

LIE GROUP INTEGRATORS FOR THE NUMERICAL SOLUTION OF DAE'S IN FLEXIBLE MULTIBODY DYNAMICS

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Abstract. The absolute coordinate method is widely used for the integrated modeling of complex flexible multibody systems including rigid bodies, nonlinear force elements as well as flexible components. In the approach proposed in [2,5], each large rotation variable is represented by three rotation parameters, which are used for the formulation of the equations of motion. This paper considers as alternative an adaptation of Lie group time integrators [4, 7, 8] in order to provide a more natural answer to the rotation parameterization problem. Lie group time integrators do not require a priori the definition of local generalized coordinates, in other words, it includes in its algorithmic structure an intrinsic and consistent strategy for the parameterization of the configuration manifold.

The motion of a flexible multibody system is described using nodal translation and rotation variables so that the system evolves on the Lie group defined by a multiple cartesian product of R^3 and $SO(3)$. The interconnections between the various bodies of the multibody system are modeled using nonlinear algebraic constraints. As a consequence, the motion of the system is restricted to a submanifold of the Lie group. In other words, the equations of motion have the structure of a differential-algebraic equation on a Lie group (DAE on a Lie group).

In order to solve DAEs on a Lie group, an extension of the generalized- α method for Lie group systems is considered [1]. This rather broad family includes as special cases the classical generalized-method for dynamic systems on a linear space [3] and the algorithm described in [9]. Compared to the classical parameterization-based approach, the remarkable simplicity of the new algorithms opens some interesting perspectives for real-time applications, model-based control and optimization of multibody systems.

Several critical benchmarks of rigid and flexible systems with large rotation speeds and

kinematic constraints are studied. In order to model flexible systems, a simple flexible beam element is developed without using any particular parameterization of rotations [6]. From these examples, it appears that Lie group methods compete with the classical "linear" generalized- α scheme from the viewpoint of accuracy and stability. Hence, these new methods are promising candidates for the development of robust, efficient and open simulation software for flexible multibody systems.

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