is around 55 mD. The inverse method is applied to determine k and ϕ. The resulting estimates are

$$k = 49 \text{ mD}, \qquad \phi = 8\%, \qquad E_{rel} = 0.98\%.$$

Therefore, the variation in permeability affects porosity estimation. If porosity remains fixed in 20%, $k = 44$mD is calculated and the minimization is worse, $E_{rel} = 2.247\%$. Pressures simulated using both sets of parameters are shown in Fig.5-a. Let us notice that the shape of the synthetic measurements cannot be reproduced with a constant permeability value.

In Example 2 the simulation is performed using a constant permeability, $k = 55$ mD, and porosity values drawn from a normal distribution ($\phi \sim N(20,5)$). Applying the fitting procedure,

$$k = 55 \text{ mD}, \qquad \phi = 21\%, \qquad E_{rel} = 0.090\%.$$

The permeability value is equal to the true one while the porosity value is very close to its normal distribution mean. Pressures simulated with those constant optimum values agree with the synthetic data, as Fig.5-b illustrates.

Finally, in Example 3, radial variations in both parameters are considered in order to generate the set of data ($\ln k \sim N(4,1)$ and $\phi \sim N(20,5)$). Estimates are very similar to those obtained in Example 1:

$$k = 50\text{mD}, \qquad \phi = 8\%, \qquad E_{rel} = 0.79\%.$$

Fig.5-c shows this match.

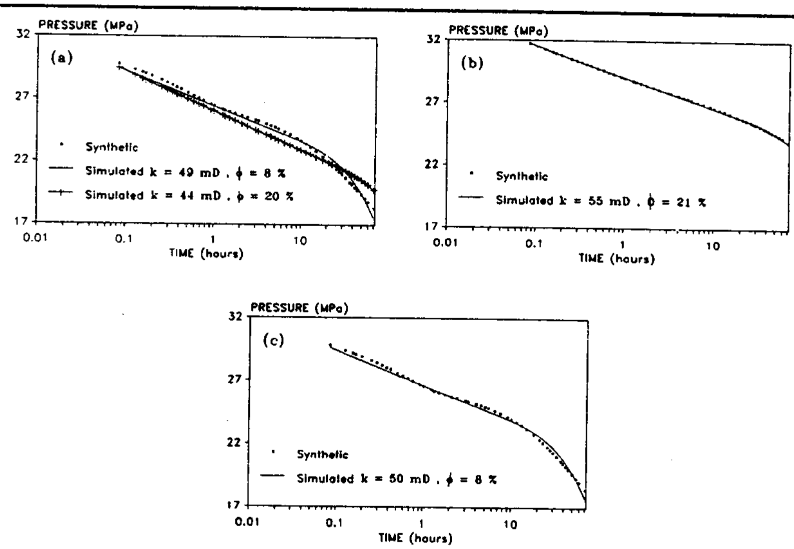


Fig. 5 : Computer match of synthetic data (a) Example 1, (b) Example 2, (c) Example 3

It should be pointed out that the constant permeability estimations (Example 1 and 3) do not agree with the mean value of the distribution, but, in Example 2 and 3, optimum porosity ϕ is approximately equal to normal distribution mean. Besides, variations in porosity affect the well pressure response slightly. Therefore, considering porosity as constant (like the statistical arithmetic mean) may not introduce large errors in the estimation procedure. On the other hand, the influence of the variation in permeability upon pressure values is significant and, in these synthetic cases, no constant value is found that can reproduce the measured data. Moreover, the attempt to find that constant estimate by means of the inverse method leads to a porosity value different to the true one.

CONCLUSIONS

A. In the first part of this work, field permeability and porosity measurements as functions of depth are analyzed in order to obtain their statistical characterization . Conclusions are:

A1 .Porosity data as function of depth fit *normal distributions;*

A2 Permeability observations must be transformed by power functions to fit a known distribution. In the field cases analyzed here, *exponential type probability distributions* are obtained. This is an unusual behavior, due to the fact that data have been measured from very heterogeneous reservoirs. They show many low and a few large permeability values. Therefore, the accepted α-normal permeability behavior must be carefully tested in each particular case because other distributions could better represent field data.

A3 Permeability measures of central tendency obtained straightforward from data are very similar to those computed using the estimated *exponential type frequency distribution functions;*

A4 It is necessary to determine whether it exists a correlation between permeability and porosity. This correlation -when it exists- not only gives important information about properties but also must be introduced in the well test fitting procedure in order to determine only independent parameters. However, correlation goodness must be quantified; meaningless values of parameters may be obtained if a poor relationship is taken into account.

B. In the second part, the influence of permeability and porosity heterogeneities on well test pressure data is studied. Synthetic and field data are used to analyze radial and vertical variations, respectively. Therefore, for the data presented hereinbefore, it is concluded that:

B1. Porosity variations, either radial or different constant values selected from vertical measurements, have a weak influence upon the pressure response. Therefore, the usual approach of considering porosity as a constant is valid and the arithmetic mean is a good porosity estimator.

B2. Permeability heterogeneities, either radial or different constant values selected from vertical data, considerably affect pressure response. Besides, in some cases, no constant permeability is found which can acceptably simulate the measured values.

B3. Permeability estimates obtained from well tests lie between the median and the arithmetic mean of permeability measures from core tests when transformed permeability fits a exponential distribution function.

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