

## MESO AND MACROMECHANIC APPROCHES FOR RATE DEPENDENT ANALYSIS OF CONCRETE BEHAVIOR

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**Abstract:** *In this work, different strategies are considered to simulate rate-dependent uniaxial tensile behavior of concrete at meso- and macromechanic levels of observation. The first one is analyzed by means of continuum Perzyna-based elastoviscoplastic interface models representing the rate dependent influence of both mortar-aggregate interfaces and mortar. The rate-dependency of this material is modeled in two different forms. On the one hand, by means of interface elements which are incorporated in the discretization of the mortar, following the approach by Carol, Lopez & Roa<sup>1</sup>. On the other hand, by means of a continuous elastoviscoplastic model, the continuum Perzyna-based Extended Leon Model, see<sup>2</sup> and<sup>3</sup>, which includes a volumetric non-associated flow rule and an isotropic softening law, embedded in a rate-dependent fracture energy formulation. The mesomechanic results are compared with those obtained from macromechanic computational analysis in which the concrete material is entirely modeled with the continuum model previously indicated. The comparative analyses lead to conclusions regarding the capabilities and features of different strategies and levels of observation to predict rate-dependent tensile failure behavior of quasi-brittle materials like concrete by means of interface and continuum models. The results in this work are part of a comprehensive research program by the authors to analyze different aspects of concrete rate-dependency under monotonic and cyclic loadings.*

## 1 RATE-DEPENDENT INTERFACE MODEL

The interface model in this work is a viscoplastic continuous Perzyna-type extension of the formulation by Carol, Lopez & Roa<sup>1</sup>. The viscoplastic yield condition is defined as

$$\bar{F} = \sigma_t^2 - (c - \sigma_n \tan(\phi))^2 + (c - \chi \tan(\phi))^2 - \dot{\lambda} \eta \quad (1)$$

where:

$$\chi = \chi(W_{ver}, \dot{W}_{ver}) : \text{traction strength (vertex of the hyperbola)} \quad (2)$$

$$c = c(W_{ver}, \dot{W}_{ver}) : \text{cohesion or shear strength} \quad (3)$$

$$\phi : \text{friction angle} \quad (4)$$

Thereby, the evolution of the work spent on rate-dependent fracture processes during formation of the crack  $W_{ver}$  is

$$\dot{W}_{ver} = \sigma_n \dot{u}_{nver} + \sigma_t \dot{u}_{tver} \quad \text{if } \sigma_n \geq 0 \quad (5)$$

$$\dot{W}_{ver} = \sigma_t \dot{u}_{tver} \left( 1 - \frac{\sigma_n \tan \phi}{\sigma_t} \right) \quad \text{if } \sigma_n < 0 \quad (6)$$

The evolution laws of the normal and tangential rate-dependent crack opening displacements are

$$\dot{u}_{nver} = \dot{\lambda}_{ver} \frac{\partial Q_{ver}}{\partial \sigma_{nver}} \quad \text{and} \quad \dot{u}_{tver} = \dot{\lambda}_{ver} \frac{\partial Q_{ver}}{\partial \sigma_{tver}} \quad (7)$$

where  $Q_{ver}$  is the viscoplastic potential function and the viscoplastic modulus  $\dot{\lambda}_{ver}$  follows from the generalized consistency condition for viscoplastic process, see<sup>3</sup>,

$$d\bar{F} = \frac{\partial \bar{F}}{\partial \sigma_n} \dot{\sigma}_n + \frac{\partial \bar{F}}{\partial \sigma_t} \dot{\sigma}_t + \frac{\partial \bar{F}}{\partial c} \dot{c} + \frac{\partial \bar{F}}{\partial \chi} \dot{\chi} + \frac{\partial \bar{F}}{\partial \dot{\lambda}} \ddot{\lambda} \quad (8)$$

which also leads to the viscoplastic consistent tangent operator and to the generalized Kuhn-Tucker loading/unloading condition during rate-dependent fracture processes. The proposed model for viscoplastic interface was calibrated with the rate-dependent tensile tests on concrete specimens by Suaris and Shah<sup>5</sup>. The results for different velocities are depicted in figure 1.

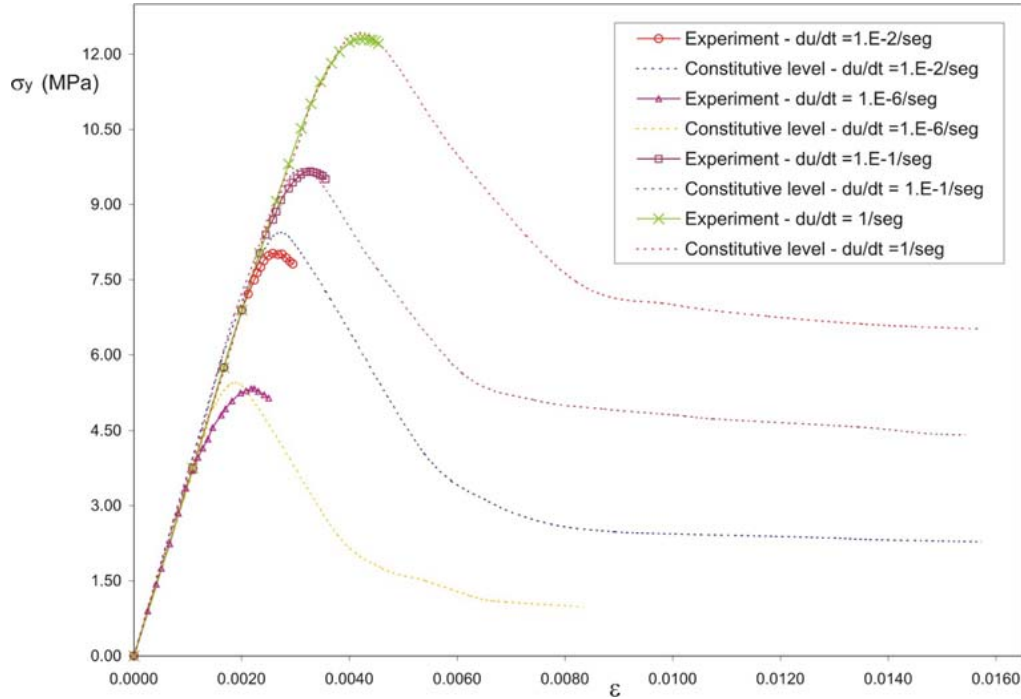


Figure 1: Rate-dependent tensile calibration tests, Suaris and Shah<sup>5</sup>.

## 2 RATE-DEPENDENT ANALYSIS OF CONCRETE TENSILE BEHAVIOR

The rate-dependent uniaxial tensile behavior of concrete material was analyzed at the mesomechanic and macromechanic levels of observation. All together, four different approaches were considered in the computational simulation. In all four cases, the discretizations indicated in figure 2 were used, which were taken from previous work<sup>1</sup> and<sup>4</sup>. The cases considered in the analysis were:

Case 1: macromechanic analysis in which all the elements of the meshes in figure 2 were modeled with the Continuum Perzyna-Leon model for concrete (PELM) by Etse & Willam,<sup>3</sup>

Case 2: mesomechanic analysis whereby the mortar elements were modeled with the PELM and the aggregate elements were considered elastic.

Case 3: mesomechanic analysis whereby the mortar elements were modeled with the PELM, the aggregate elements considered elastic and the interfaces between mortar and aggregate were modeled with the rate-dependent interface element in this paper.

Case 4: mesomechanic analysis whereby both the mortar and aggregate elements were modeled as elastic material (with different mechanical properties), while the rate-dependent nonlinear behavior was incorporated in the interfaces between mortar elements and between mortar and aggregate elements, following the approach in<sup>1</sup> and<sup>4</sup>.

The continuum model PELM was also calibrated with the experimental tests by Suaris and Shah<sup>5</sup> to

reproduce the same behavior in the rate-dependent uniaxial tensile test.

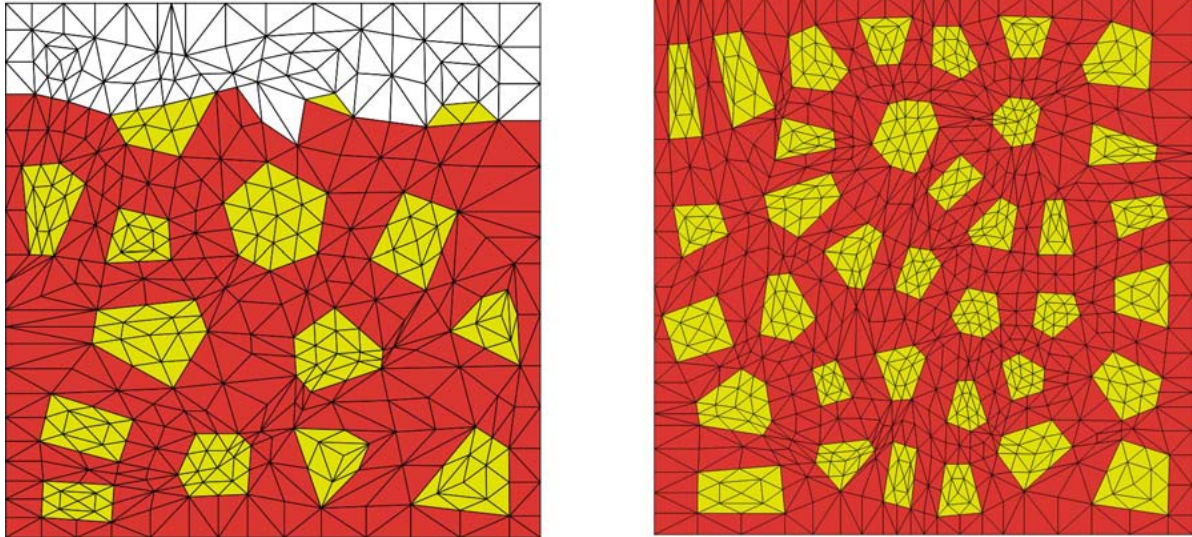


Figure 2. Discretizations used in the computational simulations of rate-dependent uniaxial tensile tests in plane stress condition. 4x4 aggregates (left) and 6x6 aggregates (right)

In figures 3 and 4 we observe the predictions of the rate-dependent uniaxial tensile test obtained with the cases 1 to 4 and with the meshes (a) and (b) indicated in figure 2. The viscosity ratios:  $\eta/\Delta t=0$  (inviscid solution),  $\eta/\Delta t=100$  and  $\eta/\Delta t=500$  were considered, while the vertical displacement rate was kept constant in all the numerical analysis:  $du/dt=1E-2$  1/seg.

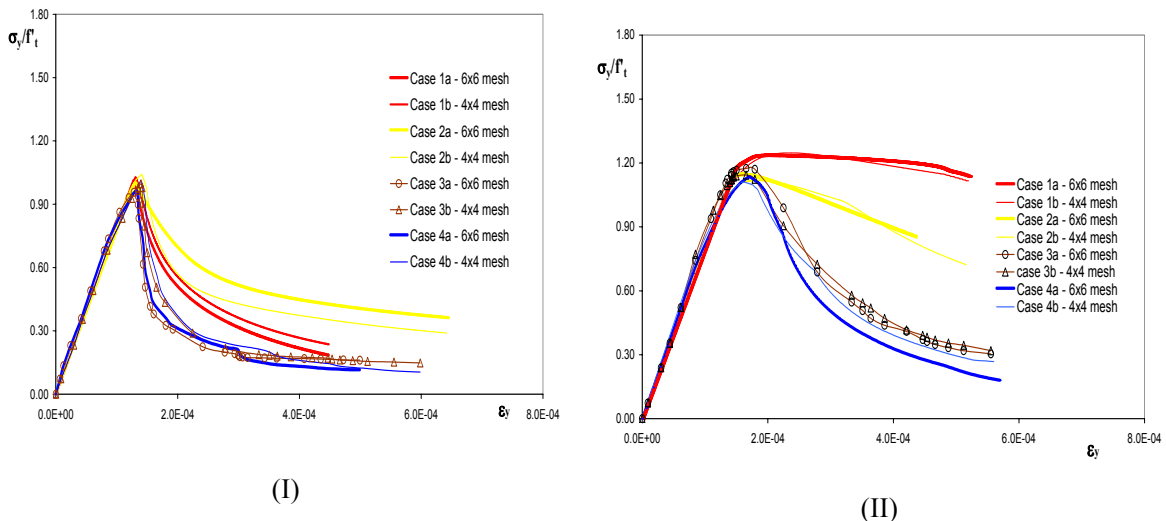


Figure 3. Rate-dependent uniaxial tensile test analysis for (I)  $\eta/\Delta t=0$  and (II)  $\eta/\Delta t=100$ .

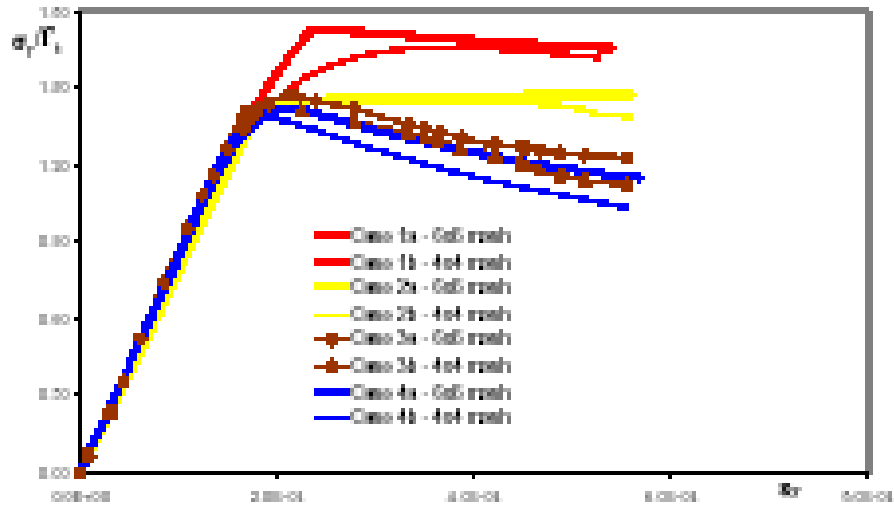


Figure 4. Rate-dependent uniaxial tensile test analysis for  $\eta/\Delta t=500$ .

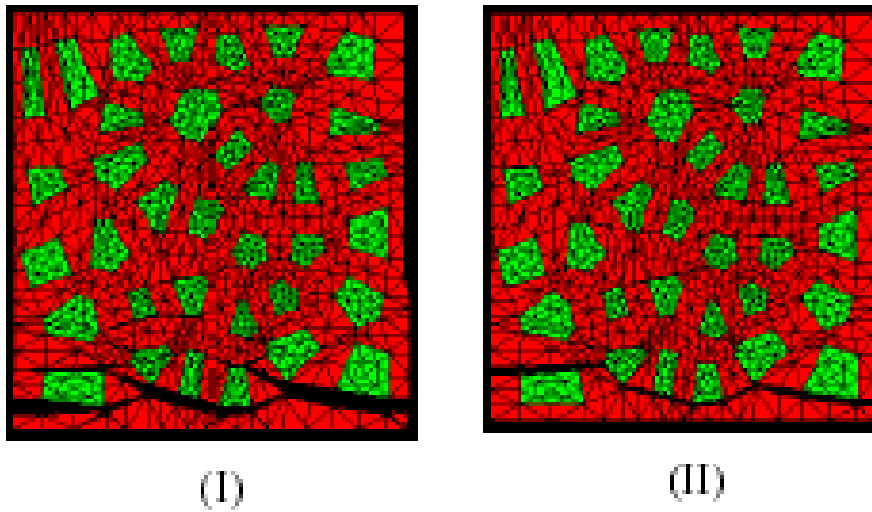


Figure 5. Deformed mesh case 4a) at residual state I)  $\eta/\Delta t=0$  and II)  $\eta/\Delta t=500$

### 3 DISCUSSION

The results in figures 3 and 4 indicate that the macromechanic strategy, denoted Case 1 in this work, is the most sensitive to the viscoplastic ratio  $\eta/\Delta t$  with regard to both the peak stress and to the post-peak behavior. Among the three different strategies for mesomechanic analysis considered here, the cases 3 and 4 are the less rate-dependent ones. So we may conclude that, the mesomechanic approaches including interface elements lead to more brittle behavior both in the inviscid or quasi-static case as well as in the rate-dependent case. The most brittle behavior is obtained in case 4, when the mortar non-linearity is also modeled with interface elements instead of the continuum PELM. In all four cases small mesh dependencies were observed.

The deformed meshes at residual stress of Case 4 with  $\eta/\Delta t=0$  and  $\eta/\Delta t=500$  are illustrated in figure 5, where the relevant difference of the response behavior corresponding to the quasi-static and high rate-dependent tensile tests can be observed. The rate-dependent solution leads to a reduction of the localization as well as to a different failure pattern.

Finally, in figure 6 we observe the predictions of the experimental analysis by Suaris and Shah <sup>5</sup> obtained with the mesomechanic strategy denoted as case 4 and the mesh (a) of figure 2. In the interfaces between aggregate and mortar elements and between mortar elements was considered the same viscoplastic parameter  $\eta$  obtained from the model calibration in figure 1. The numerical predictions match the experimental results very well both for the low and high velocity regimes.

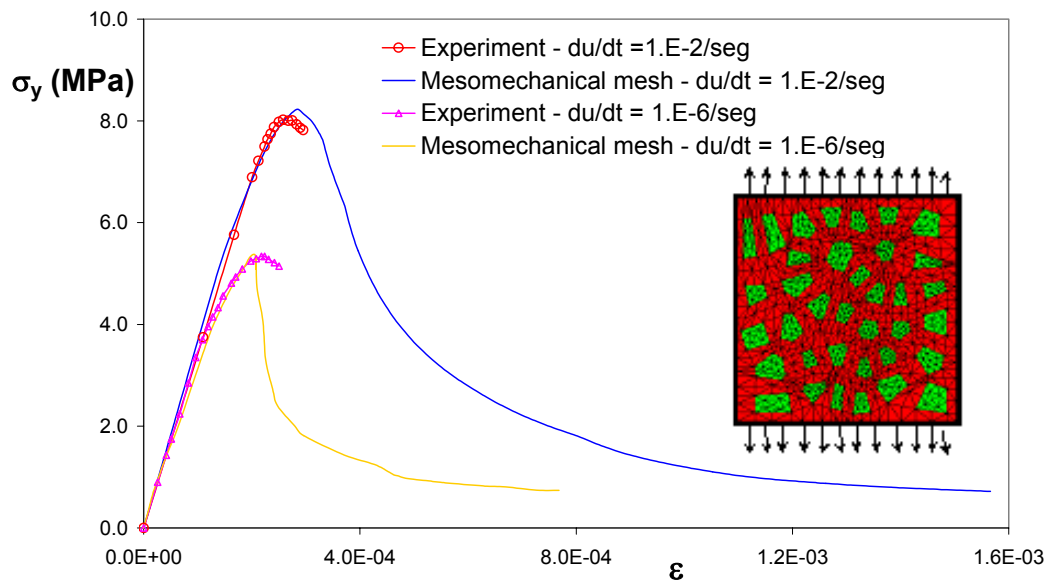


Figure 6. Prediction of Suaris and Shah <sup>5</sup> rate-dependent tensile tests with Case 4 (a).

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