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AUTOMATED CLASSIFICATION OF SYNTHESIZED LINKAGE MECHANISMS

Martín A. Pucheta and Alberto Cardona

Centro Internacional de Métodos Computacionales en Ingeniería (CIMEC), INTEC (Universidad Nacional del Litoral-CONICET), Güemes 3450, S3000GLN Santa Fe, Argentina,
e-mails: {mpucheta,acardona}@intec.unl.edu.ar, http://www.cimec.org.ar

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Abstract. In this research work we present preliminary results about the automated classification of mechanism solutions satisfying a given kinematic problem. This research area merges the conceptual and detailed design stages of mechanisms, which are strongly affected by the combinatorial explosion of this kind of problem. In general, an automated conceptual design process often consists of three stages, (i) generation of alternatives or feasible designs, (ii) evaluation of alternatives, and (iii) classification and selection of the best concepts. In terms of linkage mechanisms, the generation stage means to obtain, up to certain complexity, all non isomorphic mechanisms able to satisfy topological and design constraints. Among several design constraints, we consider the given kinematic problem and an allowed space. Alternatives are exhaustively generated by using software for type and dimensional synthesis developed by the authors, already presented in previous AMCA congresses. The evaluation stage consists in the measurement of several indexes related to the satisfaction of discrete (topological) and continuous requirements. The method used for classification is illustrated by means of a path generation example. The classification of solutions aids the designer to evaluate the influence of objectives and design constraints as well as to take complex design decisions. In future works, the selection process will also be systematized.

1 INTRODUCTION

The stages of mechanism design –as any engineering design– present a clear division between the conceptual and detailed design. The research presented in this paper pertains to the conceptual design, see Fig. 1(a); its implications on the detailed design, which are not developed in this paper, are well known. The scope of the application field is the planar linkage design.

Given a set of specifications for a product, a conceptual design process often consists of three stages, (I) generation of alternatives or feasible designs, (II) evaluation of alternatives, and (III) classification and selection of the best concept(s); see Fig. 1(a). The detailed design consists in the refinement of the product parameters by means of several detailed analyses, see Fig. 1(b). The designer is frequently assisted by optimization software to take into account the objectives and constraints not fulfilled by the concepts (IV) (Cugnon et al., 2008), and (if possible) real prototypes are build (V) and intensively tested (VI).

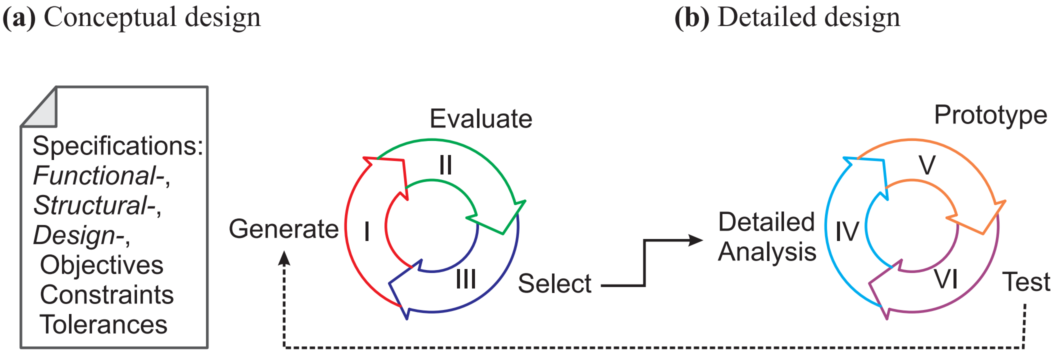

Figure 1: The cyclic nature of the design process I-VI split into two main design stages.

The nature of this process is cyclic: if none concept satisfies the requirements, a designer may explore more alternatives and even extend the design space; this return to the beginning may occur at any of the subsequent stages, e.g. from stage IV to I if requirements cannot be met by the virtual models, from stage V to I if the prototype cannot be physically built, or even more costly, from stage VI to I if tests are not satisfactory; hence the importance of the *best ranked concept(s)*.

The detailed design stage is the most clearly defined task since procedures are well-established and high-level software to aid the designer is abundant. However, the success of this stage strongly depends on the quality of the *selected concept(s)*. For each concept, solutions of detailed analysis may differ slightly. On the other hand, the conceptual design stage may present multiple solutions, either similar or radically different, and the available software to define the design space, generate, evaluate, and select the alternatives, is scarce. Tsai (2001) pointed out that given a set of requirements, the detailed design stage is alleviated if more requirements are incorporated into the generation phase. In the conceptual design stage the goal is to obtain concepts so that most of the requirements are fulfilled.

In linkage design, the generation of alternatives is developed in the type synthesis stage, and the evaluation stage in the dimensional synthesis stage. Current computer-aided software do not cover the whole process. Dimensional synthesis software, available in the market, admits to select few predefined topologies. In the present decade, some academic programs solved the problem in an automated way at the generation and evaluation stages. However, the cycle is closed by a complex decision making process.

With respect to the selection process applied to mechanism design, Erdman and Sandor

(1997, Chap. 8) presented a complete case study of a casement window mechanism, including type synthesis and patent confrontations. Sardain (1997) presented a case study for an excavator mechanism including type and dimensional synthesis. Sedlaczek et al. (2005) and also Liu and McPhee (2007) developed genetic representations of the topologies and their dimensional variables, and used Genetic Algorithms to automatically find the optimal mechanism for a given problem; both of them used big computational resources. The Freudenstein and Maki concept of "separation of structure from function" (Tsai, 2001) was used by Chen and Pai (2005) and also by Pucheta and Cardona (2007) to generate topologies satisfying the topological requirements. The last two works are *exhaustive* generation methods with testing of mechanism isomorphisms. The exhaustive search avoids missing potentially good alternatives.

In this paper, we consider the automated generation stage composed by both type and dimensional synthesis. The dimensional synthesis consists in solving a task simplified into three positions; constraints are computed only for these three positions. We call this simplified method the *initial sizing* (Pucheta, 2008), which is also known as qualitative synthesis. Then, we present preliminary results of the automated classification of mechanism solutions satisfying a given kinematic problem.

The organization of the paper is as follows: in Section 2 we briefly reference the available automated method for synthesis. In Section 3, the indexes and indicators used for classification are presented. In Section 4, we present a path following example and discuss the classification results. We conclude with some future developments.

2 TYPE AND DIMENSIONAL SYNTHESIS METHOD

In contrast with other approaches which solve the type and dimensional syntheses heuristically (Sedlaczek et al., 2005; Liu and McPhee, 2007), the proposed method is based on an exhaustive search (Pucheta and Cardona, 2007) at the type synthesis level while it is mixed –analytical and heuristic– at the dimensional synthesis stage. In this way, alternatives are analyzed case-by-case (Sandor and Erdman, 1984; Sardain, 1997).

The ***Type Synthesis*** stage is summarized as follows:

T1: Represent the sub-mechanism or prescribed parts and the kinematic task using a CAD for finite elements, discretizing the task in three or four precision positions.

T2: Convert the kinematic problem in a graph G_{ini} called *Initial Graph* which produces a mathematical model for the prescribed parts and structural restrictions.

T3: Select a desired atlas of mechanisms. This selection implies the possession of several atlases of mechanisms generated with different types of links and joints, for different degrees of freedom, and using a univocal and efficient codification.

T4: Solve the type synthesis using the search of the *Initial Graph* as a subgraph of each mechanism G_{A} of the selected atlas and identifying all the non-isomorphic occurrences.

T5: Create a schematic diagram for each alternative.

For the ***Dimensional Synthesis***, the combined use of a decomposition algorithm and precision position methods is proposed to give dimensions to each abstract topology resulting from the *type synthesis*, developing these steps computationally:

D1: Decompose the topology into single open chains (SOCs).

D2: Search an ordering of the single-open chains solvable by the available SOC solvers (dyads and triads). Identify the variables of the problem: free parameters, coordinates of new pivots and the multiplicity of each SOC. Default values for the bounds of variables are automatically computed.

D3: Solve the chains analytically using complex-numbers to represent the links (Sandor and Erdman, 1984) and reassemble the chains to reconstruct the topology.
D4: Evaluate the performances of objectives and fulfillment of the restrictions.

This method is fully detailed in the Ph.D. thesis of the first author (Pucheta, 2008) and was successfully used to solve problems with complex initial parts (Pucheta and Cardona, 2008) with application to a nozzle mechanism of a turbine engine, and used to solve bistable compliant mechanisms (Pucheta and Cardona, 2009) with application to a landing gear mechanism.

3 INDICATORS AND PREFERENCES

From now on, we call index to a set of indicators arranged in vectorial form. An indicator is an attribute, design objective or design constraint expressed in a quantitative form. A vectorial form of a mixed –discrete and continuous– performance indicators is proposed. The preferences between the characteristics are expressed by the order or position in this vector.

3.1 Discrete performance index

This index is composed of topological properties of the mechanisms: number of loops L, number of links N, number of joints J, and degree of the ground D, that is $\deg(v_0)$.

These discrete properties are attributes rather than performances, but they have associated indirect implications in the mechanism behavior. Manufacturing simplicity is associated with mechanism simplicity: smaller number of loops, smaller number of links, smaller number of joints. Weight reduction is associated with the smaller number of parts. Wear is associated with the higher number of joints, more critical in sliding than in revolute types. In order to get smooth dynamic behaviors, the relocation of masses distribution (balancing) is easier to achieve in simpler mechanisms. Sardain (1997) associated the planar stability of the linkage with the higher order of the ground, for this requirement, the latter index can be considered as $-\deg(v_0)$. Also, in multi-loop linkages, the more complex the ground is, the simpler the remaining parts of the mechanisms are.

3.2 Qualitative performance index

This index is composed of performance errors of the synthesis problem at precision positions. From the initial sizing solver we count with the quantitative information relative to the *"fitness" function* of the Genetic Algorithm used for optimization of a given topology. This function consists in four weighted functionals, the *minimization of the size of the mechanism* S_{links} together with three constraints: *minimal length of link dimensions* $L_{\min}$, *allowed space violation* A_{space}, and *non-inversion of transmission angle* I_{angle}. The functionals are balanced and weighted using heuristical rules, see (Pucheta, 2008, Chap. 6) for further details.

After a mechanism is computed, a full kinematic analysis is automatically developed and returns the number of time steps developed and, if the total time is achieved, returns the kinematic error at precision positions. Since the required kinematic error must be zero, those mechanisms which do not fulfill this target or do not complete the total time, are rejected. The total time may be not achieved when the algorithmic constraints do not completely avoid the possibility of getting locked mechanisms.

3.3 Preferences

For this vectorial form of indicators, one extreme way of providing preferences is the use of lexicographic order. Given two vectors $\mathbf{x} = (x1, \ldots, x_k)$ and $\mathbf{y} = (y1, \ldots, y_k)$, $\mathbf{x} \succeq \mathbf{y}$

if $x_i \geq y_i \quad \forall 1 \leq i \leq k$. Pairwise comparability between two indicators is clearly ensured without need for normalization.

The importance of the indicators is assigned in order, the first objective is the most important one, and only if we obtain the same results for the first objective, we then consider the second objective and so on.

Preferences are then expressed by the order in which the indicators are arranged in the vector. For minimization, the topological index can be expressed by a vector:

$$I_{\mathrm{t}} = \left[\begin{array}{cccc} L & N & J & D \end{array} \right]; \tag{1}$$

and the qualitative index by the vector:

$$I_{\mathrm{q}}^{\mathrm{lexi}} = \left[\begin{array}{cccc} S_{\mathrm{links}} & L_{\mathrm{min}} & A_{\mathrm{space}} & I_{\mathrm{angle}} \end{array} \right]. \tag{2}$$

The index I_{t} clearly shows preference for simplicity. The index $I_{\mathrm{q}}^{\mathrm{lexi}}$ expresses preference for compactness, then the degree of constraints satisfaction in order: the avoidance of too short dimensions, the allowed space, and finally, the non inversion of transmission angle.

So the global indicator of a mechanism is built as:

$$I_{\mathrm{mech}}^{\mathrm{lexi}} = \left[\begin{array}{cc} I_{\mathrm{t}} & I_{\mathrm{q}}^{\mathrm{lexi}} \end{array} \right] = \left[\begin{array}{cccccccc} L & N & J & D & S_{\mathrm{links}} & L_{\mathrm{min}} & A_{\mathrm{space}} & I_{\mathrm{angle}} \end{array} \right]. \tag{3}$$

If we use weighted preferences for the qualitative index (weights were assigned in the initial sizing stage as was mentioned in Sub-sec. 3.2), we can build a unique index as the sum:

$$I_{\mathrm{q}}^{\mathrm{sum}} = S_{\mathrm{links}} + L_{\mathrm{min}} + A_{\mathrm{space}} + I_{\mathrm{angle}}; \tag{4}$$

then, the global index has the form

$$I_{\mathrm{mech}}^{\mathrm{sum}} = \left[\begin{array}{cc} I_{\mathrm{t}} & I_{\mathrm{q}}^{\mathrm{sum}} \end{array} \right] = \left[\begin{array}{ccccc} L & N & J & D & I_{\mathrm{q}}^{sum} \end{array} \right]. \tag{5}$$

In the following test, we will show that the sum Eq. (5) is a better index than the lexicographic one Eq. (3).

4 RESULTS

A simple three-position path following example is proposed as test problem, so results are easy to understand. The problem is an adaptation to three positions (with the addition of space constraint) of the second test studied by Cabrera et al. (2002).

The precision positions of the kinematic tasks are the following (Fig. 2): $P_0 = (3.000, 3.000)$, $P_1 = (2.372, 3.663)$, and $P_2 = (1.355, 3.943)$. The allowed space is defined by a box with the lower-left corner located at coordinates $(0.0, 0.0)$ and upper-right corner located at coordinates $(6.5, 6.5)$. The imposed motorization is: $\alpha_0 = 0.0$ and $\alpha_2 = \pi/2$; the missing motion α_1 is an unknown to be computed.

Since the pivot is on the corner of the allowed space and the rotation is imposed to be $\pi/2$, the space requirement results a constraint difficult to be fulfilled. Since we impose only one pivot, the positions of the other pivots must be computed.

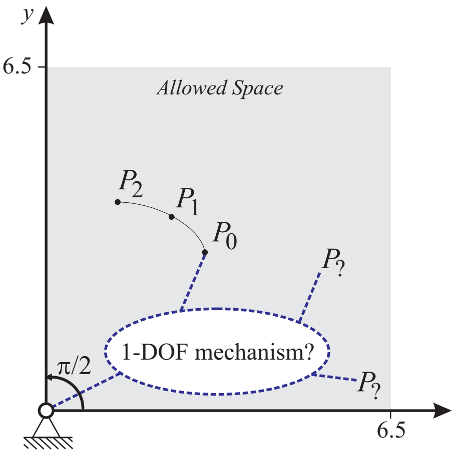

Figure 2: Path following problem subject to space constraints.

Automated type and dimensional synthesis execution

The topological space is set to planar linkages with simple joints or the revolute type. Several constraints for the search are included: distance (in terms of bodies) from the ground link to the coupler link of a value 2; avoidance of pseudo-isomorphisms; limit of the search set in 20 subgraph occurrences.

After solving the type synthesis, from 20 topologies found (not shown here), two of them, 14 and 19, cannot be decomposed into dyads and triads. All possible six-bar topologies satisfying the above constraints were included; also, some eight-bar linkages were obtained.

The parameters of the genetic algorithm used for the initial sizing solver were 120 individuals (approx. 10 individuals per variable), 120 generations, probability of crossover of 0.5, and probability of mutation of 0.01. Due to the few functions evaluations made, the results for the 20 alternatives were obtained in approximately $12\mathrm{min}$ using a personal computer (CPU Core 2 Duo, 1.86GHz with 2.00Gb RAM).

Dimensioned solutions passing through three positions are shown in Figs. 3 and 4. From these solutions, four of them, 4, 12, 16 and 17, present a locked position and do not complete the required motion.

Automated classification

The discrete indicators and performances, the objective and the constraints, are shown in Table 1 in the order of occurrence or computation.

Table 2 shows the lexicographic order of the indexes. Note in Fig. 3 that the second (mechanism 3) and third (mechanism 7) best ranked alternatives present a marked violation of the allowed space.

Finally, the weighted sum ranking is shown in Table 3. The reader can contrast this ranking with the alternatives shown in Figs. 3 and 4 to easily realize that this index is a good indicator not only for classification but also for concept selection.

From this table we should remark that the classic four-bar mechanism is on the top of the ranking. Also note in Fig. 3 the simplicity of the next four alternatives (mechanisms 10, 1, 11 and 9). It is also remarkable that one eight-bar mechanism in the sixth position (mechanism 13) is better than many six-bar mechanisms. So it was interesting to include eight-bar mechanisms in the design space.

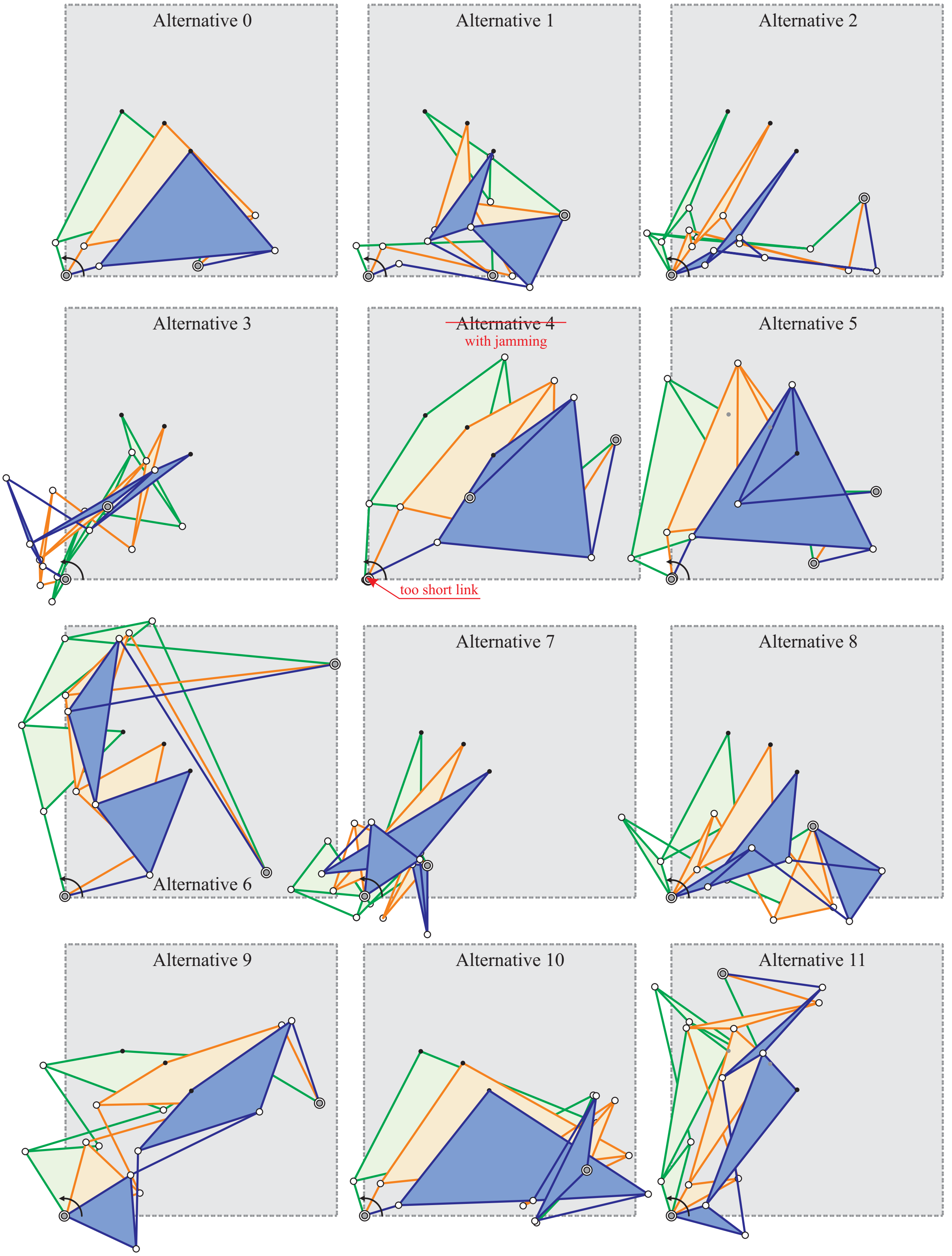

Figure 3: Alternatives for a path following example (continued in Fig. 4).

4.1 Future research and developments

In future research we will incorporate the evaluation of tolerances for objectives and design constraints and the automated selection. Many advanced selection techniques are already avail-

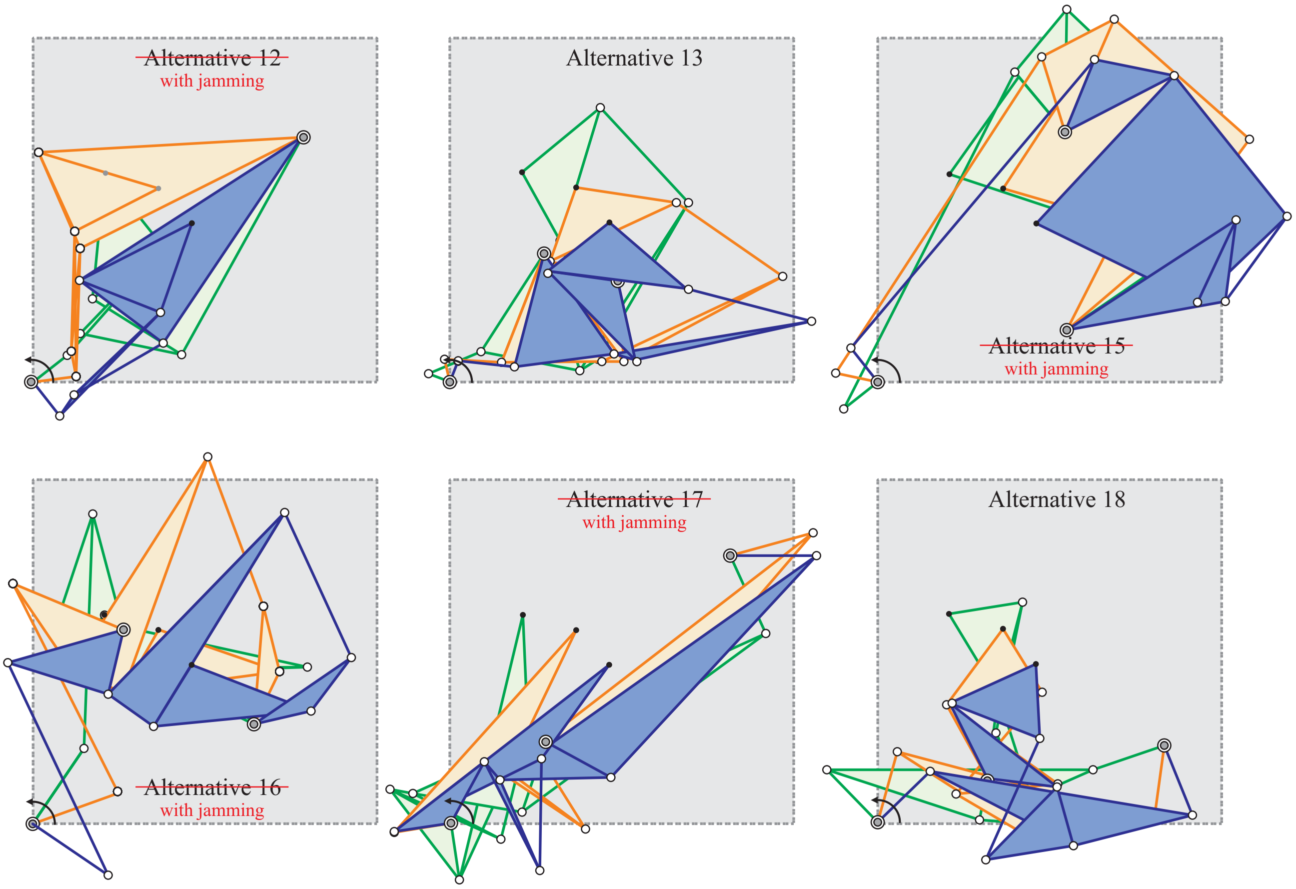

Figure 4: Alternatives for a path following example (continued from Fig. 3).

Mech.	Loops	Links	Joints	deg(v0)	Size	Length	Space	Inv.Ang
0	1	4	4	2	0.0946905	10.8197	26.7207	0
1	2	6	7	3	0.130956	20.4651	32.7774	0
2	2	6	7	2	0.133442	60.8544	64.8543	0
3	2	6	7	2	0.109014	42.6188	191.68	0
5	2	6	7	3	0.162691	13.5012	122.597	0
6	2	6	7	3	0.266498	0	112.923	0
7	2	6	7	2	0.113143	30.3851	255.064	0
8	2	6	7	2	0.150333	18.9741	129.62	0
9	2	6	7	2	0.191407	2.32895	106.087	0
10	2	6	7	2	0.181626	11.9525	33.7964	0
11	2	6	7	2	0.184218	32.6106	50.5261	0
13	3	8	10	3	0.150181	56.1797	54.0356	0
18	3	8	10	3	0.176446	32.3106	105.182	0

Table 1: Performances of the kinematically feasible alternatives in computing order.

able from other research fields (Doumpos and Zopounidis, 2006). The use of predefined rules based on previous knowledge (expert systems) are also adequate to select the best concepts.

In the dimensional synthesis stage, other useful objectives can be considered. The Genetic Algorithm can be adapted to solve a multi-objective multi-constraint design optimization problem for minimizing the continuous kinematic error and for maximizing the transmission angle,

Mech.	Loops	Links	Joints	deg(v0)	Size	Length	Space	Inv.Ang
0	1	4	4	2	0.0946905	10.8197	26.7207	0
3	2	6	7	2	0.109014	42.6188	191.68	0
7	2	6	7	2	0.113143	30.3851	255.064	0
2	2	6	7	2	0.133442	60.8544	64.8543	0
8	2	6	7	2	0.150333	18.9741	129.62	0
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6	2	6	7	3	0.266498	0	112.923	0
13	3	8	10	3	0.150181	56.1797	54.0356	0
18	3	8	10	3	0.176446	32.3106	105.182	0

Table 2: Lexicographic order.

Mech.	Loops	Links	Joints	deg(v0)	$I_{\mathrm{q}}^{\mathrm{sum}}$
0	1	4	4	2	37.6350905
10	2	6	7	2	45.930526
1	2	6	7	3	53.373456
11	2	6	7	2	83.320918
9	2	6	7	2	108.607357
13	3	8	10	3	110.365481
6	2	6	7	3	113.189498
2	2	6	7	2	125.842142
5	2	6	7	3	136.260891
18	3	8	10	3	137.669046
8	2	6	7	2	148.744433
3	2	6	7	2	234.407814
7	2	6	7	2	285.562243

Table 3: Weighted sum ranking.

the mechanical advantage, and the geometric advantage. The criteria developed to solve each linkage inside the solver can be used as classification/selection criteria of the alternatives.

5 CONCLUSIONS

In this paper we have presented preliminary results about the automated classification of mechanism solutions satisfying a given kinematic problem. We have used an automated type and dimensional synthesis solver for path following task. These results are good initial conditions for employing gradient-based optimization in order to consider the continuos kinematic task. Simple indicators based on lexicographical comparison or weighted sum automatically classified synthesized mechanisms. The latter provided a better criteria. The criteria used at the initial sizing stage can be used as classification criteria of the alternatives.

The presented method is a useful tool either for design or redesign of linkages mechanisms where a quick evaluation must be developed.

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