NUMERICAL GRID GENERATION : AN OPTIMIZATION STUDY

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ABSTRACT

This paper discusses some conditions to be met considering an optimized grid generation to be used on computational fluid dynamics. Few numerical examples considering irregularly shaped geometries mapping are presented, for discussion. Finally, a simulation of a simple 2-D conduction heat transfer problem is performed for error assessment.

RESUMO

Este trabalho apresenta algumas condições a serem satisfeitas para a geração otimizada de malhas para uso em dinâmica dos fluidos computacional. Alguns exemplos considerando o mapeamento de geometrias de formato irregular são apresentados, para discussão. Finalmente, faz-se a simulação de um problema simples bi-dimensional em condução de calor para avaliação dos erros envolvidos.

INTRODUCTION

The numerical solution of the governing equations describing fluid flow processes using finite difference methods involve the substitution of the physical domain by a point discretized one, in which the local variables are defined and are to be known. As may be expected, the localization and concentration of such points have a strong influence on the accuracy of the seeked solution, as long as it affects truncation errors, numerical boundary conditions and even the resolution of boundary layers or re-circulation regions. Consequently, it is not surprising to observe the huge amount of information available on the technical literature concerning grid generation.

Most current problems being studied on computational fluid dynamics involve the discretization of irregularly shaped regions, usually with points concentrated near solid walls. Although the usage of regular coordinates lines (e.g. cartesian, polar, etc.) may sometimes be preferred due to its simplicity, in several situations it causes extra problems and recent general purpose codes have been developed using body-fitted curvilinear generalized coordinate systems. The present paper relates to the numerical grid generation that is to be used by such codes. Although the concepts to be develop here applies equally to any 2 or 3-D grid generation technique, they are applied to the technique based on the solution of 2-D elliptic equations [1], for simplicity.

ELLIPTIC GRID GENERATION

The experience has shown that for higher efficiency, the (elliptic or any other) system of differential equations in use for grid generation needs some characteristics, such as the ability for:

i. the generation of a smooth point distribution inside the domains.

ii. presenting a one-to-one correspondence between physical and computational domain.

iii. generate concentration of points close to or far from any point or line inside the region, including boundaries.

Among others, the following system has been shown to present those characteristics:

$$\nabla^2 \xi = P$$

$$\nabla^2 \eta = Q$$

where η and ξ are generalized curvilinear coordinates, fitted to the boundaries and P and Q control grid spacing. Following differential calculus, the system above is transformed to:

$$\begin{aligned} \alpha\, x_{\xi\xi} + \gamma\, x_{\eta\eta} - 2\beta\, x_{\eta\xi} + J^2 (x_\xi P + x_\eta Q) &= 0 \\ \alpha\, y_{\xi\xi} + \gamma\, y_{\eta\eta} - 2\beta\, y_{\eta\xi} + J^2 (y_\xi P + y_\eta Q) &= 0 \end{aligned} \qquad (1)$$

where J indicates the Jacobian of the transformation, defined as:

$$J = x_\xi y_\eta - x_\eta y_\xi$$

and α , β and γ are the metric coefficients of the transformation. Their definitions are available on the literature (e.g. [1]).

Naturally, J must be non-zero, to satisfy condition ii above. Also, system (1) has to be solved simultaneously by any suitable method for solving elliptic systems. For that, suitable boundary conditions, i.e. node distribution along the boundary, must be given and the solution of this problem indicates the node distribution inside the domain. The present paper deals with the semi-automatic investigation of the node distribution along the boundaries in order to avoid dangerously small (numerical speaking, at least) Jacobians or even the collapse of coordinate lines due to irregular shape. In other words, the present paper deals with an inverse problem : it is desired to determine which boundary node distribution for a particular apllication must be used in order to minimize numerical problems. Next section discusses some of the conditions to be met.

NUMERICAL CONDITIONS TO BE MET

Consider the geometry described on figure 1, below. Although simple, this geometry may describe quite different physical situations. For instance, if inlet flow is assumed through face ED, the physical situation has been intensively studied as it describes the back-step problem, a standard model for separated flows simulation [2]. If, however, flow is allowed to enter the geometry through face FA, a convergent channel flow, i.e. pipe contraction, is described [3]. Actually, several other physical situations may be modelled using this same geometry. Although the same grid generation system may be used, regardless the physical situation, experience has indicated that a few numerical problems occur provided some conditions are not met. Naturally, this may result in non-accurate solutions and sometimes not even convergence is achieved. For example, whenever convection is dominant and grid-to-flow skewness is large, numerical viscosity effects may appear (see e.g. [4]). To minimize this, it has been necessary to use stream-line following grids and therefore different meshes for different situations. There are other problems.

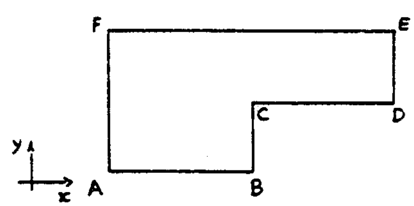

Fig. 1 Geometrical Model. Test no 1.

In many cases, e.g. for pure heat conduction problems, the mesh selection may be made considering only minimization of the coefficient of the cross-derivative terms appearing on system (1), named β . As it may be recalled, $\beta = 0$, indicates orthogonal grid, a most desired feature of such grids, specially close to boundaries. The effect of nonorthogonal grids on the accuracy of numerical solutions has been studied by Maliska & Silva [5], among others. In other cases, the mesh selection may be made considering having relatively uniform Jacobian values distribution, mainly at neighboring cells, to avoid large truncation errors due to unequal mesh sizes distribution. Actually, these two aspects may be combined to a single parameter to be controlled, the ratio between β and the square of the local Jacobian value. Also, in all cases, it is extremely important that the angle between unit vectors along the transformed coordinated lines be larger than zero, to avoid the collapse of these lines, as previously mentioned.

Unfortunately, it is not simple to satisfy all such conditions, or even only some of them, a priori. Consequently, it is highly convenient the usage of a small computer in order to hender this investigation feasible, i.e. fast and accurate. It has been the experience of the present authors, that a fully automatized search is of difficult implementation, as problem-to-problem physical insight is most important, as previously mentioned. Therefore, the intervenience of the researcher has been very helpful for that. Next, it is described some of the features developed on [6] to help this search.

PRESENT COMPUTATIONAL DETAILS

During the preliminary stages of the current study, it was felt necessary to develop an efficient yet simple search. Among some other options, the one developed to be used on [6], is based on the translation coupled to rotation of the coordinates center. That is, by moving the center of coordinates, initially at point A, say, on the figure above, along the boundary,to point B or other, several grids may be easily generated, to allow detailed investigation of the best features desired. The way this was implemented for the present study is quite simple. After specifying the nodes locations assuming point A for example, as origin, the desired translation and rotation is implemented using the formulae below:

$$\begin{aligned} xx &= XPTOLD - FX \\ yy &= YPTOLD - FY \end{aligned}$$

and

$$\begin{aligned} XPTNEW &= \quad xx * COS\ (ANGLE) + yy * SIN\ (ANGLE) \\ YPTNEW &= - xx * SIN\ (ANGLE) + yy * COS\ (ANGLE) \end{aligned}$$

where XPTOLD and YPTOLD correspond to the previous distribution, XPRNEW and YPTNEW correspond to the new distribution, FX and FY indicate the desired x and y translation and ANGLE indicates the desired rotation. As it is seen, this is a very simple technique.

It is not difficult to visualize that due to different boundary node distribution (input), the inner node distribution changes sustantially (output). Consequently, the metric coefficients are affected.

Figures 2 a,b and c indicate some of the possible selections. It is interesting to note that, in all three figures, the maximum value for β is of O (10^{-3}). However, as the areas of the smaller elements are quite small, β / J^2 maximum values are large, as shown on table I.

TABLE I : Maximum Metric Coefficients for Figure 2 case study.

Fig.	α / J^2	β / J^2	γ / J^2	$\cos\Omega$	$\cos\Omega \rvert_{max}$
2 a	882	1096	1875	0.85	0.85
2 b	1086	1251	5518	0.61	0.64
2 c	2887	596	1348	0.30	0.72

By definition, [1], the cosine of the angle between unit vectors is defined as

$$\cos \Omega = \frac{\beta}{\sqrt{\alpha}\sqrt{\gamma}} \tag{2}$$

On table I above, the fifith column indicates the value of the cosine at the point of maximum value for β / J^2 inside the region (the

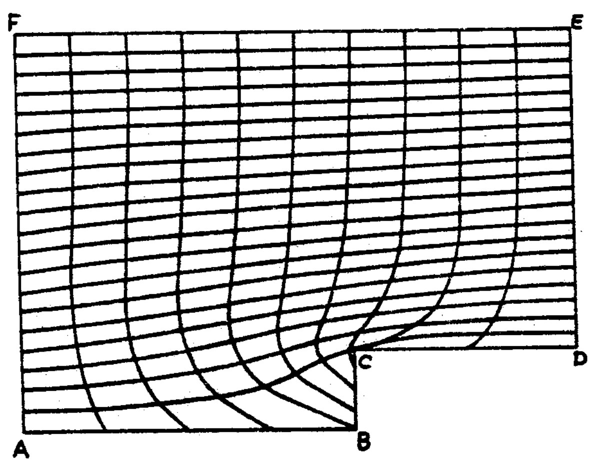

Fig. 2 a Grid Generated for 11x21 points. Center at point A

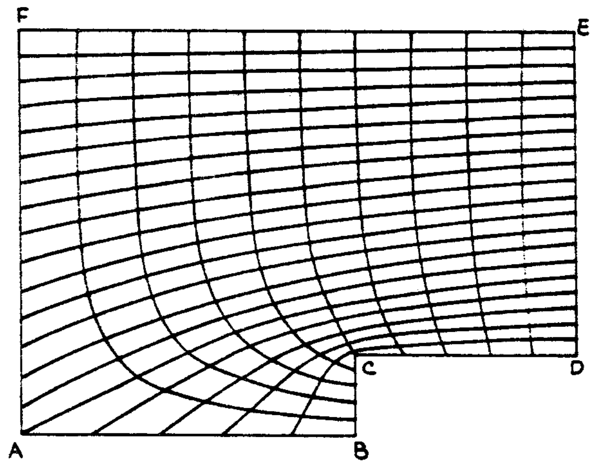

Fig. 2 b   Grid Generated for 11x21 points. Center at point B

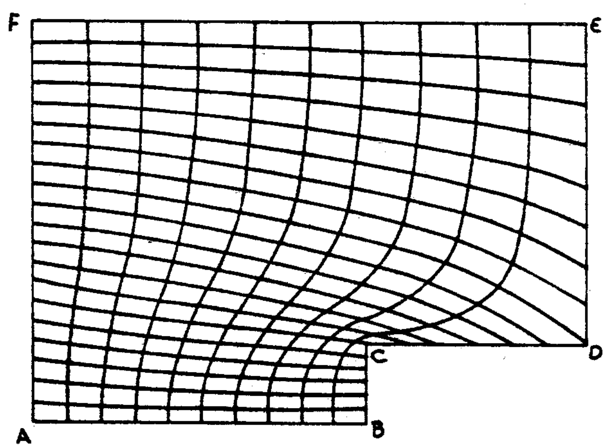

Fig. 2 c   Grid Generated for 11x21 points. Center at point A listed value). As it may be seen, for some of the cases, the maximum cosine value occur on a different position. Observing the changes ocurring on the values of the coefficients when one shifts from figure 2 a to 2 b, it is important to note that although the value of the cross-derivatives coefficient does not change much, the product changes by a factor of 3.6, therefore drastically increasing the angle between unit vectors. It is also instructive compare figures 2 a and 2 c, both with coordinate centers at point A. The differences between them occur on the definitions of the other sides of the computational domain. From what was discussed previously, figure 2 b indicates the best grid, at least, apparently. By inspection of those figures and the above table, it seems important to proceed on the investigation.

Figures 3 a,b and c constitute another example of the advantages of a computational search on the best grid for a particular application. For this case, the presence of more corner points allow more flexibilibity on the generation. Some results for this geometry will be shown on the next section.

ERROR ANALYSIS

In this section, it is desired to proceed with the evaluation of the advantages of a semi-automatic search that indicated a better grid based on the metric coefficients. For that, a simple 2-D heat conduction problem was simulated and errors analysed. The problem simulated was :

$$\nabla^2 T = S_o(x,y)$$

and the source was calculated in order that the exact temperature profile is:

$$T_{exact} = (1 - \cos 2\pi x) (1 - \cos 2\pi y)$$

Naturally, the geometry was that one indicated by figure 1 above and the grids chosen are shown on figures 2 a,b and c. Table II indicates the results, together with the required number of iterations for convergence at 10^{-5} .

Table II : Error Analysis for Geometrical Model on figure 1

Fig.	Number of iterations	percentual error
2 a	13	1.66 %
2 b	12	0.87 %
2 c	17	1.04 %

Although the three results are very small, the advantages of using the grid defined on figure 2 b are clear. In order to proceed with this investigation, this same problem will now be repeated for the geometry defined on figures 3 a,b and c. The results are shown now on table III below.

TABLE III : Maximum Metric Coefficients and Error Analysis for Figure 3 case study.

Fig.	α / J^2	β / J^2	γ / J^2	$\cos \Omega \vert_{max}$	Error
3 a	1578	5108	35062	0.86	1.5 %
3 b	8008	17761	43636	0.95	2.0 %
3 c	1469	2194	4923	0.82	1.4 %

As before, the grid that behaved smoothier resulted on smaller percentual error for the heat conduction simulation (see last column).

CONCLUSIONS

From what was observed on the simulation of a simple heat transfer problem, the search for the best grid for a particular geometry is worthwhile, if done efficiently. Apparently, the procedure indicated in the present paper was sufficient to calculate several possible grids and indicate the best one, using some numerical conditions. The present analysis should now be investigate on a more complex situation, where convective effects may dominate. It is hoped that the general concepts developed here, may be applied on that situation.

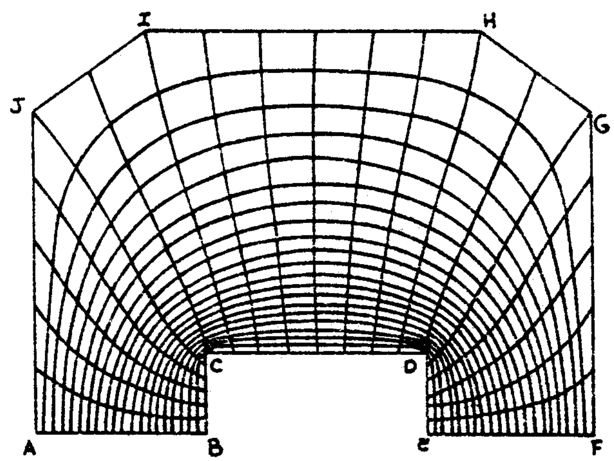

Fig. 3 a   Grid Generated for 21x21 points. Center at point A

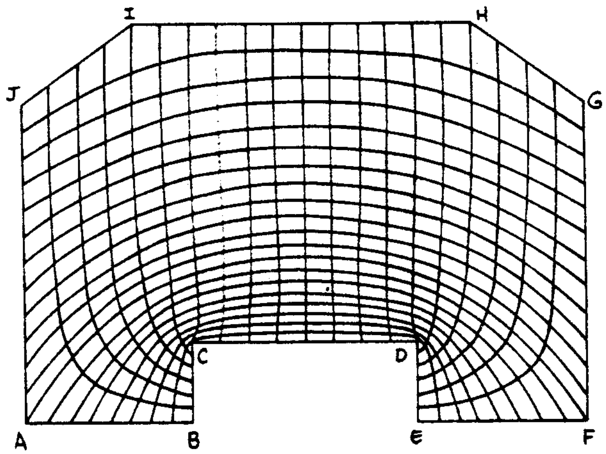

Fig. 3 b   Grid Generated for 21x21 points.   Center at point B

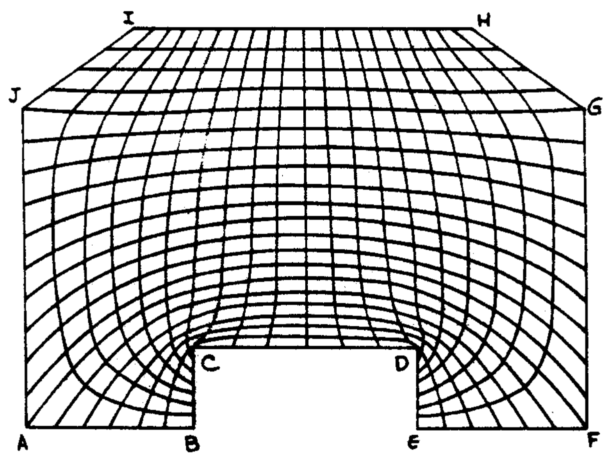

Fig. 3 c   Grid Generated for 21x21 points.   Center at point B

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