

## AN ADAPTIVE STRATEGY FOR THE NONLINEAR DYNAMIC ANALYSIS OF STRUCTURES

Breno P. Jacob, Nelson F. F. Ebecken  
COPPE/Universidade Federal do Rio de Janeiro  
Programa de Engenharia Civil - Caixa Postal 68506  
21945 - Rio de Janeiro - RJ - Brasil  
E-mail: C0C54001@UFRJ.BITNET

### ABSTRACT

This work presents the development of an adaptive time integration procedure for the nonlinear dynamic analysis of large-scale structural systems. The adaptive algorithm performs the following tasks: a) Automatic time step adjustment: the time step may vary during the analysis, according to the modal composition of the dynamic response and to the nonlinear behavior of the structural system; b) Automatic determination of stiffness reevaluations, in order to minimize the costs related to the calculation of element matrices, and assembling and decomposition of global matrices. An application on the nonlinear dynamic analysis of a compliant guyed tower for deepwater oil exploration and production is presented, and the positive characteristics of the resultant computational system in terms of efficiency, robustness, user-friendliness and quality of the response are evaluated.

### RESUMO

Este trabalho apresenta um procedimento de integração adaptativa no tempo para a obtenção da resposta não-linear dinâmica de sistemas estruturais. O procedimento adaptativo efetua as seguintes tarefas: a) Determinação automática do valor do intervalo de tempo, que pode variar ao longo da análise de acordo com a composição modal da resposta e com o comportamento não-linear do sistema estrutural; e b) Determinação automática dos instantes de tempo com reavaliações da matriz de rigidez tangente, de modo a minimizar as ocorrências de cálculo de matrizes de elementos, e de montagem e decomposição de matrizes efetivas globais. Para a avaliação das características do sistema computacional resultante em termos de economia, robustez e qualidade da resposta, apresenta-se uma aplicação na análise não-linear dinâmica determinística de uma torre complacente estaiada para exploração de petróleo em águas profundas.

### INTRODUCTION

An increasing demand for efficient numerical tools for the nonlinear dynamic analysis of structures has been observed in the last years, motivated particularly by the trend towards the development of new structural concepts which cannot be analyzed and dimensioned without proper consideration of their nonlinear dynamic behaviour. This is the case, for instance, of the Brazilian oil exploration industry, where the *compliant structure* concept has been introduced as an alternative for structural systems designed to support deepwater offshore platforms.

An efficient computational system for the nonlinear dynamic analysis of large-scale structural systems should, ideally, meet the following requirements: I) Present significant reductions in computer costs when compared to conventional systems; and II) Allow also a reduction in the "engineering time" required to obtain a desired analysis response. The objective of this work is, then, to present an adaptive time integration strategy devised having in mind these requirements, particularly oriented towards the analysis of compliant structures for deepwater oil exploration and production, and addressing the following aspects:

- The automatic and adaptive determination of the time step value, which will be free to vary along the analysis according to the modal composition of the response, and also to the nonlinear behavior of the structure at each time instant. The user is then relieved of the

task, required in conventional programs, of selecting a fixed time step following guidelines usually found in the literature, see for instance [1]. Considering first the case of a linear dynamic analysis, these guidelines request the user to estimate the range of frequencies excited by the loading; this estimation requires a free-vibration analysis of the structure, in order to assess its natural frequencies, and also a Fourier analysis of the loading to identify its significant harmonic components. In the more general case of nonlinear dynamic problems, the selection of a time step value is further complicated by the fact that the natural frequencies of the structure will vary along the analysis.

- The automatic and adaptive determination of the time instants when the tangent stiffness matrix is to be reevaluated, consequently minimizing the occurrences of reevaluation of element stiffness matrices, and of assembly and decomposition of global effective matrices. Conventional nonlinear dynamic analysis programs leaves to the user the task of defining a stiffness reevaluation policy, usually based on an input parameter arbitrarily defining the number of time instants between reevaluations.

Adaptive techniques regarding the preceding aspects should, then, constitute the most natural tool for the time integration of nonlinear structural dynamic problems; however, this field is still under active research and the use of such tools is not established. Concerning techniques for the automatic time step determination, the authors are aware of the following developments, grouped according to the particular concept employed: a) Control of time step variation based in approximate error measures [2,3,4]. In [3], for instance, this control is based on the "half-step residual", which is the error evaluated at the time instant corresponding to  $t_{n+1/2} = (t_n + t_{n+1})/2$ ; and b) Control of time step variation based in purely heuristic considerations [5]. These techniques do not properly calculate time step values, but rather adjust or control the variation of user-supplied values. In [5], for instance, the user has to supply allowable upper and lower limits for time step values, which is a requirement not very less severe than that of the selection of a fixed step value.

A third group, introduced in [6] referring to a similar concept presented in [7], consists in the determination of a "characteristic" or "dominant" frequency value at each time instant, from which a time step value can be estimated. The adaptive time integration strategy presented in this work employs this "dominant frequency" concept, allowing the modal composition of the response to be reflected in the calculation of time step values. This strategy is enhanced with new criteria for the rejection of dominant frequency values, and with the consideration of the nonlinear behavior, which is not taken into account in [6]. The nonlinear behavior is also considered through the determination of time instants with stiffness reevaluations. This adaptive strategy presented here was first outlined in [8], and is one of the developments on numerical tools and algorithms for nonlinear dynamic analysis presented in [9].

#### PRELIMINARIES ON THE NONLINEAR DYNAMIC ANALYSIS OF STRUCTURAL SYSTEMS

The mathematical representation of the nonlinear dynamic behavior of structural systems is expressed by an initial value problem consisting of a set of partial differential equations (PDE), with associated boundary conditions in space and initial values in time. The solution of this problem is in general obtained through the use of numerical methods; the usual procedure is to employ a semidiscretization technique [10], where the PDE are first discretized in space by the Finite Element Method [1,11], yielding a set of second-order ordinary differential equations (ODE) also known as the *semi-discrete equations of motion*. This set of ODE may then be discretized and solved in time by an appropriate time integration algorithm.

For nonlinear elastodynamic problems, the semidiscrete equations of motion can be written as:

$$M \ddot{U}(t) + R(u) = F(t) \quad (1)$$

where  $\ddot{U}(t)$  and  $u(t)$  are vectors containing unknown values of, respectively, accelerations and displacements corresponding to the nodal degrees of freedom;  $M$  is the mass matrix, and  $M \ddot{U}(t)$  is equivalent to a vector of inertia forces;  $R(u)$  is a vector of internal forces including contributions of elastic and damping forces, and  $F(t)$  is a vector of external loads.

Implicit algorithms are the most suited for the time discretization and integration of the class of inertial problems which comprises the applications studied in this work [9]. These algorithms operate on the following form of the equations of motion, now discretized with

respect to time:

$$M \dot{a}_{n+1} + R(d_{n+1}) = F_{n+1} \quad (2)$$

In this expression,  $a_{n+1}$  and  $d_{n+1}$  represent stepwise approximations to the "exact" values  $\dot{u}(t_{n+1})$  and  $u(t_{n+1})$  respectively. The implicit algorithm employed in the developments presented in this work is a member of the Newmark family [12], the unconditionally stable and second-order accurate algorithm known as "constant average acceleration method" or "trapezoidal rule" [1].

The strategy usually employed for the treatment of the nonlinear effects with implicit time integration algorithms consists in assuming a linearization in the neighborhood of the displaced position corresponding to the instant  $t_{n+1}$  [13]. This linearization is expressed as a truncated Taylor series, leading to the definition of a tangent stiffness matrix  $K_T$  and to the following incremental expressions for the nonlinear equations of motion:

$$M \dot{a}_{n+1} + K_T \Delta d = F_{n+1} - R(d_n) \quad (3)$$

$$d_{n+1} = d_n + \Delta d \quad (4)$$

As a consequence of the linearization assumed, the incremental equations (3) do not assure equilibrium at the end of time step  $t_{n+1}$ . Although earlier approaches ignored this fact and employed purely incremental techniques to obtain the dynamic response [1,14], it is nowadays established [1,15] that the use of iterative techniques for the solution of nonlinear systems of equations is mandatory in order to obtain reliable dynamic responses. The developments of this work employ the Modified Newton-Raphson (MNR) method [1,11], where the tangent stiffness is kept constant along the iteration process and, as will be seen in the next section, is also kept constant along a certain number of time steps.

The incremental-iterative form of the discrete equations of motion (3) is then expressed by the following expression, where damping effects are introduced through a Rayleigh proportional damping matrix  $C$  [1]:

$$M \dot{a}_{n+1}^{(k)} + C \dot{v}_{n+1}^{(k)} + K_T \Delta d^{(k)} = F_{n+1} - R(d_{n+1}^{(k-1)}) \quad (5)$$

$$d_{n+1}^{(k)} = d_{n+1}^{(k-1)} + \Delta d^{(k)} \quad (6)$$

The application of the Newmark time-integration operators to the incremental-iterative equations of motion (5) finally leads to the following set of *effective* system of algebraic equations, to be solved at each iteration of the MNR technique [9]:

$$\hat{A} \Delta d^{(k)} = \hat{b}^{(k-1)} \quad (7)$$

The *effective matrix*  $\hat{A}$  is defined as

$$\hat{A} = \hat{\omega}_0 M + K_T \quad (8)$$

where  $\hat{\omega}_0$  is a coefficient expressed in terms of the Newmark parameters, the time step value, and the Rayleigh damping parameters;  $\hat{b}^{(k-1)}$  is a vector of *effective residuals* calculated in terms of the external loads and of elastic, damping and inertia forces of the previous iteration. Details of an optimized implementation of this time integration strategy for nonlinear problems are presented in [9].

#### ESTIMATION OF TIME STEP VALUE

Many iterative techniques for the solution of generalized eigenvalue problems employ the Rayleigh quotient [1], which is defined by the following expression:

$$q(x) = \frac{x^T K x}{x^T M x} \quad (9)$$

If, in this expression, vector  $x$  is replaced by an eigenvector  $\phi$ , the corresponding eigenvalue will be given by  $q(\phi)$ . Considering now the incremental equilibrium equations (3), an expression similar to (9) may be written as:

$$\omega_{n+1}^2 = \frac{\Delta d^T K_T \Delta d}{\Delta d^T M \Delta d} \quad (10)$$

It can be seen that the value  $\omega_{n+1}$  thus obtained reflects the modal composition of the dynamic response at the time step  $t_{n+1}$  [6]. This value is considered as an estimation of a dominant or characteristic frequency for this time step, from which a corresponding value for a characteristic period can be calculated as:

$$T^{\circ} = 2\pi / \omega_{n+1} \quad (11)$$

An estimation of a time step value, which can adequately allow the integration of the mode shapes defining the composition of the response at this time instant, may then be obtained. This estimation is given as a fraction  $\lambda$  of the characteristic period  $T^{\circ}$ :

$$\Delta t^{\circ} = T^{\circ} / \lambda \quad (12)$$

The calculation of the dominant frequency at every time step by expression (10) would be computationally expensive, since it would require a matrix-vector product involving the global tangent stiffness matrix. This calculation can be performed in a more economical way by rearranging expression (8) and then replacing it in (10):

$$\omega_{n+1}^2 = \frac{\Delta d^T (\hat{A} - \hat{\alpha} \hat{\omega} M) \Delta d}{\Delta d^T M \Delta d} = \frac{\Delta d^T \hat{A} \Delta d}{\Delta d^T M \Delta d} - \hat{\alpha} \hat{\omega} \quad (13)$$

Considering now expression (7), the value of the dominant frequency will be obtained as:

$$\omega_{n+1}^2 = \frac{\Delta d^T \hat{b}}{\Delta d^T M \Delta d} - \hat{\alpha} \hat{\omega} \quad (14)$$

#### TIME STEP VARIATION

Expression (12) allows the evaluation of a new time step value for every discrete time instant of the integration process. However, the utilization of this expression without further considerations could lead either to sharp alterations of the time step, or to very small and unnecessary alterations, with the consequent reevaluations and decompositions of the effective matrix since, according to exp. (8), it depends on the time step value which enters in the calculation of the coefficient  $\hat{\alpha} \hat{\omega}$ . (This applies for linear analysis, where the reevaluation of the stiffness matrix itself is not necessary; later in this work other considerations pertinent to nonlinear analysis will be presented).

In order to avoid such undesired situations, heuristic rules are employed to determine when and how an alteration of the time step value should effectively be performed, based on the ratio between the estimated new step value and the current step value:

$$\xi = \Delta t^{\circ} / \Delta t_n \quad (15)$$

These rules are then expressed as:

$$\text{If } \xi < \xi_{\min} \text{ then } \Delta t_{n+1} = \xi_{\min} \Delta t_n; \quad \text{If } \xi > \xi_{\max} \text{ then } \Delta t_{n+1} = \xi_{\max} \Delta t_n;$$

$$\text{If } \xi_1 < \xi < \xi_2 \text{ then maintain } \Delta t_{n+1} = \Delta t_n, \text{ else } \Delta t_{n+1} = \Delta t^{\circ}.$$

Typical values for the adjustable parameters  $\xi_{\min}$ ,  $\xi_1$ ,  $\xi_2$  and  $\xi_{\max}$  may be given as, respectively: 0.5; 0.625; 1.6; 1.8. The time step value for the first time step may be given as a rough estimation, since it will be automatically adjusted in a few number of time steps.

The strategy described up to this point is basically similar to the one presented in [6], which is most suited for linear problems since only the variation of the modal composition of the response is accounted for, through the calculation of the dominant frequency, and there is no reference to the consideration of the nonlinear behavior. The following sections will present new developments: the next section presents new criteria for the determination or rejection of dominant frequency values; then follows an extension for the consideration of the nonlinear behavior in the adaptive strategy.

### CRITERIA FOR THE DETERMINATION OF THE DOMINANT FREQUENCY

At time instants of the dynamic response near to situations of maximum or minimum amplitude, the norm of the incremental displacements is relatively small, and thus the application of (14) for the calculation of the dominant frequency is prone to be affected by numerical problems. This fact is also remarked in [6], where it is suggested that the calculation of a new value for the dominant frequency, and consequently of a new estimation for the time step value, should be avoided when

$$\|\Delta d\|_{n+1} < c \|\Delta d\|_n, \quad c = 0.1 \quad (16)$$

However, in the applications considered in this work, which include environmental loads due to wave and current, the variation with time of the norm of the incremental displacements presents high-frequency components. This variation is not, then, "smooth" as the application of rule (16), based on the comparison of the norm at the current step with the norm of the previous step, would expect. The solution proposed in this work is composed by two complementary strategies a) and b) described below:

a) Instead of employing local criteria such as (16) to avoid the calculation of a new value for the dominant frequency, "global" rules are defined, based the following expression for an "average" norm of incremental displacements:

$$\|\Delta d\|_{n+1}^{\circ} = \|\Delta d\|_{n+1} / \Delta t_{n+1} \quad (17)$$

Corresponding maximum and minimum historical values are defined as respectively  $\|\Delta d\|_{\max}^{\circ}$  and  $\|\Delta d\|_{\min}$ ; in the computational implementation, situations where the loading is applied incrementally are accounted for by starting the process of determination of the minimum value only after the first maximum value is found.

The first rule checks if  $\|\Delta d\|_{n+1}^{\circ}$  is smaller than a fraction of the historical maximum:

$$\|\Delta d\|_{n+1}^{\circ} < c \|\Delta d\|_{\max}^{\circ} \quad (18)$$

The second rule checks if  $\|\Delta d\|_{n+1}^{\circ}$  is contained in an interval defined by

$$\left[ \|\Delta d\|_{\min}^{\circ}, c (\|\Delta d\|_{\max}^{\circ} - \|\Delta d\|_{\min}^{\circ}) \right] \quad (19)$$

In situations where both rules are satisfied, the proposed strategy considers that the magnitude of the quantities involved in the calculation of a dominant frequency value by expression (14) is relatively small, and thus the quality of the obtained frequency value could be prejudiced. In these situations, the time step value from the previous step is maintained. Typical values for  $c$  may range from .05 to .2 .

It should be observed that these rules were devised having in mind the applications considered in this work (compliant deepwater structures under environmental periodic loadings). Other applications, including for instance impulsive loads or plasticity effects, could need further considerations.

b) In situations where the rules (18) and (19) allow the calculation of new dominant frequency values, the following expression is employed in order to filter high-frequency components:

$$\omega_{n+1}^{\circ} = (\omega_{n+1} + \omega_n) / 2 \quad (20)$$

The estimation of the new time step value by expressions (11) and (12) then employs  $\omega^{\circ}$  instead of  $\omega$ .

### CONSIDERATION OF THE NONLINEAR BEHAVIOR

This section extends the adaptive strategy, in order to take into account the nonlinear behavior, which affects two aspects of the analysis: the first is the calculation of time step values, and the second is the determination of time instants with stiffness reevaluation.

Variation of the  $\lambda$  parameter

The effect of the nonlinear behavior on the calculation of time step values will be reflected by varying the  $\lambda$  parameter, which determines the time step estimation as a fraction of

the characteristic period, see exp. (12). The variation of this parameter is performed by monitoring the number of iterations ( $N_{it}$ ) required for convergence of the iterative MNR process at each time step.

The following adjustable parameters must now be introduced:  $N_{ot}$  = "optimum" number of iterations for convergence of the MNR process;  $\zeta$  = scale factor affecting the value of  $\lambda$ ;  $\lambda_{min}$  = minimum allowable value for  $\lambda$ ;  $\lambda_{max}$  = maximum allowable value for  $\lambda$ . Typical values for these parameters are, respectively: 3, 1.05, 20 and 500. The following rules then define the variation of  $\lambda$ :

$$\begin{aligned} & \text{Initially } \lambda = \lambda_{min}; \\ & \left\{ \begin{array}{l} \text{If } N_{it} < N_{ot} \text{ reduce } \lambda = \lambda / \zeta; \\ \text{If } N_{it} = N_{ot} \text{ maintain current value of } \lambda; \\ \text{If } N_{it} > N_{ot} \text{ increase } \lambda = \lambda * \zeta, \\ \quad \text{if } \xi < 1 \text{ reduce time step.} \end{array} \right. \quad \left\{ \begin{array}{l} \text{If } \lambda < \lambda_{min} \text{ then } \lambda = \lambda_{min}; \\ \text{If } \lambda > \lambda_{max} \text{ then } \lambda = \lambda_{max}. \end{array} \right. \end{aligned}$$

The application of these rules in the control of the time step variation comprises the following considerations:

- At time instants when the iterative MNR process is requiring more iterations for convergence ( $N_{it} > N_{ot}$ ), the value of the  $\lambda$  parameter is increased, thus leading to a more rapid reduction of the time step value, which should represent an increment sufficiently small not to disturb the ability of the MNR to treat the local linearization assumed in (3). It is observed that this rule overcomes the rules previously presented for the control of time step variation, since the time step is reduced even if  $\xi_1 < \xi < 1$ ;
- Conversely, at time instants when nonlinear effects are less severe ( $N_{it} < N_{ot}$ ), the value of the  $\lambda$  parameter is reduced, thus leading to larger time step values;
- The value of the  $\lambda$  parameter cannot be reduced beyond  $\lambda_{min}$  in order to allow the adequate integration of the modes with frequencies near or smaller than the dominant frequency;
- The value of the  $\lambda$  parameter cannot be increased beyond  $\lambda_{max}$  in order to avoid a hypothetical limit situation when the time step value is indefinitely reduced. In these cases it would be better to allow the number of iterations  $N_{it}$  to grow until it reaches a maximum value of  $N_{max}$ , thus stopping the analysis when the instability of the time integration is characterized (it should be observed that, in the applications studied by the authors, the rules defining the adaptive strategy were always able to maintain the number of iterations  $N_{it}$  not far from the optimum value  $N_{ot}$ ).

#### Control of Stiffness Reevaluations

For the determination of time instants with stiffness reevaluation, it is considered that the variation of the dominant frequency reflects not only the variation of the modal composition of the response, but also the variation of the stiffness of the structural system; moreover, the control of the  $\lambda$  parameter, as presented in the preceding section, stresses this influence of the nonlinear behavior in the calculation of the time step values.

The first criterion for the determination of a stiffness reevaluation is, thus, an alteration of the time step value as dictated by the rules previously presented. As mentioned earlier, an alteration of the time step value requires the reevaluation and decomposition of the effective matrix even in linear analyses; in nonlinear analyses this criterion will also trigger the reevaluation of the stiffness matrix.

The second criterion is more directly related to the monitoring of the iterative MNR process: stiffness reevaluations are determined whenever the number of iterations  $N_{it}$  exceeds the "optimum" value  $N_{ot}$ .

#### ADAPTIVE ANALYSIS OF A COMPLIANT GUYED TOWER

Deepwater compliant structures are characterized by their ability to undergo large displacements under the action of environmental loads. As a first consequence of this flexibility, they present a markedly nonlinear behavior due to large-displacement geometric effects. As another consequence, the first natural period is considerably greater than the characteris-

tic period of the ocean waves; inertia forces then help to support the environmental loadings and contribute in the stability of the structure.

The guyed tower is a particular concept of compliant structure, whose behavior has been extensively studied, leading to the design, fabrication and installation of the Lena Guyed Tower at the Gulf of Mexico [16,17]. The guyed tower concept is characterized by the solution employed to provide the righting forces which, along with the inertia forces, support the environmental loadings: guylines connected to the tower at a position near to the level of the resultant of these loads. An important characteristic of these guylines consists in their highly variable stiffness values, provided by a "clump weight" mechanism near the seabed touchdown point. The stiffness increases as these weights are being suspended; however, in storm condition some weights are totally uplifted, thus increasing the catenary length, reducing the guyline stiffness and limiting the maximum stress on the cable [17,18]

Parametric studies on the global behavior of a 330m-high guyed tower, under environmental loadings typical of the southeastern Brazilian coast, were presented in reference [18]. The guyed tower investigated in that work consists of an external tower with a 27-meter square section, which is supported by an internal tower with a 7-meter square section comprising the four main axial piles. Each axial pile, extending up to the top of the tower, has a diameter value of 1.20m and a thickness value of 0.10m. The connection between the external tower and the piles is made by intermittently placed guides.

The corresponding finite element model employs 468 three-dimensional nonlinear beam elements [19,20] in the discretization of the tower. The behavior of the guylines is simulated by non-linear springs with associated force functions determined by previous static analyses [18]. This discretization, referred to as the "complete model", is shown at Figure 1 and was employed in [18] for static linear, static nonlinear, and free-vibration analyses, whereas a simplified or "stick" model was employed for nonlinear dynamic analyses addressed to the study of the global behavior of the structure.



Figure 1  
Guyed Tower Finite Element Model

The objective of this section is then to present results of nonlinear dynamic analyses of that complete model, employing the computational system incorporating the adaptive strategy. The excellent performance characteristics of this strategy are demonstrated, in terms of computational economy, quality of results, and savings in "engineering time" reflected in the number of analyses needed to obtain the desired dynamic response.

These analyses correspond to environmental storm conditions, consisting of a deterministic Airy wave with a period value of 14s and a height value of 18m; current velocity at sea surface is 1.45m/s, at sea bottom is 0.25m/s; the reference wind velocity is 55m/s. The first set of analyses is referred as the "conventional analyses", employing a fixed time step and reevaluating the structural stiffness at the beginning of every time step.

The fixed time step value for the first conventional analysis is 0.025s, which is the same employed in [18] for the stick model. This step value was selected according to the guidelines presented in [1] and mentioned at the Introduction of this work; it is a relatively small value, since the parametric studies performed in [18] indicated that the first natural period could vary from 27s to 10s according to the variation of the guyline stiffness.

This first analysis presented convergence problems, observed after the integration of 15s; the divergence of the solution was characterized at the time instant of 31.4s, when the maximum specified number of 15 iterations for equilibrium in the Newton-Raphson scheme was exceeded. It is thus seen that even the reevaluation of the stiffness at every time step was

not able to allow the conclusion of the dynamic response.

A second conventional analysis is then performed, reducing the fixed time step value to 0.01 s, but nevertheless a similar divergent behavior was observed. The conclusion of a dynamic response for the desired integration time (40 s, corresponding to about three wave periods) was reached only in the third conventional analysis, with the fixed time step value specified as 0.005 s.

The time histories of the horizontal displacement at the top of the tower, corresponding to the first and third conventional analyses of the complete model, are presented in Figure 2. The divergent behavior of the first analysis is there well clear, and can also be seen at the graph presented in Figure 3, showing the number of iterations required for the convergence of the Newton-Raphson iterative process at each time instant.

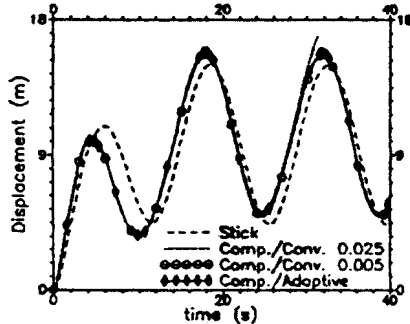


Figure 2  
Top Horizontal Displacement

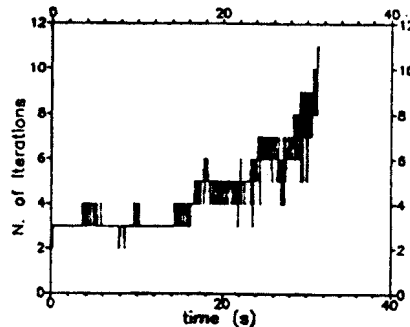


Figure 3  
Conventional Analysis Divergent Behavior

Another interesting observation can be drawn from Figure 2, which also includes the results obtained by a conventional analysis of the stick model presented in [18]. There is a reasonable agreement between the results of the stick and complete models, confirming that the stick model devised for the study of the global behavior of the tower is well calibrated.

An adaptive analysis is now performed, employing the typical values for the adjustable parameters mentioned in the corresponding sections. The integration of 40 s is completed with a total number of 1158 time instants, about seven times less than the number required by the third conventional analysis (the only one able to complete the analysis). The tangent stiffness is reevaluated in only 81 instants, which represents an average number of 14.3 instants between reevaluations, and is about a hundred times less than the number of reevaluations observed in the third conventional analysis.

Figures 2 and 4 compares the dynamic response thus obtained with the corresponding responses obtained by the conventional analyses. The time histories of the horizontal displacement at the top of the tower are compared in Figure 2. Figure 4 compares axial forces in an element at the base of one of the main axial piles, obtained by the third conventional analysis and by the adaptive analysis.

The effectiveness of the adaptive strategy in the treatment of the nonlinear behavior can be observed in Figure 5, showing the number of iterations required for the convergence of the Newton-Raphson iterative process at each time instant. It can be observed that this number was always kept around the specified optimum value of 3 iterations. This adaptation of the algorithm to the nonlinear behavior is also reflected by Figure 6, presenting the variation of the  $\lambda$  parameter along the adaptive analysis. Smaller values for this parameter were required near to instants of maximum displacements.

Figure 7 presents the variation of the time step value along the adaptive analysis. This variation reflects both the modal composition of the response, through the calculation of the dominant frequency value, and the nonlinear behavior, through the control of the value of the parameter  $\lambda$ . The two dashed straight lines represent the fixed time step values of 0.025 and 0.005 employed, respectively, in the first and third conventional analyses.



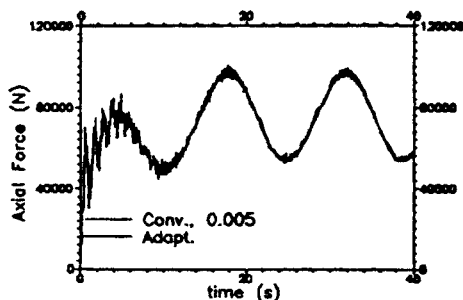


Figure 4  
Axial Force at Base of Main Pile

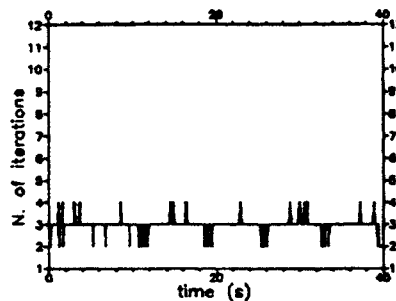


Figure 5  
Adaptive Analysis Convergence Behavior

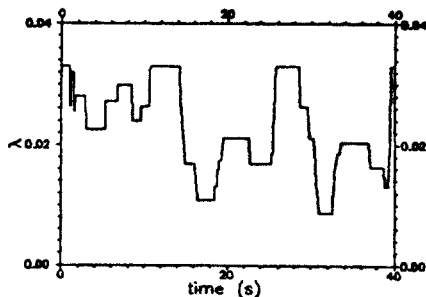


Figure 6  
Variation of  $\lambda$  Parameter

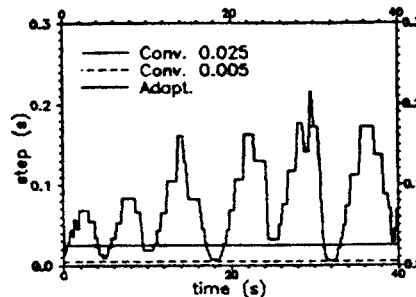


Figure 7  
Variation of Time Step Value

These results demonstrate that the adaptive strategy was able to detect the instants when smaller values of time step are required, and, conversely, the instants when larger time step values can be employed; and also that, even reevaluating the tangent stiffness in a much smaller number of instants, this strategy allowed the conclusion of the analysis without convergence problems.

The computational cost required by the only adaptive analysis necessary to obtain the dynamic response is then a fraction of the costs of each one of the conventional analyses. Table I summarizes the computing costs of the presented analyses, showing the total number of instants needed to obtain the desired response, the number of stiffness reevaluations, the total number of iterations and the total CPU time spent in a VAX 8810 computer.

Table I  
Computing Costs

| Analysis   | Number of:      |               |            | CPU(s)  |
|------------|-----------------|---------------|------------|---------|
|            | intervals       | reevaluations | iterations |         |
| Conv. .025 | (not completed) |               |            | xxx.x   |
| Conv. .010 | (not completed) |               |            | xxx.x   |
| Conv. .005 | 8000            | 8000          | 16538      | 48274.8 |
| Adaptive   | 1158            | 81            | 3418       | 4130.1  |

#### CONCLUSIONS

The application of the presented adaptive time integration strategy on the analysis of a compliant guyed tower has shown that the savings in computer time are remarkable; moreover, the comparison between the number of adaptive and conventional analyses necessary to obtain the desired dynamic response reflects other important positive characteristics of the resultant computational system.

One of these improved characteristics is the robustness, represented by the ability to conclude a nonlinear analysis without convergence problems. Also, probably still more valuable than the savings in computer time are the savings in "engineering time", or the economy in the time spent by the engineer in data preparation and submission of computer jobs; this is a characteristic of a more easier-to-use or "user-friendly" computational system.

These advantages stem from the adaptive strategy which relieves the user from the task of selecting time step values, and defining stiffness reevaluation policies. Finally, since the computational system does not depend on the user's experience and judgment in the definition of such important parameters, another positive characteristic consists in the higher reliability on the quality of the obtained dynamic response.

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