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AEROELASTIC TAILORING OF COMPOSITE PLATES THROUGH EIGENVALUES OPTIMIZATION

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Abstract. This work presents an aeroelastic tailoring approach using lamination parameters optimization. The main goal of this research is to analyse a composite plate wing subject to aeroelastic effects and the improvement in flutter speed by means of eigenfrequency maximization. First, an aeroelastic analysis is conducted with the initial plate design, to identify the eigenfrequency responsible for the flutter onset. Next, optimization problem is formulated to maximize this specific eigenfrequency. The optimization uses the reinforcing fiber orientation as design variables. The sequential linear programming (SLP) is applied and sensitivities are calculated analytically. After the optimization, flutter speed is calculated again, in order to check the optimization effect. Results presented show a marked improvement of the flutter speed.

1 INTRODUCTION

The development of laminated composite material, or fiber reinforced material, is tightly related to its use in aeronautical construction. This type of material allowed the construction of structural parts with high strength-to-mass ratio. The development of composite material, gave the designer a wide number of options for the construction of lightweight structures. The first sailplane made of fiberglass was the FS 24 Phoenix, made in West Germany, 1957 (von Gersdorff, 1981). From that time on, the use of composites in structural parts increased. The initial reluctance in using this kind of material was overcome with the determination of strength and other tests were conducted, along with development of calculation procedures (Kensche, 2003). According to Tsai (1992), the F-111, developed in the 1960's, had the first flight-worthy composite horizontal stabilizer of a fighter. The production of the F-14, F-15 and later the F-16 followed this initial design with success.

Nowadays, operational envelopes are again being extended. One important aspect that leads to this new frontier is the need for the use of smaller and smaller quantities of material. This fact is due to a number of reasons: one is the environment preoccupations; another is the economic motivation for less expensive structures; another is the need for more efficient projects, such as aircraft that fly faster and higher for longer time.

One important advent are the UAS, unmanned aerial systems, most of then small aircraft with a high relation thrust-payload. The UASs have been exploited in last decades mainly in the military field, like target drones or test aircraft. Later, it became a important tool for reconnaissance missions in the battle field, what rapidly expanded to strike missions (Sarris, 2001). In the last decade, civilian applications have also been added to the UASs. Mainly the reconnaissance capabilities revealed itself an attractive to farmers, electric lines verification, car traffic observation, environmental monitoring, etc. The list of potential applications increase rapidly with new manufacturers entering the market. The flying characteristics of such lightweight type of aircraft under the aeroelastic point-of-view are of high importance.

One characteristic of lightweight, high-span wings is the high displacement that their are subject to, much more than in the case of a conventional, rigid, short-spanned wing. Another effect is the possibility of encountering LCO during its normal operation. The LCO is a kind of aeroelastic behavior that can lead to different results, like fatigue, discomfort for the pilot or coupling with automatic control system frequencies. It can be described as a high amplitude oscillation with stationary characteristics, i.e, there is no modification on the frequency or amplitude with time, unless a perturbation occurs. According to Bunton and Denegri (2000), the LCO is closely related to flutter, except that the coupling of structural response with non-stationary aerodynamic forces are of nonlinear nature. Due to that, the aeroelastic stability analysis used to calculate the flutter speed do an excellent work to predict not only the frequency but also the speed at which the LCO appears.

The new design and mission requirements demand for new analysis methods. Structural optimization comprises an important procedure that is almost mandatory in order to achieve an efficient aeroelastic design. For the case of composite laminated wings, aeroelastic tailoring methods must be used. Aeroelastic tailoring can be defined as the design process that makes use of the directional properties of fibrous composite materials in wing skins and orients these materials in optimum directions Hertz et al. (1981). According to this, the analysis of a tailoring process must be done with special attention to the optimization methods, and not only to the aeroelastic analysis methods.

Studies of aeroelastic tailoring can be found already in late 1960's and beginning of 1970's.

A first paper that is referenced by many authors is the work of Krone (1975). Weisshaar (1980), working in the project of the X-29 fighter, continued the work. Hertz et al. (1981) presented in 1981 a revision of the studies made to validate the design of forward swept wings. This type of configuration has the inherent tendency to encounter divergence. Only with aeroelastic tailoring studies it was possible to achieve safe and lightweight designs. These studies were basically dedicated to the diverge phenomenon. Besides that, up to that time only analytical design tools were used, without consideration of transonic speeds. Neither control nor dynamic aeroelastic characteristics were evaluated at that time. Another research line investigate the use of active aeroelastic tailoring (Barrett, 1996).

Another works dealing with optimization of wing structures had the flutter speed as objective function (McIntosh and Ashley, 1978), but no composite material was used. Tripplet (1980) presents studies made with fighter aircraft with backward swept wings, with small or big angles, using a computer program dedicated to the subject, called TSO (Aeroelastic Tailoring and Structural Optimization), developed by the Air Force Flight Dynamics Laboratory, in USA. The program combined aerodynamic,static aeroelastic, structural and flutter calculations. The doublet-lattice method, as presented in Giesing et al. (1971), was used to model the unsteady oscillatory aeroelastic characteristics. The paper, however, focused on the presentation of results, and did not give much information about the mathematical methods employed. Later, the work continued with extension to the modification of flutter speed, with many different approaches. One of these was to consider the wing as a box-like beam, like in Gao and Zhang (1991), Patil (1997) or Abdalla et al. (2007). The lamination parameters are reduced to equivalent values, which become the design variables.

In this work, an approach to optimization of fiber orientation of composite laminates is presented, aiming the increase of the flutter speed of a single cantilever wing. The aerodynamic and the strucural parts of the aeroelastic problem are treated separately, and the coupling of models is made though an interpolation procedure using splines. The flutter problem is solved on the generalized aeroelastic equation of motion. The optimization process uses a sequential linear programming (SLP), with sensitivities calculated analytically.

2 AEROELASTIC SYSTEM

The term aeroelasticity comprises the coupling of aerodynamic, elastic and inertial forces. The relations between these three main disciplines evolves in various analysis types. The Collar's triangle is an efficient way for representing such relations (Collar, 1946), and better identify which are the disciplines that interact at each type of analysis, defining which are the special considerations demanded by those approaches. From that time on, many other interactions were identified as a result of the aeronautics development in last decades.

The aeroelastic equation of motion is assembled with a separation of structural and aerodynamic operators (Bisplinghoff and Ashley, 1962; Fung, 1955). It can be written as :

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}_{ext}, \tag{1}$$

where $\mathbf{M}$ is the system mass matrix, $\mathbf{C}$ is the damping matrix and $\mathbf{K}$ is the stiffness matrix, $\mathbf{q}$ is the displacement vector, including all the system's degrees of freedom - linear and angular displacements. The right side of the equation contains all the external forces $\mathbf{F}_{ext}$. For the present problem, only the unsteady aerodynamic forces are of interest, and thus, the external forces are defined as:

$$\mathbf{F}_{ext} = \mathbf{G}_s^T \mathbf{F}_a(t, \mathbf{q}, \dot{\mathbf{q}}) \;, \tag{2}$$

what means that this force depends on time and on the time derivative of $\mathbf{q}$. On the above equation, $\mathbf{G}_s$ is a matrix that interpolates the displacements form the structural model points to the aerodynamic points, given by the spline transformation presented by Harder and Desmarais (1972). The problem consists now in the formulation used to represent both parts of eq. (1): structures and aerodynamics.

2.1 Generalized structural matrices

Under a linear approach, the assumption of small displacaments is made, and the superposition principle is valid. This allows the use of generalized matrices formulation, where structural modal information is coupled to an generalized aerodynamic operator. Under this approach, the structural part is represented by eigenfrequencies and their associated eigenmodes.

The structural displacement is aproximated by the relation

$$\mathbf{q} = \mathbf{\Phi}_e \bar{\mathbf{q}}(t) \tag{3}$$

where $\bar{\mathbf{q}}(t)$ is the vector of generalized displacements and $\mathbf{\Phi}_e$ is the matrix containing the eigenvectors on each column obtained from modal analysis of the strucutral model - even Finite Element model or experimental model. The aeroelastic undamped equation of motion is then written as :

$$\mathbf{\Phi}_e^T \mathbf{M} \mathbf{\Phi}_e \ddot{\bar{\mathbf{q}}} + \mathbf{\Phi}_e^T \mathbf{K} \mathbf{\Phi}_e \bar{\mathbf{q}} = \mathbf{\Phi}_e^T \mathbf{F}_{ext}(t), \tag{4}$$

what takes to

$$\tilde{\mathbf{M}}_e \ddot{\bar{\mathbf{q}}} + \tilde{\mathbf{K}} \bar{\mathbf{q}} = \Phi_e^T \mathbf{F}_{ext}(t, \mathbf{q}), \tag{5}$$

where $\tilde{\mathbf{M}}$ and $\tilde{\mathbf{K}}$ are the generalized mass and stiffness matrices.

Again, it is important to notice that two physical models are used. The aerodynamic forces are computed based on boundary conditions defined at aerodynamic control points and the displacement vector for the boundary conditions computation is given in FEM model nodal coordinates. A displacement transformation between the aerodynamic model and the finite element model (FEM) is then used.

Mathematically speaking, the representation of the physical coordinates in another point is obtained through the transformation

$$\mathbf{x}_a = \mathbf{G}_s \mathbf{x}_e, \tag{6}$$

In this case, the transformation is applied to each modal matrix:

$$\mathbf{\Phi}_a = \mathbf{G}_s \mathbf{\Phi}_e\ . \tag{7}$$

The type of aerodynamic modelling to be used now defines how this transformation is made. This is discussed next.

2.2 Aerodynamic modeling

The aerodynamic model provides the external forcing terms of the aeroelastic equation of motion through an aerodynamic operator. The first works on the aeroelasticity, dedicated to the flutter problem, used potential flow theory with an operator that is function of the reduced frequency to represent the lift and moments on a two-dimensional airfoil (Theodorsen and Garrick, 1934; Theodorsen, 1935), what was equivalent to the indicial formulation developed by Wagner (1925), where the lift was expressed as function of a reduced time variable.

Both theories worked with two-dimensional airfoils, what was the so-called typical section. In this case, the structural behavior of a wing is condensed into only a few displacement modes, namely pitch, plunge, aileron (or flap) deflection and tab deflection. These consist in one longitudinal displacement - the pitch - and three rotations - the further, what results in 2 to 4 degrees of freedom. Later, the two-dimensional model was literally streched to the case of a finite wing, by the use of several typical sections side by side, what consists in the strip method.

However, a formulation was created to include the case of more generic planforms: the panel methods. The use of panel methods allowed a new approach, where the wing is considered as whole, and non-conventional wing configurations could be analysed. One possibility is the use of the Doublet Lattice Method - DLM - for the aerodynamic modelling. A detailed description of the method can be found in a document of the early 1970's, Giesing et al. (1971). There, a extension for non-planar surfaces to the Doublet-Lattice Method of Albano and Rodden (Albano and Rodden, 1969) is given, and the mathematics leading to that theory was gattered by Blair (1994).

In the present work, the software ZAERO is used for the aeroelastic analysis, using the DLM. One advantage of this method is its relatively computational simplicity, specially for complex configurations. In this method, a pair of continous pressure doublet sheet are replaced by a set of lattice pairs with finite length. The programming simplicity is obtained when all panels are treated equaly, independent of their proximity to wing limits (leading and trailing edges, or wing tips).

The dipole lattice is placed at 1/4 of the chord length at each panel and the upwash $w(x, y, z)$ is calculated at 3/4 of the chord length, in the midle span of each panel. Considering the displacement of the aerodynamic control points as $\mathbf{q}_a$, the resultant aerodynamic forces at the aerodynamic boxes due to this displacement is defined as

$$\mathbf{F}_a = q_\infty \mathbf{A}(ik)\mathbf{q}_a \tag{8}$$

where q_d is the dynamic pressure, and $k = \omega b/U$ is the so-called reduced frequency, where ω is the harmonic frequency, U is the free stream velocity, and b is the reference semi-chord. The aerodynamic influence coefficient matrix $\mathbf{A}(ik)$, is a function of the reduced frequency (only incompressible flow is considered here).

The generalized external forces are then given by

$$\tilde{\mathbf{F}}_{ext} = \mathbf{\Phi}_e \mathbf{G}_s^T \mathbf{F}_a = q_\infty \mathbf{\Phi}_e \mathbf{G}_s^T \mathbf{A}(ik)\mathbf{q}_a \ , \tag{9}$$

To complete the formulation of eq. (5), the displacement $\mathbf{q}_a$ is substituted by $\mathbf{\Phi}_a \boldsymbol{\eta} = \mathbf{G}_s \mathbf{\Phi}_e \boldsymbol{\eta}$ and the above equation becomes

$$\tilde{\mathbf{F}}_{ext} = q_\infty \mathbf{\Phi}_e \mathbf{G}_s^T \mathbf{A}(ik)\mathbf{G}_s \mathbf{\Phi}_e \boldsymbol{\eta} \ , \tag{10}$$

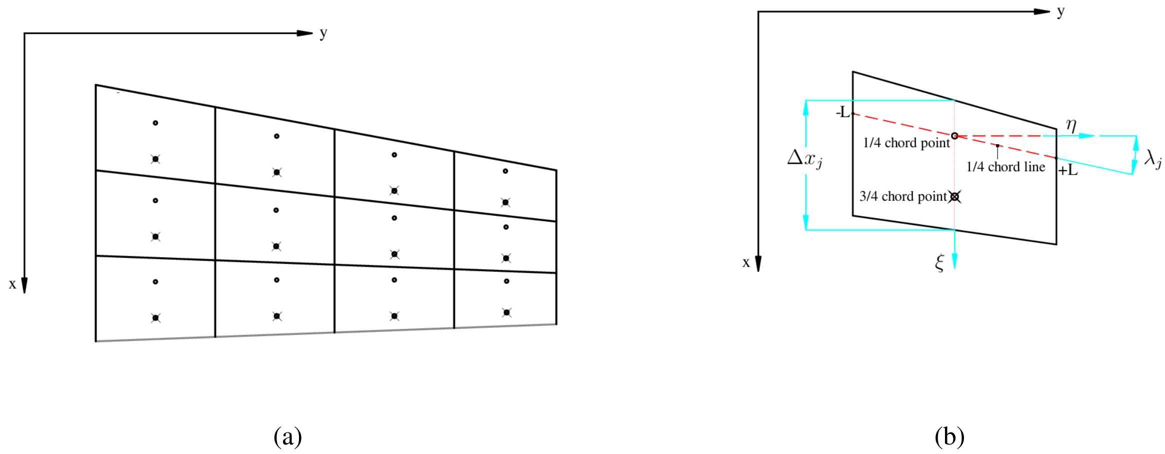

Figure 1: (a) Discretization of a wing under DLM with control points - sending and receiving. (b) Panel element with local coordinate systems.

In ZONA Technology (2007), a generalized aerodynamic influence coefficient matrix is defined as

$$\mathbf{Q}_A(ik) = \mathbf{\Phi}_e^T \mathbf{G}_s^T \mathbf{A}(ik) \mathbf{G}_s \mathbf{\Phi}_e. \tag{11}$$

Now, all matrices on eq. (5) are given in generalized coordinates. The final analysis dimension in both sides of that equation will be the number of modes taken into account. The eq. (5) may be rewritten as

$$\omega^2 \tilde{\mathbf{M}} \boldsymbol{\eta} + \tilde{\mathbf{K}} \boldsymbol{\eta} = q_\infty \mathbf{Q}_A(ik) \boldsymbol{\eta} \tag{12}$$

or, separating variables:

$$\left[\omega^2 \tilde{\mathbf{M}} + \tilde{\mathbf{K}} - q_\infty \mathbf{Q}_A(ik) \right] \boldsymbol{\eta} = 0 \tag{13}$$

This becomes a stability problem, since non-trivial solutions are searched. A detailed discussion of solution methods and procedures is given in ZONA Technology (2007).

2.3 Aeroelastic Tailoring

Aeroelastic tailoring can be defined as the design process that makes use of the directional properties of fibrous composite materials in wing skins and orients these materials in optimum directions (Hertz et al., 1981). To obtain useful results, the tailoring process must be done with special attention to the optimization methods, and not only to the aeroelastic analysis methods.

In practical terms the use of tow steering processes allows the creation of special fabric patterns. Fiber steering is a method of construction for fiber-reinforced composites that allows the unidirectional fibers to be aligned along curvilinear paths. Examples of fiber steering structures are given in http://www.adoptech.com/fibersteering/design.htm.

3 STRUCTURAL MODELING

The structural analysis is performed by the finite element method (FEM), using a eight node serendipity isoparametric quadrilateral shell element. The element formulation consists in two parts: the shell behavior is based on the degenerated solid formulation (Ahmad et al., 1970), and the laminated composite part uses an explicit integration through the thickness. The Ahmad degenerated shell element is well-known; a good review can be found for instance in Zienkiewicz (Zienkiewicz and Taylor, 1991). The thickness integration is accomplished using the third model presented by Kumar and Palaninathan (Kumar and Palaninathan, 1997), in which the isoparametric mapping inverse Jacobian matrix is assumed constant, and computations are carried only on the reference surface.

3.1 Equations of Motion

The element presents five degrees-of-freedom (three displacements and two slopes) per node. The displacement vector at the nodes is denoted by $\mathbf{q}$ and defined as:

$$\mathbf{q}^T = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \Theta_1, \Theta_2\} \quad . \tag{14}$$

The global mass matrix $\mathbf{M}$ and the global stiffness matrix $\mathbf{K}$ are assembled from the corresponding elemental matrices, and $\ddot{\mathbf{q}}$ and $\mathbf{q}$ are the acceleration and displacement vectors at the nodes, respectively. For free vibration, the equation of motion, eq. (5), becomes :

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0 \quad . \tag{15}$$

Assuming a solution for $\bar{\mathbf{q}}(t)$ from eq. (3) in the form of a periodic function given by $\bar{\mathbf{q}}(t) = \eta\, e^{-\jmath\omega_i t}$, then the displacement vector $\mathbf{q}$ is expressed by

$$\mathbf{q} = \mathbf{\Phi}_i\bar{\mathbf{q}}(t) = \mathbf{\Phi}_i\, \eta\, e^{-\jmath\omega_i t}, \tag{16}$$

where η is the generalized displacement amplitude for the i-th natural mode, $\mathbf{\Phi}_i$ is the i-th mode shape vector, and ω_i is the i-th system's natural frequency. Substituting equations (16) in relation (15) yields

$$(\mathbf{K} - \omega_i^2\mathbf{M})\mathbf{\Phi}_i = 0 \tag{17}$$

For computation purposes, eq. (17) is rewritten as a generalized eigenvalue problem,

$$(\mathbf{K} - \lambda_i\mathbf{M})\mathbf{\Phi}_i = 0\,, \tag{18}$$

using the mass normalization

$$\mathbf{\Phi}_i^T\mathbf{M}\mathbf{\Phi}_j = \delta_{ij} \qquad i,j = 1,2,...,n. \tag{19}$$

where $\mathbf{\Phi}_i$ is the eigenvector corresponding to the eigenvalue λ_i, n is the total number of degrees of freedom, and δ_{ij} the Kronecker symbol.

After imposing boundary conditions, eq. (18) can be solved for a given number of eigenvalues and their associated eigenvectors by standard algorithms.

3.2 Concept of Lamination Parameters

Classical lamination theory is an extension the standard plate bending and plane stress theories for layered plates with varying stiffness of each ply. A formal presentation of the theory is not the scope of this work, since it is easily found in many standard books about plates and shells. In this theory, the constitutive equation of laminated plate can be described as:

$$\left\{ \begin{array}{c} \mathbf{N} \\ \mathbf{M} \end{array} \right\} = \left[\begin{array}{cc} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{array} \right] \left\{ \begin{array}{c} \varepsilon \\ \kappa \end{array} \right\} \tag{20}$$

where $\mathbf{N}$ and $\mathbf{M}$ are the distributed tractions and moments, respectively, applied to the plate:

$$\mathbf{N}=\left\{\begin{array}{c} N_x \\ N_y \\ N_{xy} \end{array}\right\} \quad \mathbf{M}=\left\{\begin{array}{c} M_x \\ M_y \\ M_{xy} \end{array}\right\} \tag{21}$$

ε are midplane (membrane) strains, and κ is the vector of curvatures, second derivatives of the transverse displacements.

The submatrix $\mathbf{A}$ is called extensional stiffness matrix and is related to the influence of an extensional midplane strain ε on the in-plane traction $\mathbf{N}$. It can be described as:

$$\mathbf{A} = \sum_{k=1}^{np} \bar{\mathbf{Q}}(z_{k+1} - z_k), \tag{22}$$

where z_k is the distance from laminate midplane to the bottom of the k-th ply, $\bar{\mathbf{Q}}$ is the transformed stiffness matrix, and np the number of plies.

The submatrix $\mathbf{B}$ is a coupling stiffness matrix , this one computes the contribution of the curvature κ to the traction, and it occurs when the plate is not symmetric. This term can be described as:

$$\mathbf{B} = \frac{1}{2}\sum_{k=1}^{np} \bar{\mathbf{Q}}(z_{k+1}^2 - z_k^2), \tag{23}$$

and $\mathbf{D}$ is a bending stiffness matrix defined as:

$$\mathbf{D} = \frac{1}{3}\sum_{k=1}^{np} \bar{\mathbf{Q}}(z_{k+1}^3 - z_k^3). \tag{24}$$

It's important remember that, for symmetric plates, the term related to the coupling matrix $\mathbf{B}$ is zero, once applying an in-plane traction to the plate will not generate a curvature, or a bending moment will not generate an extensional strain. A good review about the formulation of all matrices presented above can be found in many books about composite materials, as in Jones (Jones, 1975).

4 OPTIMIZATION PROCEDURES

Optimization can be described as the search for the best outcome of a given operation provided some known restrictions are satisfied. There is a considerable bibliography describing optimization techniques and methods, since Maxwell and Michel published their groundbreaking works on structural weight reduction. Later, with advent of linear programming techniques, optimization techniques arose in many fields, generating a vast body of literature. Nowadays, optimization is a pervasive tool in many sectors in engineering, economics and almost every other field of knowledge. In engineering area, one can notice a fundamental interest in aeronautical construction to components weight reduction. And with advent of advanced composites, optimization techniques has been developed together aeroelastic tailoring in order to find the best structural design. By means of choosing the right lamination parameters, layup and topology optimization becomes a powerful tool for designing new structures.

4.1 Formulation of the optimization problem

In this work, we obtain the laminated ply configuration to maximize the eigenvalue related to flutter effects of a cantilevered laminated plate. The eigenfrequency chosen is maximized in an unconstrained formulation, using the wing shell ply fiber orientations of the discretized model as design variables, i.e., the number of design variables is the element numbers times ply numbers. The optimization problem can be stated as follows:

$$\text{maximize} \qquad \left(\frac{\omega^2}{\omega_0^2}\right) \tag{25}$$

where ω_0^2 is the non-optimized eigenvalue evaluated in first structural analysis, and ω^2 is the updated eigenvalue of the optimization process.
This formulation allows the algorithm to find the laminate configuration that increases the gap between the selected eigenvalues, whose interaction is causing flutter effects. The goal in analysis is to set the lamination parameters in a way that, the interaction that causes flutter effects will shows up in higher speeds.

4.2 Optimization Algorithms

The flowchart presented in figure 2 shows the optimization procedure used in this work, including the procedure applied to compute the eigenmodes and eigenvalues used as input files for ZAERO. The routine first loop is not exactly part of the optimization, it just assembles the stiffness and mass matrices of the plate after the finite element discretization is performed and obtain the initial eigenvalues and eigenmodes through modal analysis.

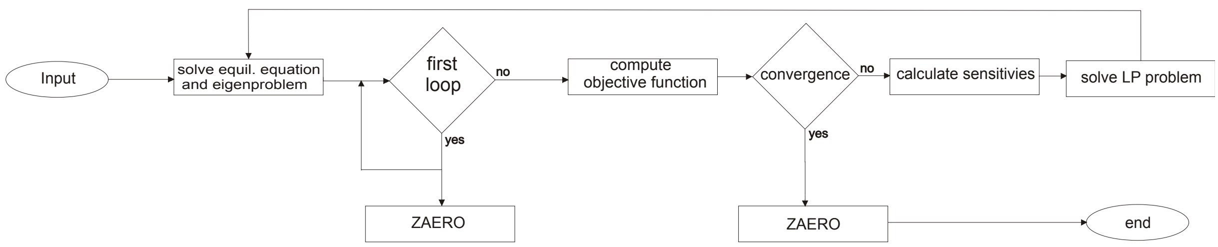

Figure 2: Flowchart of optimization and analysis procedure.

In the first step, the equilibrium equations and the eigenvalue problem are solved, and an input file containing eigenvalues and eigenmodes is created. In the second step, the optimization process starts and the objective function is computed. If the objective function converges, the optimization algorithm stops, otherwise the next step is the computation of eigenvalues sensitivities with respect to design variables. As final step, the design variables are updated by the sequential linear programming (SLP). Then the procedure returns to first step. When convergence is reached, a new set of eigenfrequencies and its eigenvectors are obtained for aeroelastic analysis of the optimized structure. The sensitivity of the j-th eigenvalue λ_j where $\lambda_j = \omega_j^2$ with respect to the i-th design variable θ_i is obtained by

$$\frac{\partial \lambda_j}{\partial \theta_i} = \frac{\mathbf{\Phi}_j^T \left(\dfrac{\partial \mathbf{K}}{\partial \theta_i} - \lambda_j \dfrac{\partial \mathbf{M}}{\partial \theta_i} \right) \mathbf{\Phi}_j}{\mathbf{\Phi}_j^T \mathbf{M} \mathbf{\Phi}_j} \tag{26}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrices, respectively. $\mathbf{\Phi}_j$ is the discretized j-eigenmode, and θ is the fiber angle of composite ply. In this case, is useful to notice that, as the mass of the elements do not vary with the angle of the fibers, then the derivative of $\mathbf{M}$ with respect to θ is zero. And since the eigenvectors are normalized by mass, the equation (26) becomes:

$$\frac{\partial \lambda_j}{\partial \theta_i} = \mathbf{\Phi}_j^T \left(\frac{\partial \mathbf{K}}{\partial \theta_i} \right) \mathbf{\Phi}_j \tag{27}$$

In the step where design variables are updated using SLP in optimization, the relation between the eigenvalue chosen to be updated and the next one needs to be monitored. Since the chosen eigenvalue is increasing at the updates, its value can reach the next eigenvalue. Once a repeated eigenvalue problem is identified, the calculation for the eigenvalue derivatives is not straightforward.

In this research, a method for repeated eigenvalue problem proposed by Pedersen (Pedersen, 1999) is used. For identical eigenvalues $\lambda_i = \lambda_{i+1}$, there are two different eigenvectors. For this eigenvalue sensitivity, it is necessary to find a region, represented by a plane, where maximum and minimum values for derivative are sought.

Let $\mathbf{P}$ be a 2 x 2 matrix, its components can be described by:

$$\mathbf{P}_{mn} = \mathbf{\Phi}_m^T \left(\frac{\partial \mathbf{K}}{\partial \theta} \right) \mathbf{\Phi}_n \tag{28}$$

then the eigenvalues of $\mathbf{P}$ are the maximum and minimum gradient values. The matrix has the size of the numbers of repeated eigenvalues, and the choice for minimum or maximum gradient value depends on whether it is a maximization or a minimization procedure.

5 RESULTS

For the analysis, a 0.45 $[m]$ x 0.08 $[m]$ x 0.002 $[m]$ single cantilevered wing made of Graphite-Epoxy is applied and each ply thickness is 0.0005 $[m]$. The initial configuration of the composite plate is $[0/90]_s$ with the x-axis along the length. As shown in figure 3, the structural analysis model has 288 elements (36 x 8). While in the aeroelastic model the plate is divided in 36 panels for ZAERO analysis (6 x 6).

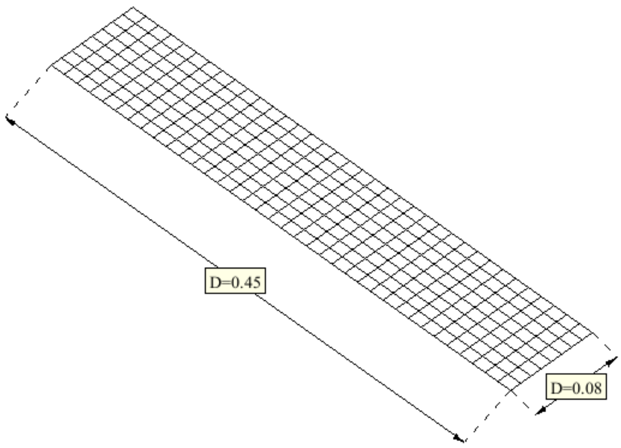

Figure 3: Single cantilevered wing model

The elastic properties of the model are presented in table 1. The plate material is Gr-Ep (graphite-epoxy) and the density is 2100 $[Kg/m^3]$. It is important to emphasize that the values in the table below are ply properties.

Table 1: Laminate Properties

$E_1 = 137.9\ GPa$	$G_{12} = 7.101\ GPa$	$\nu_{12} = 0.30$
$E_2 = 8.963\ GPa$	$G_{13} = 7.101\ GPa$	$\nu_{13} = 0.30$
$E_3 = 8.963\ GPa$	$G_{23} = 6.205\ GPa$	$\nu_{23} = 0.49$

The modal analysis results of the non-optimized structure are in the figure 4. The first eight modal shapes and their frequencies are represented.

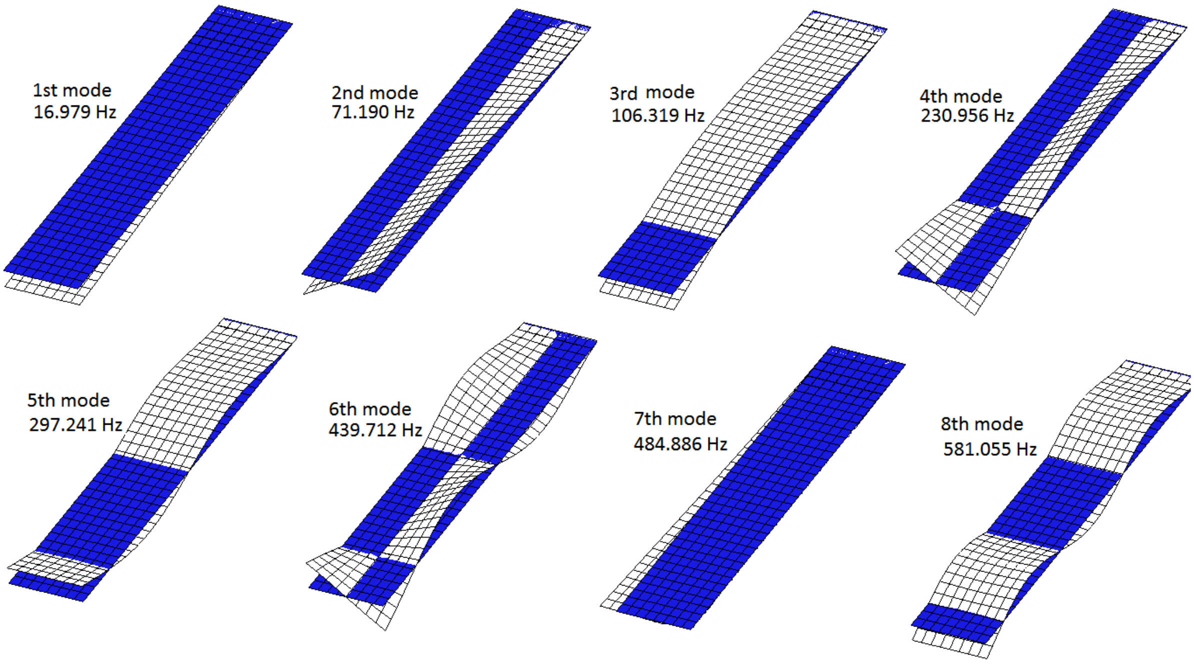

Figure 4: Initial structure frequencies and their modal shapes

The gain in frequency is plotted in the objective function evolution (figure 5). Algorithm convergence is reached in 27 iterations. During the optimization process, the repeated eigenvalue problem shows up. This problem is shown as the slower convergence region of the graph, since the repeated eigenvalue optimization is significantly more difficult.

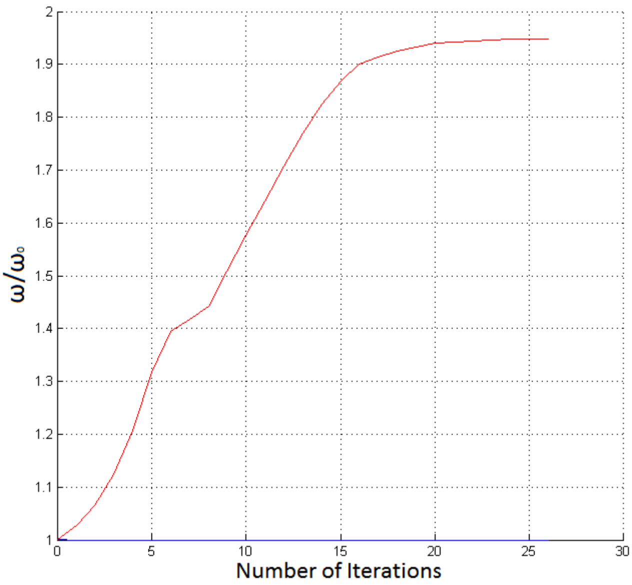

Figure 5: Objective Function

After optimization, the new set of eigenfrequencies and modal shapes are obtained. The figure 6 shows the improvement in the first torsional modal shape frequency, it was previously the second frequency modal shape and now it shows up in the third frequency modal shape.

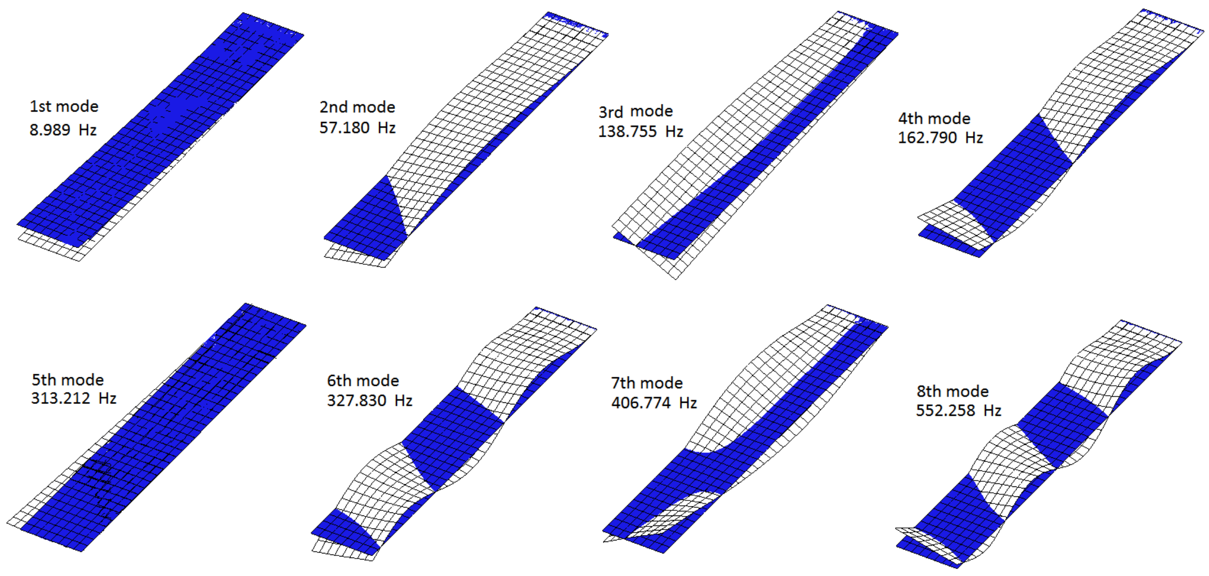

Figure 6: Optimized structure frequencies and their modal shapes

The figure 7 consist in the VGF curves (velocity versus damping and frequency) obtained by ZAERO analysis showing the frequency and damping evolution with velocity. The curves are obtained through the flutter analysis using the modal analysis of the structural model, before and after the optimization procedure.

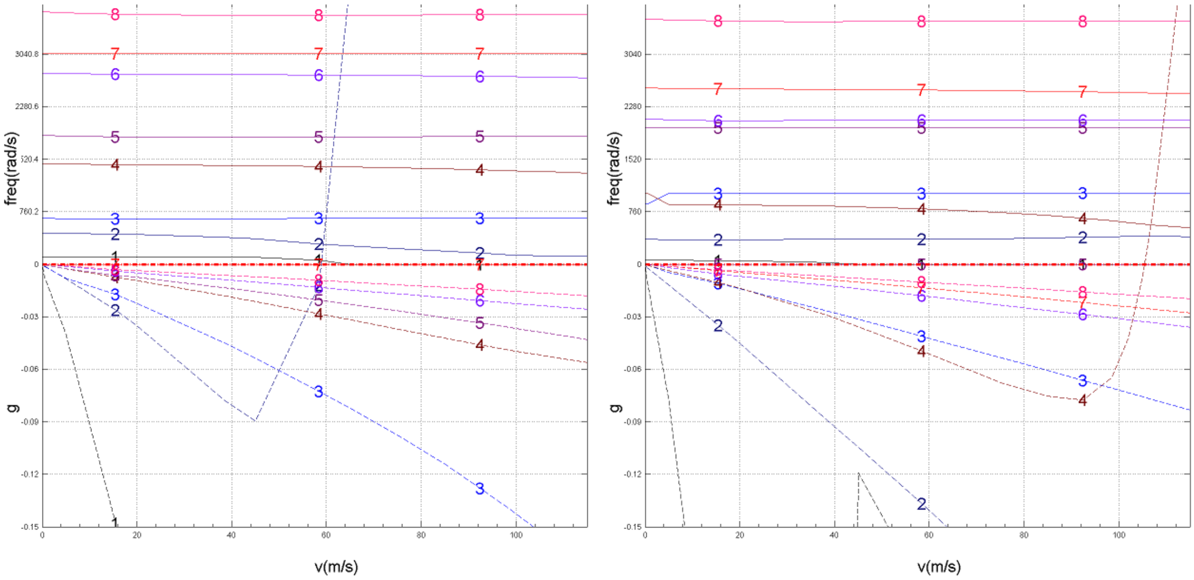

Figure 7: Flutter analysis before and after the optimization procedure

These results show a marked improvement in flutter speed when eigenvalues optimization is applied in the composite wing, and the frequency responsible for such effects is maximized.

The fiber orientation of non-optimized and optimized structure is presented in figure 8. The upper ply is that one with fibers initially oriented along the length and the lower ply has the fiber orientation transversally the wing length.

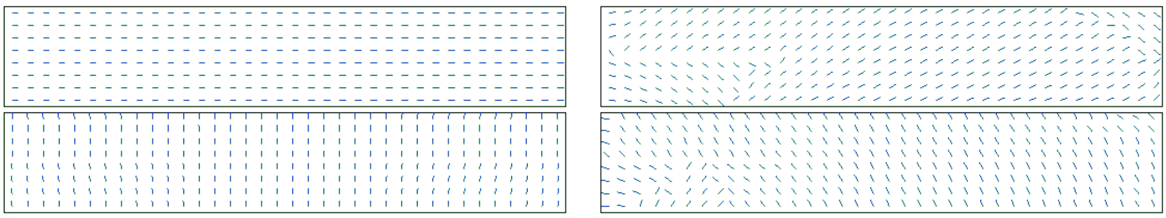

Figure 8: Fiber orientation before (left) and after (right) the optimization procedure

6 CONCLUSIONS

The optimization procedure proposed in this article succeeds in improving the simulated fluid-structure interaction behaviour, increasing the flutter speed. It is important to remark that the simple model used in the simulation does not take into account divergence effects. Although further analyses must be done using a better aeroelastic model, the result indicates a remarked improvement in the wing behaviour. The design obtained in this research is important to validate the design methodology. It can be seen mainly as an academic result, since the varying fiber orientation introduces costly manufacturing complexities. In spite of that, the development of modern fiber-reinforced composite manufacturing techniques indicates that these structures will soon be affordable enough to be widely used. This work reinforced the role of the eigenvalue optimization as a great tool in composite wing designs, without aerodynamic analysis in the problem formulation, but using aeroelastic analysis as input data.

The result of this research helps to stablish aeroelastic tailoring as a fundamental procedure in composite material wing designs. Additionally, it shows that special attention must be due to the optimization methods and not only to the aeroelastic analysis.

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