THE MODELLING OF STRUCTURAL DYNAMICS USING BOND GRAPHS

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ABSTRACT

This work discusses the model of dynamic structural problems through the generalized technique of Bond Graphs. It is presented a methodology for the graph construction when a continuous system is modeled by the finite element method. We discuss also the facility to obtain the system graph when the structure must be considered as a dynamic subsystem, i.e., when the continuous media is modeled by several finite elements and is a part of the whole system.

INTRODUCTION

The use of finite element method in structural analysis has received new researches each day, and the developments of complex models are presented in great number of works.

There are, however, difficulties in the modeling of structural dynamic behavior when it must be considered as subsystem of the whole system. In the problem with interactions among subsystems of distinct domains, we must consider the structure as an element that exchange energy with other systems.

Then, for the high technology used in the machine and equipment designs, we need a systematic approach to model theirs components. This methodology only is possible with the use of techniques that permit the modeling of subsystems of distinct physical nature interacting each other [1].

The technique that in last years has been used with efficiency in the physical modeling, simulation and analysis stages is the Bond Graphs. Its unified approach to several physical domains permits the unions among subsystems.

In this work we establish the bond graph representing structural models obtained by the Finite Element Technique. It is shown the procedures to model linear structural systems.

We present, as example, the methodology used to model a beam element by the bond graphs technique. It is considered in the model the mass,

the stiffness and dissipative effects to the beam structure. The modularity of the technique, permitting the inclusion of lumped elements in the model, is also explored.

ELASTIC COUPLING AMONG DEGREES OF FREEDOM

The bond graphs technique considers a physical system being represented by lumped parameters with coupling through the forces equilibrium and the kinematic restrictions referred to the problem. Then if we have a continuous represented by the finite element technique, we can find the stiffness parameters among two any degrees of freedom. Figure 1 shows a continuous with two generic displacements of nodes represented.

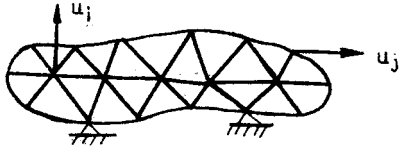

Figure 1. A continuous system with two degrees of freedom.

The global stiffness matrix representing the system has two lines and two columns related to the u_i and u_j degrees of freedom, and can be represented by the matrix:

$$[K] = \begin{bmatrix} \cdots & \cdots & \cdots \\ \cdots & K_{ii} & \cdots & K_{ij} & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & K_{ij} & \cdots & K_{jj} & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix} \qquad (1)$$

Considering only these two degrees of freedom we can write the following matrix:

$$[k] = \begin{bmatrix} k_{ii} k_{ij} \\ k_{ij} k_{jj} \end{bmatrix} \qquad (2)$$

In this matrix each stiffness coefficient is defined considering the elastic effects of the elements associate to the u_i and u_j degrees of freedom.

Particularly to these two degrees of freedom we have the corresponding velocities that, from the bond graph technique, are represented by 1-junctions. If we consider inertias m_i and m_j and forces F_i and F_j referring to these degrees of freedom, the graph representing the dynamic among these velocities can be shown in figure 2.

With the procedures to determine de state equations [2] we have:

Equations for the 1-junctions:

$$\begin{aligned} e_1 &= e_6 - e_5 - e_7 \quad \text{and} \quad f_6 = f_7 = f_5 = f_1 \\ e_2 &= e_0 + e_8 - e_4 \quad \text{and} \quad f_0 = f_8 = f_4 = f_2 \end{aligned} \qquad (3)$$

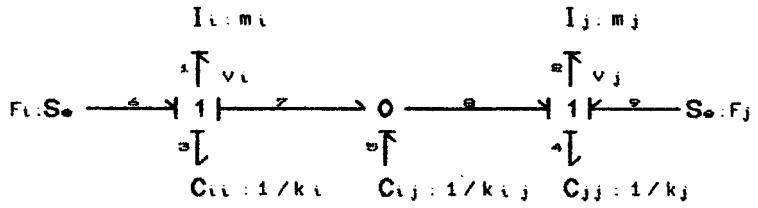

Figure 2. Bond graph representing the coupling among two degrees of freedom.

Equations for the 0-junctions:

$$f_5 = f_8 - f_7 \qquad \text{and} \qquad e_7 = e_8 = e_5 \tag{4}$$

Constitutive Relations:

$$\begin{aligned} f_1 &= p_1/m_i & e_3 &= k_i q_3 \\ f_2 &= p_2/m_j & e_4 &= k_j q_4 \\ & & e_5 &= k_{ij} q_5 \end{aligned} \tag{5}$$

Sources:

$$e_6 = F_i \qquad \text{and} \qquad e_9 = F_j \tag{6}$$

In the relations (3), (4), (5) and (6) e, f, p and q are the generalized variables of effort, flow, momentum and displacement respectively. With these relations we can obtain the state equations, wrote as a function of the energy variables p and q. The algebraic manipulation leads to

$$\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -k_i & 0 & -k_{ij} \\ 0 & 0 & 0 & -k_j & k_{ij} \\ 1/m_i & 0 & 0 & 0 & 0 \\ 0 & 1/m_j & 0 & 0 & 0 \\ -1/m_i & 1/m_j & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ q_3 \\ q_4 \\ q_5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_6 \\ e_9 \end{bmatrix} \tag{7}$$

It is easy identify in the state equations the relations among the state variables and the lagrangean coordinates of the problem. Equations (8) show these relations.

$$\begin{aligned} p_1 &= m_i \ddot{u}_i & q_3 &= u_i \\ p_2 &= m_j \ddot{u}_j & q_4 &= u_j \\ & & q_5 &= u_j - u_i \end{aligned} \tag{8}$$

If we substitute equations (8) in the two first state equations the classical equations of motions used in the finite element method can be obtained.

$$\begin{bmatrix} m_i & 0 \\ 0 & m_j \end{bmatrix} \begin{bmatrix} \ddot{u}_i \\ \ddot{u}_j \end{bmatrix} + \begin{bmatrix} k_i - k_{ij} & -k_{ij} \\ -k_{ij} & k_j - k_{ij} \end{bmatrix} \begin{bmatrix} u_i \\ u_j \end{bmatrix} = \begin{bmatrix} F_i \\ F_j \end{bmatrix} \tag{9}$$

In this matrix form the k_i and k_j elements must be defined by the following relations:

$$k_i = k_{ii} + k_{ij}$$
$$k_j = k_{jj} + k_{ij} \qquad (10)$$

The graph of figure 2 represent, then, the elastic coupling among two degrees of freedom. Analog development can be obtained for the dissipative effects as viscous damping. The graph considering these effects is shown in figure 3.

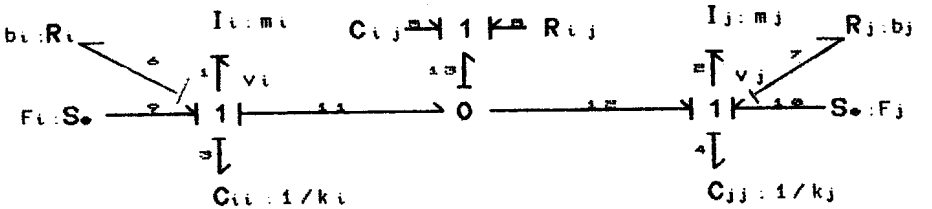

Figure 3. Bond graph for two degrees of freedom including elastic and dissipative effects.

The manipulation of the energy variables for this graph leads to the following state equations:

$$\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} = \begin{bmatrix} (b_{ij}-b_i)/m_i & -b_{ij}/m_j & -k_i & 0 & -k_{ij} \\ -b_{ij}/m_i & (b_{ij}-b_j)/m_2 & 0 & -k_j & k_{ij} \\ 1/m_i & 0 & 0 & 0 & 0 \\ 0 & 1/m_j & 0 & 0 & 0 \\ -1/m_i & 1/m_j & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ q_3 \\ q_4 \\ q_5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_0 \\ e_1 0 \end{bmatrix} \qquad (11)$$

Using the relations (10) and their analogy to dissipative effects, we can shown that the state form (11) represents the classical problem of the dynamic, expressed mathematicaly as

$$[m]\ddot{\underset{\sim}{u}} + [b]\dot{\underset{\sim}{u}} + [k]\underset{\sim}{u} = \underset{\sim}{F} \qquad (12)$$

The [m], [b] and [k] are respectively the mass, damping and stiffness matrices.

THE BOND GRAPH FOR THE BEAM ELEMENT

The classical beam element used in plane structure analysis has four flexure degrees of freedom as shown in figure 4.

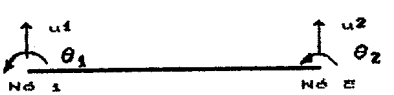

Figure 4. The classical beam element.

Considering an element with length L, elasticity modulus E and moment of inertia I we can write its stiffness matrix.

$$[k] = \frac{2EI}{L^3}\begin{bmatrix} 6 & 3L & -6 & 3L \\ 3L & 2L^2 & -3L & L^2 \\ -6 & -3L & 6 & -3L \\ 3L & L^2 & -3L & 2L^2 \end{bmatrix} \qquad (13)$$

In this matrix the four rows and columns are related to the four degrees of freedom u_1, θ_1, u_2 and θ_2 in this order.

With the presence of terms out of the principal diagonal, the bond graph for this element must consider the coupling to all degrees of freedom, then we must use the graph topology of figure 3.

Considering also the beam inertias as lumped parameters concentrated in the element nodes, we can represent the bond graph for this element as shown in figure 5.

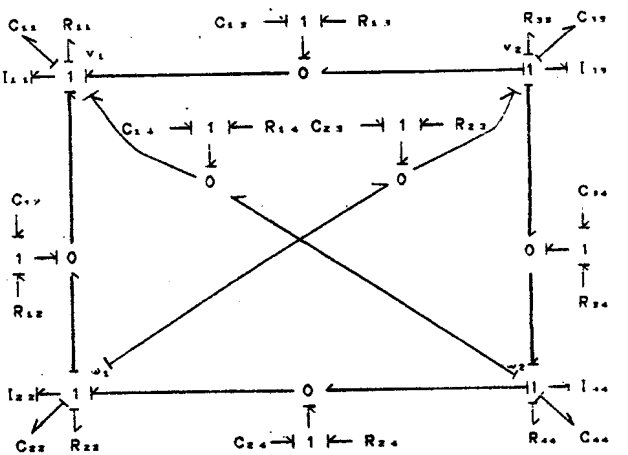

Figure 5. Bond graph for the beam element.

The generalized capacitors represent the beam flexibility and are defined by the expressions:

$$C_{ij} = 1/k_{ij} \qquad p/\ i \neq j \quad e$$
$$C_{ij} = 1/k_i \qquad p/\ i = j \qquad (14)$$

In expressions (14) the k_i elements are defined by equations (10).

$$k_i = \sum_{k=1}^{4} k_{ik} \qquad (15)$$

The generalized resistors represent the dissipative effects which expressions are similar to (14) and (15).

$$R_{ij} = b_{ij} \qquad p/\ i \neq j \quad e$$
$$R_{ij} = b_i \qquad p/\ i = j \tag{16}$$

Where,

$$b_i = \sum_{k=1}^{4} b_{ik} \tag{17}$$

The 1-junctions in the graph of figure 5 represent the absolute velocities related to the four degrees of freedom. In this typical graph the beam is free, without boundary conditions and applyed forces.

The geometric boundary conditions can be easily considered in the graph through the elimination of 1-junctions referred to the null velocities. Then, for example, if this element represents a clamped-free beam, we must consider v_1 and ω_1 nulls. The 1-junctions referred to these velocities and the corresponding bonds must be eliminated. If the beam is also forced in the v_2 direction of node 2, we can consider a effort source in the corresponding 1-junction. The bond graph for this problem can be constructed as shown in figure 6.

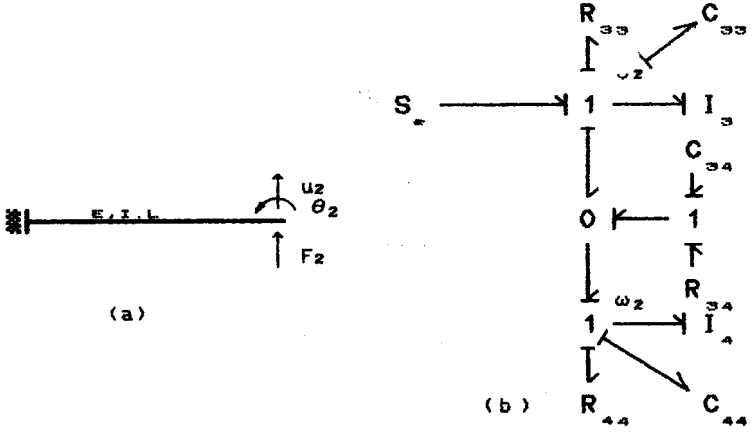

Figure 6. a) Physical model of a clamped-free beam with one beam element.
b) Bond graph of the model.

Reference [2] presents the methodology for the establishment of the mathematical model based on the bond graph. With this methodology we can find the state equation form for the problem. The simulation of these equations leads to the response of the system expressed through its energy variables. We can shown that the state equation form provides the same answers that of the conventional form used by the finite element method.

INTERACTIONS AMONG LUMPED AND CONTINUOUS SYSTEMS

An important advantage of the bond graph approach is the facilities to

represent the interactions among systems of different types. Next we consider a beam with single supports, without dissipative effects and represented by three beam elements. The beam interacts with a lumped mass-spring-damper system and is forced as show in figure 7.

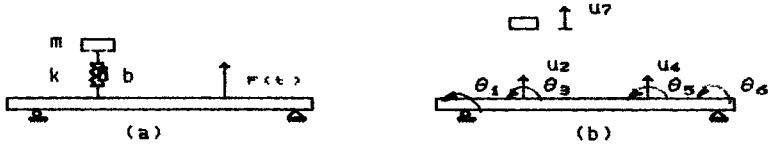

Figure 7. a) Structural system modeled by three beam elements interacting with lumped parameters system.
b) Conventional degrees of freedom adopted by the finite element method.

The bond graph for this system can be represented as illustrated in figure 8, where all degrees of freedom couplings were considered. We can observe also the coupling among subsystems made through the 1-junctions referred to the velocity in the contact point among subsystems. This procedure allows the knowledge of the charge effects between subsystems, i.e., we can determine the actual dynamic influence of one system in other.

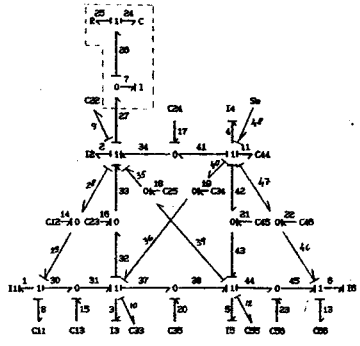

Figure 8. Bond graph for the model of figure 7.

With the bond numbers adopted in figure 7, we have the follow state vector with twenty four energy variables:

$$\underset{\sim}{X}^t = [p_1\ p_2\ \ldots\ p_7\ q_8\ q_9\ \ldots\ q_{24}] \tag{18}$$

The state matrix can be put in the form:

$$[A] = \begin{bmatrix} [A_{pp}] & [A_{pq}] \\ [A_{qp}] & [A_{qq}] \end{bmatrix} \tag{19}$$

Where,

$$[A_{pp}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -b/I_2 & 0 & 0 & 0 & 0 & b/I \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b/I_2 & 0 & 0 & 0 & 0 & -b/I \end{bmatrix} \tag{20}$$

$$[A_{pq}] = \begin{bmatrix} -k_1 & 0 & 0 & 0 & 0 & 0 & k_{12} & -k_{13} & 0 & 0 & 0 \\ 0 & k_{22} & 0 & 0 & 0 & 0 & -k_{12} & 0 & k_{23} & k_{24} & k_{25} \\ 0 & 0 & -k_3 & 0 & 0 & 0 & 0 & k_{13} & -k_{23} & 0 & 0 \\ 0 & 0 & 0 & -k_4 & 0 & 0 & 0 & 0 & 0 & -k_{24} & 0 \\ 0 & 0 & 0 & 0 & -k_5 & 0 & 0 & 0 & 0 & 0 & -k_{25} \\ 0 & 0 & 0 & 0 & 0 & k_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k \\ k_{34} & -k_{35} & 0 & 0 & 0 & 0 \\ -k_{34} & 0 & -k_{45} & -k_{46} & 0 & 0 \\ 0 & k_{35} & k_{45} & 0 & -k_{56} & 0 \\ 0 & 0 & 0 & k_{46} & k_{56} & 0 \\ 0 & 0 & 0 & 0 & 0 & k \end{bmatrix} \tag{21}$$

$$[A_{qp}]^t = \begin{bmatrix} 1/I_1 & 0 & 0 & 0 & 0 & 0 & 1/I_1 & -1/I_1 & 0 & 0 \\ 0 & 1/I_2 & 0 & 0 & 0 & 0 & -1/I_2 & 0 & 1/I_2 & 1/I_2 \\ 0 & 0 & 1/I_3 & 0 & 0 & 0 & 0 & 1/I_3 & -1/I_3 & 0 \\ 0 & 0 & 0 & 1/i_4 & 0 & 0 & 0 & 0 & 0 & -1/I_4 \\ 0 & 0 & 0 & 0 & 1/I_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/I_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/I_5 & 0 & 0 & 0 & 0 & 0 & 1/I_2 \\ 0 & 1/I_3 & -1/I_3 & 0 & 0 & 0 & 0 \\ 0 & -1/I_4 & 0 & -1/I_4 & -1/I_4 & 0 & 0 \\ -1/I_5 & 0 & 1/I_5 & 1/I_5 & 0 & -1/I_5 & 0 \\ 0 & 0 & 0 & 0 & 1/I_6 & 1/I_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/I \end{bmatrix} \tag{22}$$

$$[A_{qq}] = [0] \tag{23}$$

The mathematical system, then, can be represented in its conventional state form.

$$\dot{\underset{\sim}{X}} = [A]\,\underset{\sim}{X} + [B]\,\underset{\sim}{U} \tag{24}$$

The $\underset{\sim}{U}$ vector here is a scalar with the value of the forcing and the [B] is a vector with all null terms, except B(4)=1.

The simulation of equations (24) permits the dynamic analysis of the involved variables. The results are similar to that presented by Margolis [3] where he uses the modal analysis concepts.

Figure 9 shows the representative graphic to some velocity points of the system. (1) Suspended mass; (2) The forcing point and (3) The interaction point.

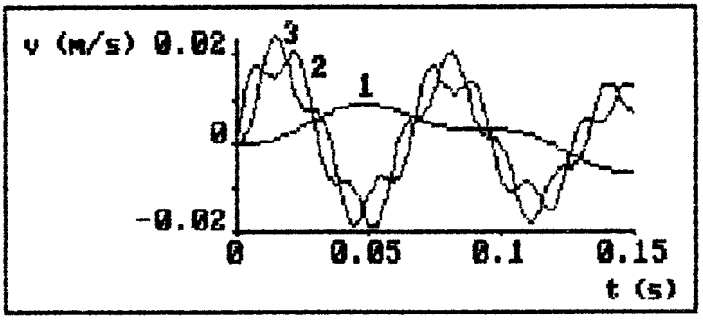

Figure 9. Simulation Results

FINAL COMMENTS

The methodology presented in this work is absolutely general. With the limitations here presented we can make the dynamic analysis of any structural system interacting with others physical domains.

It is importante to note that the number of state variables for this procedure can be very large sometimes. Here, for example we had twenty four state variables for a system with seven degrees of freedom. This is a particular characteristic of the technique that, in a first analysis, do not provide the minimum number of state variables to the problem. This problem can be solved if we adopt some order reduction criterions for the system. An other way to reduce the number of state variables is to define a methodology relating the lagrangean coordinates used by the finite element method with the energy variables of bond graphs technique.

If we use the store and dissipative fields of the technique we don't need reduce the order of the system, because the number of state variables will be exactly twice the lagrangean coordinates for the continuous system.

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