OPTIMUM DESIGN OF TRUSSES WITH DISCRETE VARIABLES AND BUCKLING CONSTRAINTS

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ABSTRACT

An efficient method to solve the least weight design of plane and space trusses is developed. A realistic truss model, including discrete variables and buckling constraints is used.

The solution is obtained solving a sequence of approximate problems in the dual space.

The difficulties introduced by the discrete variables and the buckling constraints are successfully overcome.

A couple of examples shows the effectiveness of the method.

RESUMEN

Se desarrolla un eficiente método para resolver el diseño óptimo de enrejados planos y espaciales. Se usa un modelo realista de enrejado, que incluye variables discretas y restricciones de pandeo.

La solución se obtiene resolviendo una secuencia de problemas aproximados en el espacio dual.

Las dificultades introducidas por las variables discretas y las restricciones de pandeo se resuelven exitosamente.

Un par de ejemplos muestra la efectividad del método.

INTRODUCTION.

The field of optimum design of structures is of recent development. At the beginning of the 1960 decade the problem was formulated as one of nonlinear constrained minimization with inequality constraints [1]. During the following years, the major effort was devoted to develop efficient methods since the problem of design optimization is normally characterized for having a large number of degrees of freedom, design variables ans constraints.

In the study of truss design, to emphasize the search for efficiency, a simplified model has been traditionally used. In this model the variables are assumed continuous and the allowables values for compression stresses are constant; see, for example, Refs. 2,3.

This work approaches the problem of least weight truss design using a more realistic model. It includes discrete variables and buckling constraints. The introduction of discrete variables leads to the necessity of solving a nondifferentiable problem. In turn, buckling constraints have allowable values which, instead of being constant, are complicated functions of the design variables. Moreover, they incorporate the radius of gyration as a design variable, in addition to the cross sectional area, thus defining two independient design variables per member.

The problem is solved through a sequence of approximate subproblems in the dual space. The approximations make the solution method more efficient [4,5] and consist of deletion of noncritical constraints and use of first order Taylor series expansions instead of the exact constraints. The dual space has the advantage of defining continuous variables. The difficulties introduced by the buckling constraints are overcome without losing the essential efficiency advantages of the simplified model. An algorithm that fits the special features of the dual problem is used.

DEFINITION OF THE DESIGN VARIABLES.

Buckling constraints in trusses depend on two kind of variables, namely, the cross sectional areas and the radius of gyration associated with the maximum slenderness. Even though these two types of variables are independent with respect to each other, for engineering design purposes, it is practical and reasonable to assume that they are dependent. With this assumption the dimension of the design space may be reduced to half. The dependence can be established by empirical formulas from data provided by standard steel sections [6-10].

In Ref. 7 an empirical function of the form

$$r_m = \alpha \sqrt{A} \qquad (1)$$

was found, where r_m is the cross sectional minimum radius of gyration, A is the cross sectional area, and α is a parameter determined by the least squares methods. Using Eq. (1), the following values of α were obtained :

α = 0,55 for equal-leg angle shapes,
α = 0,75 for equal-leg two angles back to back shapes.

If the radius of gyration corresponding to maximum slenderness (which controls the design) is not the minimum, an equation of the type of Eq.(1) can still be used to relate it with the area. In effect, for a given type of cross-sectional shape, it is found that both radius of gyration may be related by an almost constant factor [11].

PROBLEM FORMULATION.

The problem of least weight truss design subjected to several static load conditions with mixed (continuous and discrete) variables and constraints on stresses, including buckling, displacements, and bounds on the variables can be formulated as follows :

Problem (P)

Find $\vec{A}$

such that $W(\vec{A}) = \sum_{i=1}^{n} \gamma_i \, \ell_i \, A_i \rightarrow \min$

subjected to

$$\begin{aligned} \underline{u}_j \leq u_{jr}(\vec{A}) &\leq \bar{u}_j && j = 1,n_d \\ & && r = 1,n_c \\ -\underline{\sigma}_k(A_k) \leq \sigma_{kr}(\vec{A}) &\leq \bar{\sigma}_k && k = 1,n \\ \underline{A}_i \leq A_i &\leq \bar{A}_i && i \in I_c \\ A_i &\in \Omega_i && i \in I_d \end{aligned} \qquad (2)$$

where

$\vec{A}$ = vector of cross sectional areas,
n = number of truss members,
n_d = number of displacement constraints,
n_c = number of load conditions,
I_c = set of indices associated with the continuous variables,
I_d = set of indices associated with the discrete variables,
A_i = cross sectional area of bar i,
ℓ_i = length of bar i,
γ_i = specific weight of bar i,
u_{jr} = joint displacement j due to load condition r,
$\underline{u}_j$, $\bar{u}_j$ = lower and upper bounds for joint displacements u_{jr},
σ_{kr} = stress in bar k due to load condition r,
$\bar{\sigma}_k$ = allowable tension stress for bar k (it is constant),
$\underline{\sigma}_k(A_k)$ = allowable compression stress for bar k (it depends on A_k),
$W(\vec{A})$ = total truss weight,

$\underline{A}_i$, $\bar{A}_i$ = lower and upper bounds for the continuous design variable A_i ($i \in I_c$),

Ω_i = set of admissible values for the discrete design variable A_i ($i \in I_d$),

Ω_i = $\{A_i^{(q)}, \; q = 1, n_i\}$,

n_i = number of discrete values for the design variable A_i, $i \in I_d$.

The value of the allowable compression stress $\underline{\sigma}_k$ is taken from the Code AISC-78 [12], and has the expression :

$$\underline{\sigma} = \begin{cases} \dfrac{\left[1 - \frac{1}{2}\left(\frac{\lambda}{C_c}\right)^2\right]\sigma_y}{F.S.} & \lambda < C_c \\ \dfrac{12}{23}\dfrac{\pi^2 E}{\lambda^2} & \lambda \geq C_c \end{cases} \tag{3}$$

in which

$$F.S. = \frac{5}{3} + \frac{3}{8}\frac{\lambda}{C_c} - \frac{1}{8}\left(\frac{\lambda}{C_c}\right)^3 \tag{4}$$

is the factor of safety for plastic buckling,

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} \tag{5}$$

is the slenderness limit between elastic and plastic buckling, σ_y is the yielding stress of the material,

$$\lambda = \frac{\ell_p}{r} \tag{6}$$

is the maximum slenderness, ℓ_p is the buckling length, and r the associated radius of gyration.

The AISC-78 code states bounds on the slenderness values according to

$$\begin{aligned} &\lambda \leq 200 \text{ for bars in compression,} \\ &\lambda \leq 240 \text{ for bars in tension.} \end{aligned} \tag{7}$$

Since the buckling lengths do not vary in the design process, these constraints may be indirectly imposed on the radius of gyration, and, by Eq. (1), on the cross sectional areas, according to

$$\begin{aligned} &A \geq \left(\frac{\ell_p}{200\alpha}\right)^2 \text{ for members in compression,} \\ &A \geq \left(\frac{\ell_p}{240\alpha}\right)^2 \text{ for members in tension.} \end{aligned} \tag{8}$$

Problem (P) is of mixed and nonlinear type having behavior constraint functions that are implicit in the design variables.

It is customary to reduce the notation of problem (P) by including the stress and displacement constraints in a unique group of constraints g_j. In this way, problem (P) can be presented as :

Problem (P1)

Find $\vec{A}$

such as $W(\vec{A}) = \sum_{i=1}^{n} \gamma_i \ell_i A_i \rightarrow \min$

subject to

$$g_j(\vec{A}) \geq 0 \qquad j = 1, n_b \qquad (9)$$

$$\underline{A}_i \leq A_i \leq \bar{A}_i \qquad i \in I_c$$

$$A_i \in \Omega_i \qquad i \in I_d$$

where n_b is the number of all behavior constraints.

PROBLEM SOLUTION

In the literature of the optimum structural design it is well known that, in general, a direct application of a mathematical programming algorithm to problem (P1) leads to a very inefficient solution. Several successful approximations to improve the efficiency has been introduced and now are widely used [3,4,5]. They have been mostly applied to the truss simplified model. In this work, the efficiency approximations are applied to the model with discrete variables and buckling constraints. They are :

a) Design variable linking, to reduce the space dimension;

b) Nonpotentially critical constraint deletion, to reduce the number of constraints; and

c) Use of explicit approximate constraint functions generated by expanding the retained structural response constraint functions in first order Taylor series in terms of reciprocal variables.

The foregoing measures define an approximate problem which is convex (if it has only continuos variables), is separable and has explicit functions (easy to compute). These favorable properties substantially facilitate the solution of the problem. However, the answer to the original exact problem is obtained by the convergence of the solutions of a sequence of the approximate subproblems.

Although there exist very appropriate primal algorithms to solve the approximate problem (especially algorithms of the gradient projection type), it has been experienced that the solution in the dual space is, in general, more efficient. In this work, the latter approach is adopted.

It has been shown [3] that the first order approximations behave well when applied to stress or displacement constraints with constant allowable values. In this work linear approximations of buckling constraints are obtained by the same procedure used for the other constraints. Since these approximations usually will not perform as as the others, a special move limit technique, introduced in Refs. 13 and 14, is used. By this technique, called "shrinking-expanding", the constraint surface is moved closer to the point about which the linear expansions have been made; this is done with the purpose of reducing the working region in the design space at this stage, keeping the approximate functions as close as possible to the exact ones. In this way, the advance to the minimum may be controlled.

APPROXIMATE PROBLEM

According to the aforementioned approximations, problem (P1) leads to the following approximate problem in terms of the linked reciprocal variables :

Problem (PA)

Find $\vec{x}$

such that $W(\vec{x}) = \sum_{i=1}^{m} \frac{w_i}{x_i} \rightarrow \min$

subject to

$$\tilde{g}_j(\vec{x}) \geq 0 \qquad j = 1,n_r \tag{10}$$

$$\underline{x}_i \leq x_i \leq \bar{x}_i \qquad i \in I_{xc}$$

$$x_i \in \Gamma_i \qquad i \in I_{xd}$$

where

$\vec{x}$ = vector of reciprocal variables,

x_i = $T_{ik}/A_k \quad k \in I(i)$, (11)

T_{ik} = linking factor between A_k and x_i,

$I(i)$ = set of indices of the areas linked in the group i,

m = number of linked reciprocal variables,

n_r = number of retained constraints,

I_{xc} = set of the indices of the continuous linked reciprocal variables,

I_{xd} = set of indices of the discrete linked reciprocal variables,

w_i = unit weight associated with bar group i,

$$w_i = \sum_{k \in I(i)} \gamma_k \ell_k T_{ik}, \tag{12}$$

$\underline{x}_i, \bar{x}_i$ = lower and upper bounds for x_i. They correspond to the most restrictive bound values within the group i,

Γ_i = set of discrete values for the variable x_i. It contains the intersection of the sets of discrete values for the variables in the group i,

Γ_i = $\{x_i^{(q)}, \; q=1, \ldots, n_{xi}\}$

n_{xi} = number of admissible discrete values for the linked reciprocal variable x_i, $i \in I_{xd}$.

It should be noted that x_i represents a group of variables A_k linked in such a way that their relative values are specified and they do not vary during the optimization process.

Functions $\tilde{g}_j(\vec{x})$ represent linear approximations of the exact constraints g_j, and have the form

$$\tilde{g}_j(\vec{x}) = g_j(\vec{x}_o) + \sum_{i=1}^{m} \frac{\partial g_j}{\partial x_i}(\vec{x}_o) \quad (x_i - x_i^o) \tag{13}$$

This expression can be written in the form

$$\tilde{g}_j(\vec{x}) = \bar{g}_j - \sum_{i=1}^{m} c_{ji}\, x_i \tag{14}$$

To compute $\tilde{g}_j(\vec{x})$, tension and displacement derivates are obtained according to usual methods of implicit differentiation [3,5]. In this work it is necessary to calculate, in addition, the compression allowable stress derivatives, which are obtained from the explicit formulas, Eqs. (1), (3), (4), and (6).

DUAL FORMULATION

The approximate primal problem (PA) leads to the following associated dual problem [3] :

Problem (DA)

Find $\vec{y}$

such that

$$d(\vec{y}) = \sum_{i=1}^{m}\left[\frac{w_i}{x_i(\vec{y})} + x_i(\vec{y}) \sum_{j=1}^{n_r} y_j\, c_{ji}\right] - \sum_{j=1}^{n_r} y_j\, \bar{g}_j \rightarrow \max \tag{15}$$

subject to

$$y_j \geq 0 \qquad\qquad j = 1, n_r$$

in which

$$x_i(\vec{y}) = \begin{cases} \underline{x}_i & \text{if } x_{imin} \leq \underline{x}_i \\ x_{imin} & \text{if } \underline{x} < x_{imin} < \bar{x}_i \\ \bar{x}_i & \text{if } x_{imin} \geq \bar{x}_i \end{cases} \tag{16}$$

$$i \in I_{xc}$$

where

$$x_{imin} = \left[\frac{w_i}{\sum_{j=1}^{n_r} y_j\, c_{ji}}\right]^{1/2} \tag{17}$$

and

$$x_i(\vec{y}) = x_i^{(q)} \quad \text{if} \quad \frac{w_i}{x_i^{(q)}\, x_i^{(q-1)}} < \sum_{j=1}^{n_r} y_j\, c_{ji} < \frac{w_i}{x_i^{(q)} x_i^{(q+1)}} \qquad (18)$$

$$i \in I_{xd}$$

and

$\vec{y}$ = vector of dual variables.

In relation (18)it is assumed that the $x_i^{(q)}$, $q=1,n_{xi}$, are ordered according to decreasing values. If in relation (18) the left hand inequality becomes equality, the discrete variable may be either $x_i^{(q-1)}$ or $x_i^{(q)}$; in turn, if the right hand inequality becomes equality, the discrete variable may be either $x_i^{(q)}$ or $x_i^{(q+1)}$.

From Eqs. (16), (17), and (18) the primal variables x_i are obtained in terms of the dual variables y_j.

Properties of the Dual Function.

Function $d(\vec{y})$ has the following characteristics [3,15,16] :

a) It depends on continuous variables y_j.

b) It is continuous and concave.

c) It has first order discontinuities in hyperplanes in the dual space associated with changes of admissible values in the discrete variables.

d) It has second order discontinuities in hyperplanes in the dual space associated with change of values of the continuous variables x_i from x_{imin} to $\underline{x}_i$ or $\bar{x}_i$.

For continuous primal variables, the solution $\hat{y}$ of problem (DA) yields the optimum solution $\hat{x}$ for problem (PA) through Eqs. (16) and (17). This is because in this case (PA) is a convex programming problem having a Lagrangian function with a saddle point [17].

In the case of discrete primal variables, the solution $\hat{y}$ of problem (DA) gives, through Eq. (18), a solution $\vec{x}(\hat{y})$ which is ε-optimal if $\vec{x}(\hat{y})$ is feasible [16]. That is, if $W(\hat{x})$ is the solution of problem (PA), the following relation is satisfied.

$$W(\vec{x}(\hat{y})) \leq W(\hat{x}) + \varepsilon \qquad (19)$$

The scalar ε measures a bound of the theoretical error of the solution $W(\vec{x}(\hat{y}))$. An expression for ε is given in Ref. 16 :

$$\varepsilon = \sum_{j=1}^{n_r} \hat{y}_j\, g_j(\vec{x}(\hat{y})) \qquad (20)$$

Problem (DA) has two additional favorable properties that makes it attractive in comparison with the primal problem (PA). They are : i) the number of dual variables is, in general, small compared with that of primal variables, because it is equal to the number of retained constraints; ii) problem (DA) has very simple constraints consisting of conditions of non negativity on the dual variables.

SOLUTION ALGORITHM

According to the foregoing considerations, the algorithm proposed in the sequel is based on the following points :

1. Given a feasible initial point in the linked reciprocal variables space, a feasible region is defined according to the potentially critical constraints retained at that stage.
2. That feasible region is "shrunk" towards the initial point by the shrinking- expanding technique. To do that, the retained constraint functions g_i are replaced by reduced constraints δg_i given by

$$\delta g_i(\vec{x}) = g_i(\vec{x}) - (1 - \rho) \quad g_i(\vec{x}_o) \tag{21}$$

where ρ is a constant less than or equal to 1 [13,14].
3. Constraints δg_i are linearized to define problem (PA). This problem is solved through its dual (DA).
4. The solution point of (PA) is used as initial for a new problem. The sequence of points so obtained tends to the solution of the original problem (P1).

Accordingly, the algorithm is :

Step 1 : Choose $\varepsilon > 0$, $\rho \leqslant 1$, $\vec{\Lambda}^{\circ} > \vec{0}$.
Compute the linked reciprocal variables $\vec{x}^{\circ}$ (Eq. (11)).
Perform a structural analysis at $\vec{x}^{\circ}$; $t=0$.

Step 2 : If $t=0$, go to Step 3. If not, perform a structural analysis at $\vec{x}^t$.

Step 3 : Scale $\vec{x}^t$ up to the constraint surface if it is infeasible. Compute the weight W^t.

Step 4 : If $t=0$, go to Step 5.

$$\text{If} \quad \left| \frac{W^t - W^{t-1}}{W^t} \right| \leqslant \varepsilon \text{ , Stop.}$$

Step 5 : Compute the constraints at $\vec{x}^t$.
Delete the non critical constraints.
Reduce the retained constraints according to the shrinking-expanding technique.
Construct first order approximations of the reduced constraints at $\vec{x}^t$.

Step 6 : Construct the approximate primal problem and its dual.

Step 7 : Solve the dual problem, getting $\vec{x}_f^t$, (Eqs. (16),(17), and (18)).
$t = t+1$; $\vec{x}^t = \vec{x}_f^t$.
Go to Step 2.

The optimizer used in Step 7 is a projected supgradient algorithm specially suited to cope with the non differentiable character of the dual objective function. It is fully described in Refs. 15 and 16. It consists of determining the maximum ascent direction at a nondifferentiable point by choosing the vector of minimum euclidean norm in the supdifferential $\partial \ell(\vec{y})$ at this point.

SCALING FACTOR

Step 3 of the algorithm states that if the final point $\vec{x}^t$ from the preceding stage lies out of the feasible region, it must be amplified by a scaling factor so that a new point on the constraint surface is obtained, which will be feasible. The infeasibility of $\vec{x}^t$ is measured with respect to the exact constraints, and it may occur when the approximate problem is solved, even though $\vec{x}^t$ may be feasible with respect to the approximate constraints.

Scaling is performed to construct the new approximate problem $(DA)^t$ on a convenient basis, since the approximations improve when a point is closer to the constraint surface; in addition, constraint deletion may be made more rationally. For the purpose of constructing problem $(DA)^t$, the scaled $\vec{x}^t$ does not need to have discrete components with admissible values.

For efficiency reasons, the response function values corresponding to the scaled design must not be determined at the expense of a structural analysis, but in terms of their initial values, before scaling. In the case of displacement and tension stress constraints, which have constant allowable values, the computation of the scaling factor to the constraint surface and of the scaled values is simple. In effect, it is well known that in a truss if all the areas are modified by a factor μ, stresses and displacements change with its reciprocal $1/\mu$. Therefore, the scaling factor to reach a constraint allowable value is :

$$\mu_i = \frac{v_i}{v_{iad}} \qquad (22)$$

where v_i represents a tension stress or displacement and v_{iad} its constant corresponding allowable value.

The scaling factor for compression stress constraints, whose allowable values depend on the design variables (see Eqs. (1),(3),(6)), is of non trivial computation. Closed form expressions for this factor are derived in Ref. 11. They are :

$$\mu_i = \left[\frac{23\ \lambda_i^2 \sigma_i}{12\ \pi^2\ E}\right]^{1/2} \quad \text{for} \quad \lambda_i \geq C_c \left[\frac{23\ \sigma_i}{6\sigma_y}\right]^{1/2} \qquad (23)$$

and

$$\mu_i = \frac{4}{3}\left[\frac{\lambda_i^2}{2\ C_c^2} + \frac{5}{3}\ \frac{\sigma_i}{\sigma_y}\right] \cos^2\left(\frac{\phi}{3}\right) \text{ for } \lambda_i < C_c \left[\frac{23\ \sigma_i}{6\sigma_y}\right]^{1/2} \qquad (24)$$

in which ϕ is the smaller positive angle such that

$$\cos\phi = \left[\frac{3}{\frac{\lambda_i^2}{2C_c^2} + \frac{5\sigma_i}{3\sigma_y}}\right]^{3/2} \frac{\lambda_i}{8C_c}\frac{\sigma_i}{\sigma_y} \tag{25}$$

Considering all the constraints, the scaling factor μ to the most critical constraint intersected by the scaling straight line is computed by

$$\mu = \max_i \mu_i \tag{26}$$

NUMERICAL EXAMPLES

A computer program, based on the algorithm proposed herein, is applied to the computation of the least weight design of two classical trusses. The program was written in FORTRAN G and the examples were processed in the IBM 370/3031 computer at the University of Chile.

Example 1. Ten bar plane truss.

The least weight design of 10 bar plane truss showed in Fig. 1 is solved.

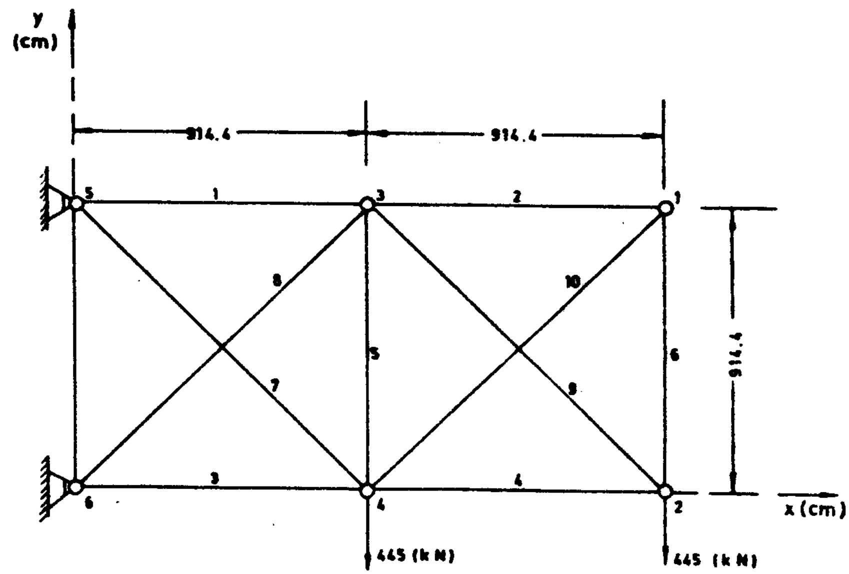

Fig. 1. Ten bar plane truss.

The truss is subjected to one load condition described in Fig. 1. The constraints are as follows :

a) The horizontal and vertical displacements at joints 1,2,3, and 4 are limited to be less than 5.08 cm.

b) The tension allowable stress is 172 MPa.

c) The allowable value for compression stress is taken from AISC-78 Code (Eq. 3).

d) The cross sectional areas have minimum values constraints coming from AISC-78 Code slenderness restriction (Eq. 8).

All bars are of the same material having the following properties:

Density : $\gamma = 0.00277\ kg/cm^3$
Modulus of Elasticity : $E = 68963$ MPa.

Bar cross sectional shapes consist of two angles with legs back to back (α=0.75). Two values of the shrinking - expanding parameter ρ are taken; namely, ρ=0.8 and ρ=1.0.

Two cases are solved. One considers mixed variables, in which the areas corresponding to bars 1,3,5,7, and 9 are assumed discrete. The second case includes only discrete variables. Table I shows the admissible values for the discrete variables in both cases.

Table I. Example 1. 10 bar truss.
Admissible values for discrete variables.

Bar	Areas (cm^2)							
1,2,5, 6	25.81	32.26	38.71	45.16	48.11	51.61	58.06	64.52
	70.97	77.42	83.87	90.32	96.77	98.19	103.23	109.68
	116.13	122.58	129.03	135.48	135.74	138.90	141.94	148.39
	149.68	154.84	161.29	167.74	174.19	180.65	187.10	193.55
	196.90	200.00						
3,4	38.71	45.16	48.11	51.61	58.06	64.52	70.97	77.42
	83.87	90.32	96.77	98.19	103.23	109.68	116.13	122.58
	129.03	135.48	135.74	138.90	141.94	148.39	149.68	154.84
	161.29	167.74	174.19	180.65	187.10	193.55	196.90	200.00
7,9	51.61	58.06	64.52	70.97	77.42	83.87	90.32	96.77
	98.19	103.23	109.68	116.13	122.58	129.03	135.48	135.74
	138.90	141.94	148.39	149.68				
8,10	77.42	83.87	90.32	96.77	98.19	103.23	109.68	116.13
	122.58	129.03	135.48	135.74	138.90	141.94	148.39	149.68
	154.84	161.29	167.74	174.19	180.65	187.10	193.55	196.90
	200.00							

Iteration history related to weight progression and the final design for both mixed and pure discrete cases are detailed in Tables II and III, respectively.

Analyses	Mixed case (1)		Discrete case	
	ρ = 0.8	ρ = 1.0	ρ = 0.8	ρ = 1.0
1	6607.32	6607.32	6607.32	6607.32
2	3590.01	3629.43	4468.50	4823.17
3	3478.98	3438.77	3746.82	4002.90
4	3359.32	3397.55	3663.11	3803.56
5	3331.38	3244.56	3637.42	3735.54
6	3252.42	3163.25	3590.12	3678.77
7	3258.66	3278.30	3515.90	3548.37
8	3246.36	3246.64	3506.41	3511.12
9	3234.40	3231.48	3506.76	3571.61
10	3232.58	3221.46	3502.92	3509.91
11	3221.02	3231.82	3487.19	3499.91
12	3220.66	3297.76	3449.01	3491.21
13	3220.43	3200.28	3449.01	3469.09
14		3200.14		3449.03
15				3449.01
Time (sec-CPU)	3.50	4.84	5.57	6.63

(1) Variables 1,3,5,7, and 9 are discrete.

Table III. Example 1. 10 bar truss.
Final design.

Member	Area (cm^2)	
	Mixed case (1)	Discrete case
2	25.88	51.61
3	187.10	200.00
4	131.42	109.68
5	25.81	25.81
6	25.88	51.61
7	51.61	149.68
8	268.63	200.00
9	103.23	70.97
10	79.63	154.84
Final weight (kg)	3220.14	3449.01

(1) Variables 1,3,5,7, and 9 are discrete.

Table II shows that, for this case, the method works well for both values of the parameter ρ (the values ρ=1 means no shrinking). However, ρ=0.8 required a shorter time and a smaller number of analyses to converge. In turn, the mixed case was solved in a shorter time compared to the discrete case.

In Table III the final weight corresponding to the mixed case is smaller than that of the pure discrete case. This result is logical since the continuous variables, which are half of the total number of variables in the mixed case, have less restricted values than the discrete ones.

Example 2. 72 bar space truss.

Fig. 2 shows the 72 bar space truss whose least weight design is sought. The structure is subject to two load conditions described in Table IV.

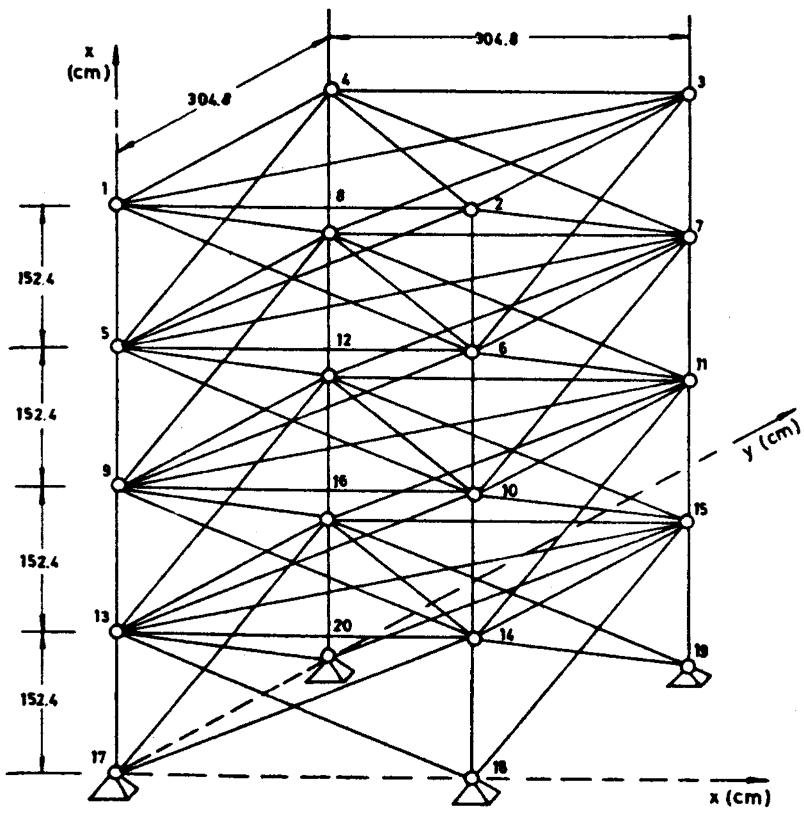

Fig. 2. 72 bar space truss.

Table IV. Example 2. 72 bar space truss.
Load conditions.

Load condition	Number of loaded joints	Joint Number	Load components (N)		
			P_x	P_y	P_z
1	1	1	22270	22270	- 22270
2	4	1	0.0	0.0	- 22270
		2	0.0	0.0	- 22270
		3	0.0	0.0	- 22270
		4	0.0	0.0	- 22270

Table V specifies the topology of the bar members.

Table V. Example 2. 72 bar space truss.
Topology of members.

Bar	Initial joint	Final joint	Bar	Initial Joint	Final joint	Bar	Initial joint	Final joint
1	1	5	25	7	10	49	9	10
2	2	6	26	6	11	50	10	11
3	3	7	27	8	11	51	11	12
4	4	8	28	7	12	52	11	9
5	2	5	29	5	12	53	9	11
6	1	6	30	8	9	54	10	12
7	3	6	31	5	6	55	13	17
8	2	7	32	6	7	56	14	18
9	4	7	33	7	8	57	15	19
10	3	8	34	8	5	58	16	20
11	1	8	35	5	7	59	14	17
12	4	5	36	6	8	60	13	18
13	1	2	37	9	13	61	15	18
14	2	3	38	10	14	62	14	19
15	3	4	39	11	15	63	16	19
16	4	1	40	12	16	64	15	20
17	1	3	41	10	13	65	13	20
18	2	4	42	9	14	66	16	17
19	5	9	43	11	14	67	13	14
20	6	10	44	10	15	68	14	15
21	7	11	45	12	15	69	15	16
22	8	12	46	11	16	70	16	13
23	6	9	47	9	16	71	13	15
24	5	10	48	12	13	72	14	16

In this example the variables are linked (with $T_{ij}=1$) according to the groups defined in Table VI.

Table VI. Example 2. 72 bar space truss.
Linking of truss members.

Group member	N° of members in the group	Member numbers
1	4	1- 4
2	8	5-12
3	4	13-16
4	2	17-18
5	4	19-22
6	8	23-30
7	4	31-34
8	2	35-36
9	4	37-40
10	8	41-48
11	4	49-52
12	2	53-54
13	4	55-58
14	8	59-66
15	4	67-70
16	2	71-72

All bars are of the same material, with the following properties:

Density : $\gamma = 0.00277$ kg/cm³
Modulus of Elasticity : E = 68963 MPa.

The constraints are :

a) The displacements at joints 1 through 16 must be smaller than 0.634 cm, along the 3 directions x, y, and z.

b) Tensile stresses must be less than the allowable value of 172 MPa.

c) The allowable value for compression stress is taken from AISC-78 Code (Eq. 3).

d) The cross sectional areas have minimum value constraints according to AISC-78 Code slenderness restrictions (Eq. 8).

Equal leg angle shapes (α=0.55) are used for the truss members. Two values of the shrinking - expanding parameter are used; namely, ρ=0.8 and ρ=1.0.

A mixed variables case is solved. The variables 1-4, 13-16, 19-22, 31-34, 37-40, 49-52, 55-58, and 67-70 are assumed discrete with admissible values according to Table VII.

Table VII. Example 2. 72 bar space truss.
Discrete variables admissible values.

Bar numbers	Areas (cm^2)
1- 4,19-22,37-40 55-58	2.00 2.50 3.00 3.30 3.50 4.00 7.30 8.50 9.00 9.50 10.00 12.00 12.70
13-16,31-34,49-52 67-70	8.50 9.00 10.00 12.00 12.70

Weight progression at each stage and the optimum design are reported in Tables VIII and IX, respectively.

TABLE VIII. Example 2. 72 bar space truss.
Weight progression

Number of Analyses	Weight (kg)	
	ρ = 0.8	ρ = 1.0
1	683.12	683.12
2	682.58	671.02
3	680.58	641.50
4	679.53	650.38
5	663.37	638.23
6	650.03	638.10
7	638.10	631.62
8	631.62	630.74
9	620.04	620.04
10	620.04	620.04
Time (sec-CPU)	32.38	35.20

Table IX. Example 2. 72 bar space truss. Final design.

Group	Area (cm^2)
1	12.70
2	11.36
3	8.50
4	15.05
5	8.50
6	9.40
7	8.50
8	10.46
9	12.00
10	9.40
11	8.50
12	15.05
13	12.70
14	10.34
15	8.50
16	10.46
Final weight (kg)	620.04

In this example, similarly to Example 1, the method behaved well with both values of the parameter ρ.

CONCLUSIONS

An efficient algorithm to solve the least weight design of plane and space trusses is developed. A realistic truss model, including discrete variables and buckling constraints is used.

Standard approximations, usually applied to simpler models, are successfully used.

The minimum weight design is found solving a sequence of approximate problems in the dual space.

The difficulties introduced by the discrete variables and the buckling constraints are solved as follows :

1. The approximate primal problem with discrete variables leads to a dual problem with continuous variables having a concave objective function that has first order discontinuities. An algorithm, specially suited for this nondifferentiable problem, is implemented.

2. The linear approximations of buckling constraints do not behave as well as those related to constraints with constant allowables. To avoid convergence instabilities in the sequence of approximate problems, the shrinking - expanding move limit technique is applied. It should be noted, however, that in the two examples included in this work, all the approximations showed a good behavior.

3. The troublesome problem of scaling of variables to the constraint surface of buckling constraints, is neatly solved by determining closed form solutions for the scaling factor.

4. Buckling constraints introduce the radius of gyration as a new design variable, in addition to the cross sectional area. To avoid doubling the design space, approximate empirical relations between these two variables are introduced.

A couple of examples proves the effectiveness of the method.

ACKNOWLEDGMENT.

The authors are grateful to the Department of Research Development, University of Chile, for the partial financing of this work through Project I-1747-8533.

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