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CRACK DETECTION IN A SPINNING BEAM

Marta B. Rosales[*†], **Carlos P. Filipich**[*‡], **and Miguel A. Conca**[*§]

[*]Department of Engineering-Universidad Nacional del Sur- 8000 Bahía Blanca, Argentina
mrosales@criba.edu.ar

[‡] Mechanical System Analysis Group- FRBB, UTN- 8000 Bahía Blanca, Argentina

[†] CONICET

[§] Becario CIC - maconca@uns.edu.ar - Buenos Aires, Argentina

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Abstract. *This paper deals with the detection of a crack in a spinning beam (rotor) by means of the measured frequencies method. This technique as a crack detection criterion has been extensively applied in the last decade meanly due to the fact that frequencies are, among other dynamical parameters, easily measured. However the inverse problem of determination of the crack parameters (location and depth) for a given set of measured frequencies is not simple. An efficient numerical technique has to been employed so as to obtain acceptable results. In this study the effect of the crack is modeled through the introduction of intermediate flexional springs in a spinning beam of circular cross section and rotating around its longitudinal axis with constant angular velocity. The beam-springs analytical model is first stated and the power series method is employed to obtain the solution for a given set of data, say the springs constants, the crack location or the frequency. It should be noted that the springs and the crack depth may be related by some expression from Fracture Mechanics. Here a systematization of the series gives rise to an efficient numerical method. An algorithm is then written and prepared to solve the inverse problem. Then experimental frequencies are measured in a cracked spinning beam. At this stage, this experiment is performed numerically, with a spinning beam with a notch. The flexural frequencies are obtained. These are the input for the previous numerical algorithm to find the solution of the inverse problem: i.e. predict the crack depth and location resp., given the measured frequencies. Numerical examples are included with an evaluation of the errors in the results. The methodology has been tested previously in an non spinning Euler-Bernoulli beam with very promising results.*

1 INTRODUCTION

Cracks in structural elements may indicate a fatigue problem, mechanical defects or others faults from the manufacturing process. In any case they represent a threat to the reliable behaviour of this part or structural element. Then its detection is an important issue.

It is well known that a structural element shows changes in its behaviour due to the presence of a crack. Many detection methods are based in some structural parameters as the mass, the stiffness, the Young's modulus and in modal parameters such as the frequencies, the mode shapes, the mode damping.

Several researchers have tackled the problem with diverse techniques. Many works are available on crack detection in beams[1–6] . The cracked rotor (or spinning beam) has been also dealt with in several works. See for instance.[7,8]

The location and depth estimation of a crack using the changes in the measured frequencies of a cracked member has been an extended criterion in the last years. One of the reasons is that frequencies are, among other dynamic parameters, easily obtained from measurements. So their experimental determination for a given cracked element is rather direct.

However the inverse problem of determination of the crack parameters (location and depth) for a given set of frequencies, in a damaged element, is not as simple. So in order to obtain meaningful results an acceptable model and an efficient numerical technique have to be adopted.

In this work the crack detection of a damaged spinning beam ("rotor") is presented. The authors have dealt with the vibrations of spinning beams with various complexities before.[9–11] These papers addressed the cases of a beam with different principal moment of inertia of the cross section.

This study tackles the detection problem by modelling the crack in the spinning beam with two intermediate springs in each principal plane. The algebra for the spinning beam with intermediate springs is first stated. The resulting differential system is solved by means of a power series technique.

In the direct problem, if the springs constants and their location were known, one would be able to obtain the natural frequencies of this structural system.

Since the aim of the crack detection is the determination of both its location and depth, an inverse problem has to be stated. The above-mentioned algorithm is used as follows. The measured frequencies of the damaged element are input as data. Once solved, the location and springs constant are obtained. The Fracture Mechanics equivalence[12,13]will allow to find the depth of the crack.

The power series technique are a useful means to have an efficient numerical tool. The authors have solved several ordinary nonlinear problems using a similar approach.[14–17] Also boundary value problems were approached with power series.[18–20]

The methodology is shown with an illustration. The natural frequencies have to be experimentally measured. However, in this stage, the damaged spinning beam (a rotor) is modeled as a stepped beam of three spans, the intermediate one representing the notch. This problem was solved using the equations of the spinning beam[9] and its algebra is included in the Appendix.

From its solution three frequencies are obtained which are then input in the detection algorithm to solve the inverse problem. A numerical example illustrates the methodology and the errors are evaluated. These results along with the ones obtained in a previous work on a Bernouilli-Euler beam[21] are encoraging. At present the authors are addressing the same spinning beam problem with other crack configurations and complexities.

2 STATEMENT OF THE SPINNING BEAM VIBRATIONAL PROBLEM

As stated in a previous work of the authors[9] the transverse, vibrational behaviour of a beam rotating with constant spin about its longitudinal axis, assuming that its cross section possesses only one axis of symmetry, is governed by the following partial differential equations:

$$u'''' + a^2(\ddot{u} - \Omega^2 u - 2\Omega\dot{v}) = 0 \tag{1}$$

$$v'''' + A^2(\ddot{v} - \Omega^2 v + 2\Omega\dot{u}) = 0 \tag{2}$$

where $u(Z,t)$ (in x direction) and $v(Z,t)$ (in y direction) are the transverse displacements of the beam in plane xy of the cross section, $a^2 \equiv \frac{\rho F}{EJ_x}$, $A^2 = \frac{\rho F}{EJ_y}$, ρ is the mass density of the beam, F is the cross-sectional area of the beam, J_x and J_y are the moments of inertia with respect to the x and y axes, respectively, E is the Young's modulus, Ω is the constant angular velocity around the longitudinal axis Z. Dots denote time differentiation and primes denote differentiation with respect to Z. If normal modes are assumed equations (1) and (2) may be written as

$$H'''' - a^2((\lambda^2 + \Omega^2)H + 2\Omega\lambda f) = 0 \tag{3}$$

$$f'''' - A^2((\lambda^2 + \Omega^2)f + 2\Omega\lambda H) = 0 \tag{4}$$

where $H(Z)$ and $f(Z)$ are the mode shapes, unknowns of the problem. λ are the circular natural frequencies

On the other hand the detection algorithm is based in a spinning beam with a intermediate springs which will be stated in the next section.

3 SPINNING BEAM WITH INTERMEDIATE SPRINGS

The spinning beam with intermediate springs is depicted in Figure 1.

The governing equations (3) and (4) stand for each part of the beam: $H_1(z_1)$ and $f_1(z_1)$ for the first part and $H_2(z_2)$ and $f_2(z_2)$ for the second one, $0 \leq z_j \leq 1$, z_j : are the non-dimenssionalized variable $(j = 1, 2)$. The boundary conditions for $H_1(z_1)$ and $H_2(z_2)$ are (assuming a simply supported beam)

$$H_1(0) = 0 \quad H_2(1) = 0 \tag{5}$$

$$H_1''(0) = 0 \quad H_2''(1) = 0 \tag{6}$$

and the continuity conditions are the following

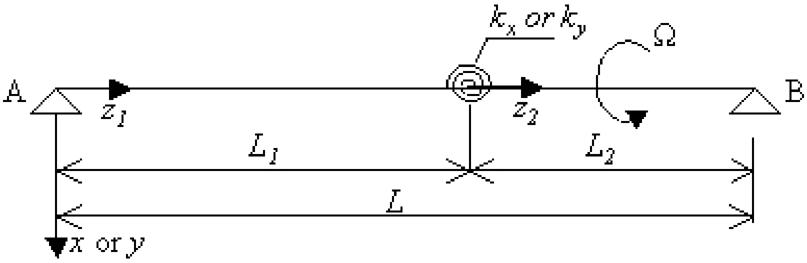


Figure 1: Spinning beam with intermediate springs

$$H_1(1) = H_2(0) \tag{7}$$

$$\frac{EJ_y}{L_1^2} H_1''(1) + k_y \left[\frac{H_1'(1)}{L_1} - \frac{H_2'(0)}{L_2} \right] = 0 \tag{8}$$

$$\frac{EJ_y}{L_2^2} H_2''(0) + k_y \left[\frac{H_1'(1)}{L_1} - \frac{H_2'(0)}{L_2} \right] = 0 \tag{9}$$

$$\frac{EJ_y}{L_1^3} H_1'''(1) - \frac{EJ_y}{L_2^3} H_2'''(0) = 0 \tag{10}$$

Similar expressions are found for $f_1(z_1)$ and $f_2(z_2)$. As mentioned before the power series is a well-known technique to solve differential problems.

The basic unknowns in the direct problem are the mode shapes $H_1(z_1)$, $H_2(z_2)$, $f_1(z_1)$ and $f_2(z_2)$. They are expanded in power series as follows:

$$H_j(z_j) = \sum_{i_0}^{\infty} A_{(j,i)} z_j^i \tag{11}$$

$$f_j(z_j) = \sum_{i_0}^{\infty} B_{(j,i)} z_j^i \tag{12}$$

$$0 \le z_j \le 1; \; j = 1, 2$$

where i_0 denotes $i = 0$.

After the replacement of the expansions (11) and (12) in the differential system (3) and (4), the following recurrence system is obtained.

$$\begin{aligned} A_{(j,i+4)} &= \frac{\Lambda_{y_j}[(1+\eta^2)A_{(j,i)} - 2\eta B_{(j,i)}]}{\varphi_{4i}} \\ B_{(j,i+4)} &= \frac{\Lambda_{x_j}[(1+\eta^2)B_{(j,i)} - 2\eta A_{(j,i)}]}{\varphi_{4i}} \end{aligned} \tag{13}$$

where:

$$\Lambda_{x_j} = \frac{\rho F L_j^4 \lambda^2}{E J_x}$$

$$\Lambda_{y_j} = \frac{\rho F L_j^4 \lambda^2}{E J_y}$$

$$\eta = \frac{\Omega}{\lambda}$$

$$\varphi_{4i} = (i+1)(i+2)(i+3)(i+4)$$

So this way, the eigenproblem may be solved and the natural frequencies and mode shapes are found.

The above-described algorithm is appropiate to solve the direct problem; i.e.: given a spinning beam with intermediate springs, the natural frequencies and mode shapes may be obtained.

The same algorithm will be used to solve the inverse problem as will be described in the next section.

4 CRACK DETECTION IN A SPINNING BEAM

If one is able to measure the natural frequencies in a damaged spinning beam, then the previous algorithm gives a means to detect a crack, its location and its magnitude.

At this stage of the study and to validate the methodology, a numerical experiment is carried out in order to simulate it. A three-span beam is employed as a model of a cracked beam (Appendix). The crack was assumed symmetric. Although this situation is not the most frequent, it is valid to test the methodology. The non-symmetric crack is under study at present.

The spinning beam has a particular behaviour as reported by Bauer[22] and Filipich *et al.*[9] among other authors. For a given spin (angular velocity) the sequence of natural frequencies (ordered numerically), in general, alternate modes. Thus, if we choose an example (see Filipich *et al.*[9]) (J_x=J_y=J , Ω_{ND}=70, where Ω_{ND}=$\Omega\sqrt{\frac{\rho F}{EJ}}$) the first frequency corresponds to the third mode, the second frequency to the second and so on.

5 NUMERICAL EXAMPLE

As mentioned before, the physical experiment is replaced here with a computational experiment.

The simply-supported spinning beam has a circular cross-section (J_x=J_y=J) of 0.05 m of radius and length 1.00 m. The mass density is ρ=7850 kg/m^3 and the Young's modulus E=2.1x10^{11} N/m^2. The angular velocity is set to Ω=3879.15 rad/seg and Ω=9051.34 rad/seg (which corresponds to nondimensional velocities Ω_{ND}=30 and Ω_{ND}=70 respectively).

The cracked zone is modeled by a very short span 5x10^{-4}m wide and a circular cross-section with radius 0.03m (i.e. a crack with $a = 0.02$ m of depth), located at Z=0.3 m.

The values of frequencies of the "damaged" beam obtained for the present example are depicted in Tables 1 and 2 found with Ω_{ND}=30 and Ω_{ND}=70 respectively.

Table 1: First four natural frequencies of a cracked beam. Ω_{ND}=30

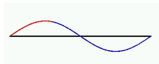
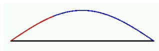
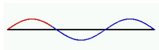
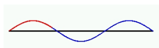
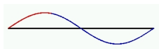
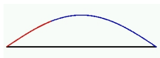
i	λ_i	Mode shape	Used
1	1211.71722951021		Yes
2	2605.48654642698		Yes
3	5152.80468312496		No
4	7603.25283041748		Yes

Table 2: First four natural frequencies of a cracked beam. Ω_{ND}=70

i	λ	Mode shape	Used
1	2431.05867738177		Yes
2	3960.47692352495		Yes
3	7777.68069946014		Yes
4	10324.9988361644		No

For each example the first three values of frequencies are input in the beam-springs algorithm. As is observed in the Tables 1 and 2, there are two different frequencies (among the first four ones) which correspond to a one semi-wave mode. It was found that both values lead to the same result in crack detection. Then, the three selected frequencies are each one corresponding to one semi-wave, two semi-waves and three semi-waves. In this particular problem of the spinning beam they are not in sequential order.

After the frequencies are input in the computational program, three curves L_1 (crack location) vs. k (argument stiffness) (Figures 2 and 3) are obtained. The intersection of the three curves is the solution of the crack detection problem. It may be observed that due to the symmetry of the problem, two locations are found. In this case L_1=0.3 m and L_1 =0.7 m (both at 0.3 m from the ends).

The intersection points are depicted in Table 3.

As may be observed in Table 3, the location were found with very small errors (negligible) with $\Omega_{ND} = 30$ and $\Omega_{ND} = 70$.

Table 3: Values of location and springs constants found with the crack detection algorithm.

Ω_{ND}	L_1 (m)	Error %	k (=k_x=k_y) (Nm)
30	0.3001	0.033	$3.3985\text{x}10^8$
70	0.3001	0.033	$3.3985\text{x}10^8$

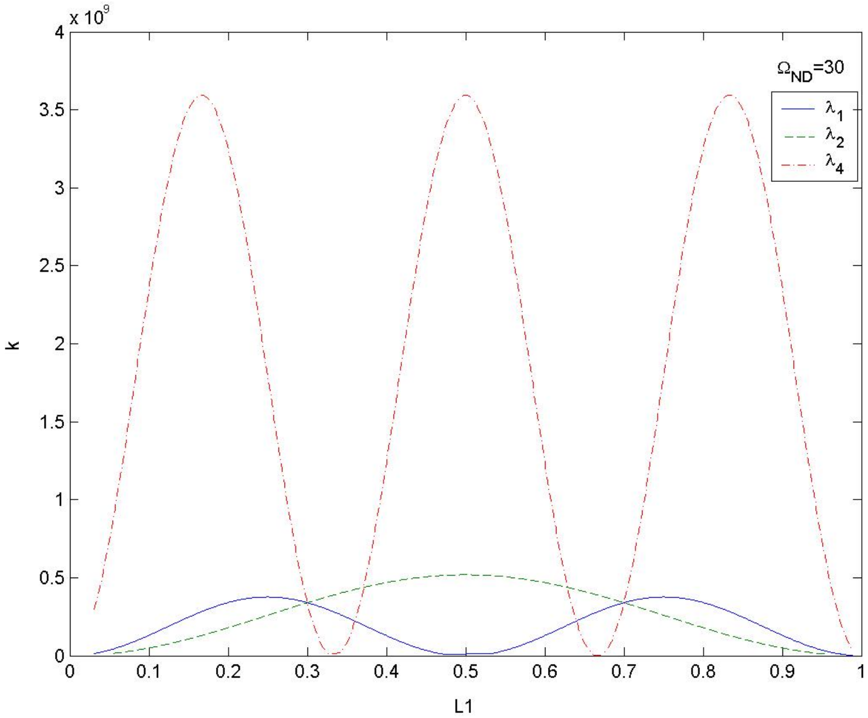


Figure 2: Curves corresponding to frequencies λ_1, λ_2 and λ_4. $\Omega_{ND} = 30$

Table 4: Crack depth found with equivalence with flexibility.

Ω_{ND}	k (Nm)	a (m)	Error %
30	$3.3985 \text{x} 10^8$	0.0181	-9.5
70	$3.3985 \text{x} 10^8$	0.0181	-9.5

In order to obtain the depth a equivalence between the springs constants and the crack depth should be used. However to the authors knowledge no such relationship is reported for a symmetric crack as the one studied here. Consequently the stepped model of the Appendix was employed to tabulate different values of crack depths and the equivalency of a springs. This appears as an alternative way to find the depth. The results are summarized in Table 4

6 FINAL COMMENTS

A crack detection method in a damaged spinning beam (rotor) was presented.

The detection criterion employed is that of the measured frequencies.

The inverse problem is solved by means of an algorithm developed with a spinning beam having intermediate springs to simulate the crack. A power series technique is employed to tackle the solution. This well-known tool provides an efficient and accurate numerical method necessary in order to obtain meaningful results.

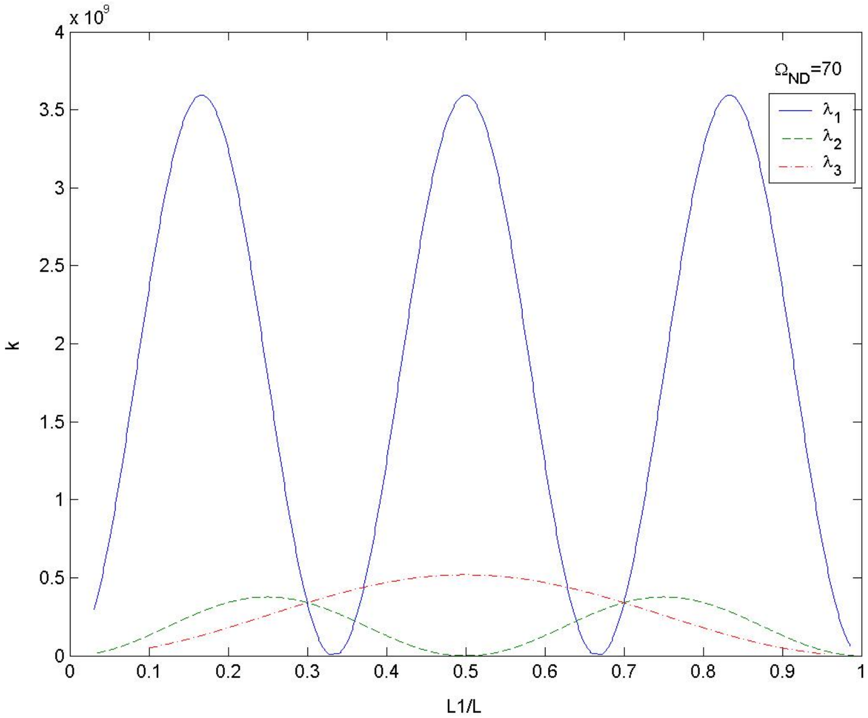


Figure 3: Curves corresponding to frequencies λ_1, λ_2 and λ_3. Ω_{ND}=70

The case of a spinning beam presents certain particularities that increases the complexity in any crack detection method based in measured frequencies.

Its vibrational behaviour shows an alternation of mode shapes that changes with the angular velocity values.

In the present work, a numerical experiment replaces the cracked spinning beam. Although this is not the real situation, it is useful to validate the inverse solution.

The results are excellent in the location value an with acceptable errors in the depth. It was observed that the width of the crack (in the Z direction) affects the accuracy of the depth resulting value. Also, as was expected, the angular velocity value does not affect the crack detection.

The authors are at present improving the numerical experiment simulation considering a nonsymmetric crack.

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APPENDIX

The vibrational problem of a stepped spinning beam of three spans (Figure 4) is governed by equations (1) and (2), valid for each span. Once normal modes are assumed, equations (3) and (4) must be solved for each span, where $H_1(z_1)$, $f_1(z_1)$, $H_2(z_2)$, $f_2(z_2)$, $H_3(z_3)$, $f_3(z_3)$ are the mode shapes; $0 \leq z_j \leq$; $j = 1, 2, 3$.

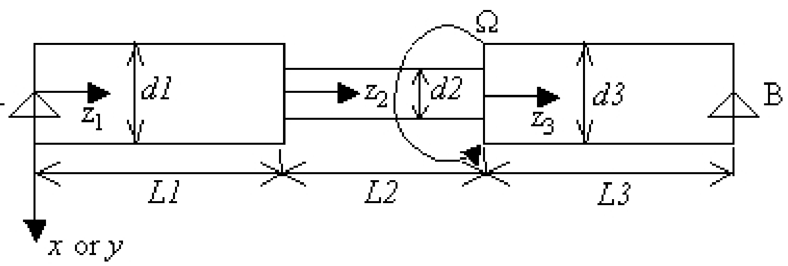

Figure 4: Three span stepped beam with constant angular velocity Ω

The boundary conditions are:

$$
\begin{aligned}
H_1(0) = 0; H_3(1) = 0 \\
H_1''(0) = 0; H_3''(1) = 0
\end{aligned}
\tag{A.1}
$$

and the continuity conditions are:

$$
\begin{aligned}
H_1(1) &= H_2(0) \\
H_2(1) &= H_3(0) \\
\frac{H_1'(1)}{L_1} &= \frac{H_2'(0)}{L_2} \\
\frac{H_2'(1)}{L_2} &= \frac{H_3'(0)}{L_3} \\
\frac{EJ_y^1}{L_1^2} H_1''(1) &= \frac{EJ_y^2}{L_2^2} H_2''(0) \\
\frac{EJ_y^2}{L_2^2} H_2''(1) &= \frac{EJ_y^3}{L_3^2} H_3''(0) \\
\frac{EJ_y^1}{L_1^3} H_1'''(1) &= \frac{EJ_y^2}{L_2^3} H_2'''(0) \\
\frac{EJ_y^2}{L_2^3} H_2'''(1) &= \frac{EJ_y^3}{L_3^3} H_3'''(0)
\end{aligned}
\tag{A.2}
$$

Governing equations (3) and (4) are written in terms of the unknowns (mode shapes H_j and f_j with $j = 1, 2, 3$). The system is then solved after proposing the following expansions in power series:

$$
\begin{aligned}
H_j(z_j) = \sum_{i_0}^{\infty} A_{(j,i)} z_j^i \\
f_j(z_j) = \sum_{i_0}^{\infty} B_{(j,i)} z_j^i \\
j = 1, 2, 3
\end{aligned}
\tag{A.3}
$$

The natural frequencies and the corresponding mode shapes are then obtained for the desired geometry .

In the present crack detection problem, the intermediate span represents the cracked section and consequently L_2 is assumed small.

On the other hand, the equivalent spring constant of second span (cracked section) may be found as follows:

$$
\frac{EJ_y^2}{L_2^3} \int_0^1 [H_2'']^2 dz_2 = k_{eq} \frac{[H_2'(1) - H_2'(0)]^2}{L_2^2}
$$

Then for each value of crack depth a, i.e. $d_2 = d_1 - a$, it is possible to find the equivalent spring constant k_{eq}. Inversely, given a constant k one is able to find the value of J (=J_x=J_y) and from it, the corresponding radius of the intermediate span. The value of a is then derived directly.