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PROBABILISTIC METHODS IN ENGINEERING: QUANTIFYING UNCERTAINTY AND IMPLEMENTING PERFORMANCE-BASED DESIGN

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Abstract. The last decades have seen great advances in computational models and in the ability to tackle previously intractable problems in mechanics. Yet, these models normally incorporate many variables which are not deterministic, introducing uncertainty in the calculated output. The problem of estimating the probability distribution of the output, given the probabilistic description of the input, has also received much attention since the 1960's and many computational techniques have now been developed to account for the probabilistic nature of the problem. This paper discusses the integration of analysis and reliability assessment, including examples of large Canadian projects and design codification. Current objectives in probabilistic methods include the development of performance-based design. This paper discusses this topic in the context of earthquake engineering, with an application to the performance-based design of a steel pile under seismic excitation.

1. INTRODUCTION

The last decades have seen great advances in computational models and in the ability to tackle previously intractable problems in mechanics. The development of finite element calculations models, for example, has made it possible to obtain solutions for increasingly difficult problems in solid or fluid mechanics. Yet, these models normally incorporate many variables which are random or non- deterministic, introducing uncertainty in the calculated output. The problem of estimating the probability distribution of the output, given a probabilistic description of the input, has also received attention since the 1960's and many computational techniques have now been developed (Melchers, 1987; Der Kiureghian, 1986; Ditlevsen, 1981). In particular, these techniques have allowed the development of software applicable to complex practical problems, expanding the capabilities of the analyses in order to obtain solutions in a more realistic setting.

Why are probabilistic approaches really necessary? Of course, they provide a reasoned way of taking uncertainties into account. However, it could be argued that older, deterministic design methods, coupled with traditional and experience-based "safety factors", also led to designs that have withstood the challenge of many years in service. One only needs to consider the achievement of beautiful designs like the 1937 Golden Gate bridge in San Francisco. On the other hand, major disasters could be attributed to the lack of a proper probabilistic analysis, as was the case for the adequacy of the flood protection levees for New Orleans during hurricane Katrina. The use of probabilistic methods in engineering does not, per se, improve the safety of specific designs, but the safety level achieved is part of the calculation and could thus be controlled to a desired target value. The introduction of these methods provides a means of achieving a more even distribution of safety as demand conditions change from one situation to another. The probabilistic framework also embodies the treatment of uncertainties in optimization and decision making in all phases of engineering. Also implicit is the advantage of improvements in the economy of the designs and a better utilization of materials, along with a fairer comparison of different design alternatives.

Probabilistic problems in engineering are generally stated in terms of the demand D on a system and the capacity C to withstand that demand. The function $G = C(\mathbf{x_c}) - D(\mathbf{x_d})$, also called safety margin or limit state or failure or performance function, depends on sets of random variables $\mathbf{x_d}$ and $\mathbf{x_c}$ associated with, respectively, the demand and the capacity. C and D may be explicitly given functions, but usually they are only known as the result of discrete numerical calculations using complex models. For example, C could implement a material fatigue model while D is the effect from a randomly applied stress history. D could also be the output from a finite element model calculation, in which the variables $\mathbf{x_d}$ could represent random variables for a spatial variation in material properties (stochastic finite elements). In earthquake engineering, C may be the displacement limit for the structure and D the maximum displacement calculated by a nonlinear dynamic analysis for a corresponding input ground motion.

Given this formulation, the probability of system non-performance equals the probability of the event $G < 0$. The calculation of this probability can be achieved by simulation procedures (with different levels of efficiency), or by approximate approaches like First or Second Order Reliability Methods (FORM or SORM), both based on the calculation of a reliability index β (Melchers, 1987; Ditlevsen, 1981). These methods have now been implemented in computer programs which are commercially available, with the user only being required to provide the definition of the problem (that is, the capacity and demand functions). The reliability of a system can be calculated either for the event coinciding with the application of the demand (as when an earthquake occurs) or over a service life (when the occurrence of the demand may itself be uncertain). When either D or C, or both, are only known as discrete results from numerical calculations for specific values of the intervening variables (for example, from a finite element or a nonlinear dynamics analysis), then D and/or C can be represented by approximating functions in terms of those variables, a technique that includes the use of general response surfaces or neural networks (Bucher, 1990; Zhang, 2004; Möller, 2003; Möller et al., 2009, 2010a, 2010b; Ghalibafian, 2006). As the approximating functions may not fit the discrete results exactly, additional random variables may be introduced to represent the regression error.

Some design characteristics of the system (e.g., geometric dimensions, statistical parameters) are usually part of the model for either C or D. Although the reliability can be calculated for a system of given characteristics, an important objective in probabilistic design is to find values for those parameters so that the corresponding probability of non-performance (or Prob($G<0$)) be less than a tolerable value. At the same time, optimization objectives can be added so that a minimum weight, or minimum total cost, are achieved under the required reliability constraints.

Probabilistic design then involves making full use of our computational tools in the context of uncertain inputs, in order to ensure, with required confidence, that the performance of the system during its service life be as prescribed. This objective has now become known as "*performance-based design*", and different techniques for its implementation are still being discussed and applied (Wen, 2001; Bertero and Bertero, 2002; Möller 2010).

Applications of full probabilistic approaches can now be increasingly found in practice. In Canada, they have been applied to different types of important problems, as the following examples detail.

1) Hibernia is an oil gravity off-shore platform located in the Atlantic, on the Grand Banks off the coast of Newfoundland, on a depth of about 80m (Figure 1). A reinforced concrete structure had to be designed to withstand possible collisions with icebergs drifting from the north, possibly in an unfavorable sea state. Collision loads

were to be estimated for an annual exceedence probability of $1.0x10^{-4}$. Random variables were the iceberg size and speed, the sea or wave state, and the crushing properties of ice (dependent on the size of the collision area). Models included consideration of energy balance during the collision, plus hydrodynamic interaction between the waves and the ice mass (Foschi, Isaacson,et al.,1996,1998).

2) Figure 2 shows another example, the Confederation bridge, a 17km structure spanning the Northumberland Strait between the Canadian provinces of New Brunswick and Prince Edward Island. At this site, the sea freezes in winter and ice sheets travel downstream impacting the piers near the surface, with a level arm of 50m with respect to the sea bottom, Figure 3. Thus, the weight of the pier foundation had to be designed to withstand the ice impact without resulting in tilting of the piers or foundation sliding at the bottom. A design requirement for this bridge was a target reliability index β = 4.25 for the maximum demands in 100 years.

3) Figure 4 shows a new cable-stayed bridge constructed recently across the Fraser river near Vancouver, British Columbia, and Figure 5 shows the new bridge across the Pitt river, an important tributary of the Fraser. Both of these rivers are navigable, with flows controlled by seasonal fluctuations and the tides. Of concern was the maximum force that could be imposed on the bridge piers as the result of a collision with an out-of-control vessel.

Figure 1. Hibernia oil platform

Figure 2. Confederation bridge

Figure 3. Ice impacting with pier, Confederation bridge

According to requirements of the Canadian Highway Bridge code, that force must be calculated in correspondence with a $1x10^{-4}$ annual exceedence probability. Random variables involved were the vessel traffic, vessel size, displacement and speed, vessel position within the navigation channel when in distress, the river current at different times of the year, and the effect of the tides (all bridges are located at a relatively short distance from the sea).

Figure 4. Golden Ears bridge, British Columbia, 2009

Figure 5. Pitt River bridge, British Columbia, 2010

4) Other applications have involved the simulation of manufacturing processes for composites (from wood resources or utilizing carbon fibers), studying the variability in the mechanical or geometric properties of the composite as functions of random variables associated with the manufacture (amount of resin used, fiber size, fiber orientation, temperature and pressure history, cooling time, etc.).

Earthquake engineering is a topic that naturally involves large uncertainties, mostly associated with the dependence of the demand D on the ground motion. Probabilistic approaches have been proposed recently to implement performance-based design, and the use of neural networks or response surfaces to represent D at different performance levels have been the subject of several recent works (Zhang et al., 2004; Möller et al., 2003, 2009), and of other papers at this conference (Möller et al., 2010a, 2010b).

All the previous examples utilized probabilistic approaches and have in common that they were not covered by codified calculation procedures. All the applications specified just the performance criteria and an accepted probability with which they might not be met.

It is argued sometimes that full probabilistic methods may impose a substantial demand on the design engineer, depending on his/her familiarity with reliability theory, specialized software and probabilistic problem formulation. Modern design codes have attempted to circumvent this apparent difficulty by developing procedures that *partially* achieve the goals of probabilistic design, while maintaining a framework similar to the traditional deterministic approaches. Let us now discuss this aspect of the problem.

In design codes, the random variables are represented by fixed "characteristic or design" values. Thus, for example, the 5th-percentile of the tensile strength of wood may be adopted as the "characteristic strength", or the acceleration spectrum, for a given structural period and with a 2% exceedence in 50 years, may be adopted to represent the "design intensity" of an earthquake. A deterministic design equation is then used to calculate the design parameters. In this equation, as generally shown in Eq.(1), the characteristic values are multiplied by factors ("load factors" or "load combination factors" for the demands, "resistance factors" for the capacities),

$$\alpha_D D_N + \alpha_Q (Q_{N1} + \gamma\, Q_{N2}) = \varphi\; R_N\, S \tag{1}$$

in which: D_N = effect of the characteristic dead (or permanent) load D;
α_D = load factor for dead loads;
Q_{N1}, Q_{N2}= effect of the characteristic design live loads Q_1 and Q_2;
α_Q = load factor for live loads;
γ = live load combination factor;
φ = resistance factor;
R_N = characteristic resistance;
S = design parameter (e.g., cross-sectional modulus for bending).

Given the characteristic values and the different factors, Eq.(1) can be used to calculate the design parameter S. This calculation, in fact, is not substantially different from older, deterministic approaches. The objective of the design code is to permit the calculation of S so that its associated reliability, under uncertain loads D, Q_1 and Q_2 , matches a target for the code. The reliability corresponding to the calculated S from Eq.(1) can be evaluated with the performance function

$$G = R\,S - (Q_1 + Q_2) \tag{2}$$

The different factors in Eq.(1) are adjusted until the target reliability is matched. However, if the uncertainties in the loads are changed (for example, when the load statistics are changed), different factors will be obtained. Thus, since the factors are normally meant to cover all design situations, the load and resistance factors must be calibrated or optimized over a number of applications. Obviously, with only a small number of optimizing factors, the achieved reliability would vary from application to application. The reliability would remain uncertain for applications outside the range used for the calibration. Thus, codified approaches cannot guarantee a target reliability and, in this sense, they only partially fulfil the objectives of probability-based design. If properly calibrated, they can show a tolerable narrow range of reliabilities across design situations. To improve the results, codes may introduce more factors under different names, for example, "load combination factors". At the moment, not all codes have been calibrated in all countries, and even within a specific code there might be sections which have not been calibrated. Earthquake engineering is an area in which little calibration has taken place, as the large uncertainties involved are not consistent with a small number of factors desired in a code design procedure. Earthquake engineering is, therefore, an area in which full probabilistic approaches offer a wide range of application possibilities, at the structural, geotechnical or soil/structure interaction level.

To close this introduction, it should be apparent that the results from a probabilistic analysis are conditional on the computational models used for either capacity or demand. Although "model error" could be acknowledged by introducing additional random variables, the robustness of the approach calls for always using the "best" computational tools, rather than simplifications which will require a greater, and more uncertain, model error correction.

2. PERFORMANCE-BASED DESIGN IN EARTHQUAKE ENGINEERING

This section discusses reliability assessment and performance-based design in the context of earthquake engineering. As with any other engineering application, the problem requires explicit satisfaction of multiple performance criteria, with associated levels of confidence over a service life. The structural or geotechnical analysis used must reflect the actual behavior in a realistic manner. All the major uncertainties involved must be taken into account: the ground motion, the material nonlinearities, the hysteretic behavior and energy dissipation of members and connections, and the approximate nature of the analytical models used to estimate either the responses or the induced damage. Structural reliability analysis must be applied to evaluate the probability of non-performance for each of a set of performance or limit state functions, related either to collapse or to different damage levels. A discussion of the need for a reliable, comprehensive approach to performance-based seismic engineering has been offered by Bertero et al. (2002). The probability estimations may be carried out by different methods. Direct simulation may be used to estimate the probability of the event G<0 (Monte Carlo trials) but, as the probability of non-performance in a well-designed structure is generally small, the simulation may entail a great number of performance function evaluations, each requiring the execution of a nonlinear dynamic analysis. This task could quickly become computationally very intensive. Other techniques start by computing conditional probabilities for a fixed level of the hazard (for example, a fixed level of the peak ground acceleration). These conditional probabilities, called fragilities, are then used to compute the total probability by integration over all possible hazard levels. The implementation of approximate methods like First or Second Order Reliabilities may create numerical problems as the corresponding algorithms depend on a determination of the gradient of the function G, which may not be smooth.

In order to make more efficient the use of simulations, a method has been proposed that implies a functional representation of the nonlinear dynamic results, in terms of the intervening variables and design parameters. This representation could take the form of: 1) a mathematical function (response surface) adjusted to a database of discrete response results (Möller,2003); or 2) a neural network (Zhang et al., 2004, Möller et al., 2009) trained with the input-output results from the dynamic analysis. In any case, the representation can be used as a substitute for further dynamic analyses, simplifying and making more efficient the task of simulation. Neural networks are used to represent complex, unknown input-output relationships. Here they are used to represent such a relationship between the input variables (including the design parameters) and the response outputs obtained from the dynamic analysis. This paper presents an application of this method in reliability assessment and performance-based design in seismic engineering, using as a case study the design of a pile foundation. Other case studies are also presented at this conference (Möller et al., 2010a, 2010b).

In the context of earthquake engineering, the structural response will change every time that the accelerogram record of the ground motion is changed. Thus, for a given peak ground acceleration, the response will change with the frequency content of the record and the duration of the strong motion. Let $\boldsymbol{X}$ be the vector of random variables excluding the record, $\boldsymbol{d}$ the vector of design parameters and r a nominal variable representing the individual record. A response R (for example, maximum drift or maximum shear force) will then be a function

$$R = f(\mathbf{X}, \mathbf{d}, r) \tag{3}$$

For fixed combinations of $\boldsymbol{X}$ and $\boldsymbol{d}$, the responses R are obtained for a set of records r, calculating the mean response value $\overline{R}(\mathbf{X}, \mathbf{d})$ and the standard deviation $\sigma_R(\mathbf{X}, \mathbf{d})$ *over all records*. Each of these statistics is then represented by a corresponding *neural network*, for which the input variables are $\boldsymbol{X}$ and $\boldsymbol{d}$.

If the distribution of R over the records is assumed to be Lognormal (since one would be interested in the absolute value of R), then

$$R(\mathbf{X},\mathbf{d}) = \frac{\bar{R}}{\sqrt{1+(\sigma_R/\bar{R})^2}} \exp\left(R_N \sqrt{\ln\left\{1+(\sigma_R/\bar{R})^2\right\}}\right) \quad (4)$$

in which R_N is a Standard Normal variable.

Performance-based design requires the optimal satisfaction of multiple performance objectives, with associated confidence levels. A typical performance function is assumed in the form

$$G(\mathbf{X},\mathbf{d}) = R_{LIM} - R(\mathbf{X},\mathbf{d}) \quad (5)$$

in which the response $R(\boldsymbol{X},\boldsymbol{d})$ is represented by Eq.(4), implementing the corresponding neural networks for $\bar{R}$ and σ_R. R_{LIM} is the limiting capacity for the response R. This utilization of the neural networks permits a very efficient calculation of the probability of failure corresponding to a given vector of design parameters ***d*** and statistics for ***X***, either by Monte Carlo simulation or by Importance Sampling simulation around an "anchor point" in the Standard Normal, uncorrelated variable space. An anchor point could be calculated by fitting first a quadratic response surface around the mean point (Bucher, 1990), using responses obtained when each of the variables, in turn, is set at its mean ± 2 standard deviations. In each case, the performance function is evaluated from the neural networks. Using this quadratic surface and starting from the mean point, one iteration of FORM yields a first approximation for a design point and the corresponding vector joining it with the origin (mean point). Subsequently, the anchor point is found by searching along that vector direction until the performance function $G(\boldsymbol{X},\boldsymbol{d}) \sim 0$.

The vector ***X*** sampled during the simulation must be checked to ensure that it is situated in the space enclosed by the bounds of the random variables in the database used to develop the neural networks, and the sampling density functions must be appropriately censored. Finally, the probability of failure P_f obtained by simulation is expressed in terms of a corresponding reliability index $\beta(\boldsymbol{d})$ using the correspondence

$$P_f=\Phi(-\beta) \quad (6)$$

Current earthquake design codes consider only one performance requirement, related to life safety and preventing structures from collapse. This criterion has proven insufficient, however, since recent experiences have shown that structures subjected to moderate earthquakes can sustain levels of damage which, although not resulting in collapse, can imply very high costs of repair. Thus, more than one performance level should be considered, including serviceability, controlled damage and survivability. For serviceability levels under frequent minor earthquakes, the structure should remain basically in the elastic range. The deformation, for example, inter-story drift, should be checked for damage levels under occasional, moderate earthquakes. In this case, the structure may respond nonlinearly. For survivability under rare, severe earthquakes, the structural performance is fully nonlinear and may be at the edge of collapse.

Performance-based design should then be formulated as an optimization problem: to find the design parameter vector ***d*** by minimizing an objective function (e.g., total cost), subject to specific reliability constraints for each of the performance criteria considered. If total cost is not minimized, then the optimization could consider only the reliability constraints and the following objective function,

$$\Psi = \sum_{j=1}^{ND} (\beta_j^T - \beta_j(\mathbf{d}))^2 \tag{7}$$

subject to

$$d_i^l \leq d_i \leq d_i^u \tag{8}$$

in which

ND is the number of performance criteria;

β_j^T is the target reliability index for performance level j;

$\beta_j(\mathbf{d})$ is the calculated reliability index for performance type j , given the design parameter vector ***d***;

d_i^l is the lower bound of the design parameter d_i;

d_i^u is the upper bound of the design parameter d_i.

The optimization can be done by any constrained optimization procedure. A gradient-free search algorithm (Möller et al., 2009) has been shown to be effective and robust.

Using fragilities to calculate the probabilities of non-performance (conditional on a hazard level) leads to the same results that would be obtained with the method described here. However, for design optimization, fragilities would have to be obtained for different design parameter vectors ***d***. Here, the vector ***d*** is part of the input for the neural networks and, therefore, the consequences of changes in design can be quickly evaluated. The computational work required to obtain all the fragilities may be equal to, or larger than, that required to develop the databases to train the neural networks.

3 APPLICATION EXAMPLE: STEEL PILE FOUNDATION

Figure 6 shows a pile steel tube, of diameter D, wall thickness t and length L, supporting a mass M. The pile is embedded into a sandy soil layer with relative density D_R . Under earthquake excitation, the mass will displace an amount Δ. Of interest is an assessment of the probability with which the displacement Δ (the response of interest) will exceed levels given as fractions of the pile diameter D. That is, the performance function G is defined as

$$G(X,\mathbf{d}) = \lambda \, \mathrm{D} - \Delta(\mathrm{a}_G, \omega_S, T, M, D_R, r) \tag{9}$$

in which

λ = fraction of D defining the limiting displacement;
a_G = peak ground acceleration;
ω_S = soil frequency in the Clough-Penzien Power Spectral Density function;
T = duration of the strong motion part of the accelerogram record;
M = applied mass;
D_R = soil relative density;
r = nominal variable indicating the accelerogram record.

The pile has a diameter D = 0.356m, with wall thickness t = 0.10m, and a length L = 30m. Yield strength and elastic modulus were assumed deterministic and to have nominal values for mild steel (respectively, 250 MPa and 200000 MPa). Twenty earthquake records were simulated as stationary processes using a spectral representation based on the Clough-Penzien Power Spectrum Density function (Clough and Penzien, 1975), an envelope modulation function (Amin and Ang, 1968), and twenty different sequences of random phase angles.

For different combinations of the intervening variables, databases were constructed for the mean response Δ and its standard deviation over the twenty records. These databases were then used to train corresponding neural networks, as previously discussed. Finally, the response Δ in Eq.(9) was represented using the format shown in Eq.(4).

The structural analysis of the pile was done considering the dynamic equilibrium of the mass M as a single degree of freedom system. The restoring hysteretic force $F(\Delta)$ can be calculated using a beam finite element model of the pile, allowing for its elasto-plastic behavior, and calculating the deflected shape w. The nonlinear soil reactions $p(w)$ shown in Figure 6 can be represented by p-w curves, keeping track of the development of gaps between the pile and the surrounding soil. This model (Foschi, 2000) depends solely on mechanical properties of the pile and the soil, and produces the hysteretic loop for any input excitation, automatically developing the pinching characteristics.

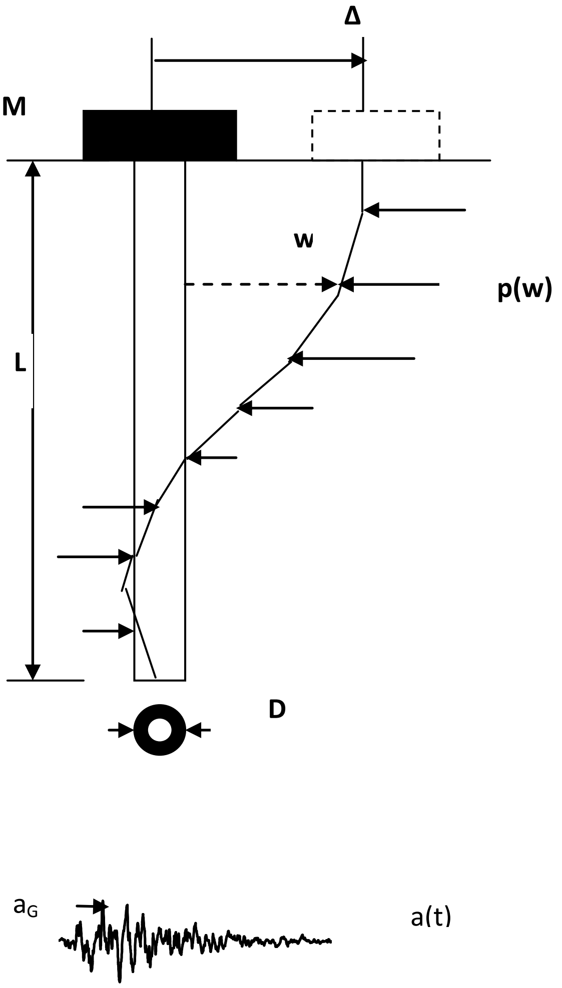

Figure 6. Pile steel tube under earthquake excitation

The *p*-*w* relationship used was (Yan and Byrne, 1992),

$$p(w)=\begin{cases} E_{max}w & \text{if } w \le \alpha^2 D \\ E_{max}\alpha D\left(\dfrac{w}{d}\right)^{0.5} & \text{if } w > \alpha^2 D \end{cases} \tag{10}$$

in which $\alpha = 0.5\,(D_R)^{-0.8}$, and D_R is the soil relative density. The modulus E_{max} depends on the specific weight of the soil and the depth of the soil layer (Yan and Byrne,1992). In this work only the relative density was considered to be a random variable. In order to explore the importance of the analysis model used, an alternate formulation of the hysteretic properties was considered. In this formulation, the response to a prescribed pile head cyclic displacement history is obtained first (by calculation or by testing), and it is then fitted with a specified format for the hysteretic loop. Many such formats are contained in dynamic analysis packages but, although one could obtain a good fit of the cyclic response, there is no guarantee that the good representation would also be achieved for any other history or earthquake record. Perhaps the most sophisticated approach of this kind is the model commonly known as BWBN (Baber et al., 1981, 1985). This is a first-order differential equation, containing 13 parameters. The solution of this equation, for a given history, can represent loops with pinching and degradation characteristics. The parameters are adjusted to match a given cyclic response. Although very versatile, this approach cannot guarantee that the same parameters would generate a proper loop for excitations other than the one used for their calibration. One of the objectives in this application example is a comparison of the two hysteretic formulation approaches, both for reliability assessment as well as for performance-based design.

The pile was subjected to a cyclic displacement history *Δ(t)* as shown in Figure 7. The finite element approach developed a corresponding hysteresis loop as shown in Figure 8. This response was used to calibrate the parameters of the BWBN model, with the resulting loop shown in Figure 9.

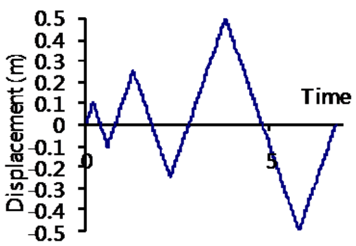

Figure 7. Cyclic displacement history

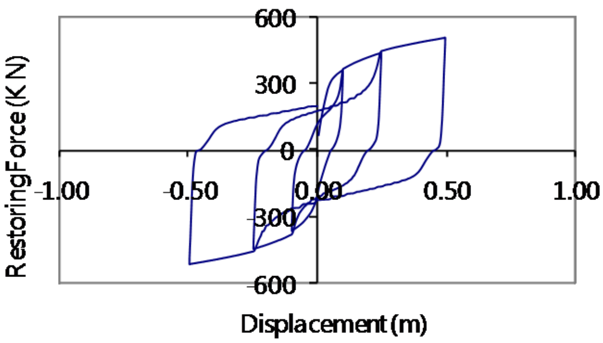

Figure 8. Calculated loop, cyclic displacement history, finite element approach.

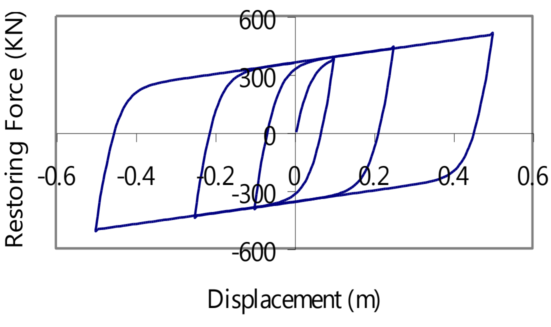

Figure 9. BWBN-developed loop for cyclic displacement history

Dynamic analyses for the twenty different earthquake records were carried out with the BWBN representation of the hysteretic restoring force and, alternatively, calculating each time the hysteresis via the finite-element model. In both cases, the neural network methodology previously described was applied. Figures 10 and 11 show, respectively, the degree of agreement between the neural network predictions and the analysis data for the mean and the standard deviation of the response *Δ* over the records . In each figure, $\sigma_{\varepsilon r}$ is the standard deviation of the relative error.

Reliability evaluations were carried out for different values of the parameter λ. The use of neural networks, as described, facilitates the use of simulations as these become very efficient. Table 1 shows the statistical data for the intervening variables, and Table 2 the reliability results.

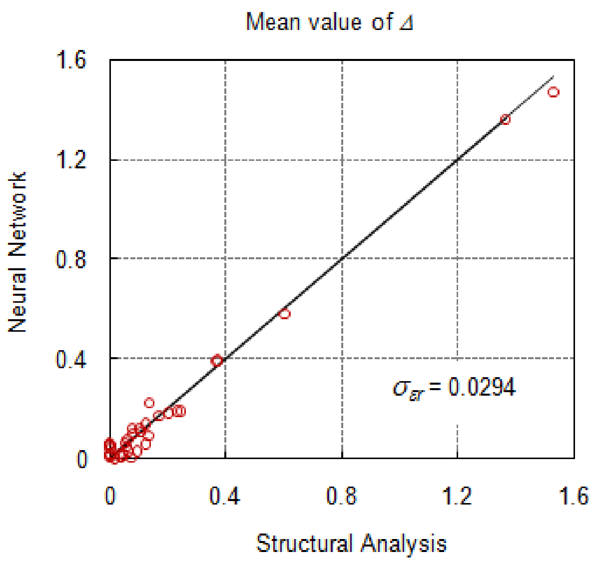

Figure 10. Neural network representation, mean response

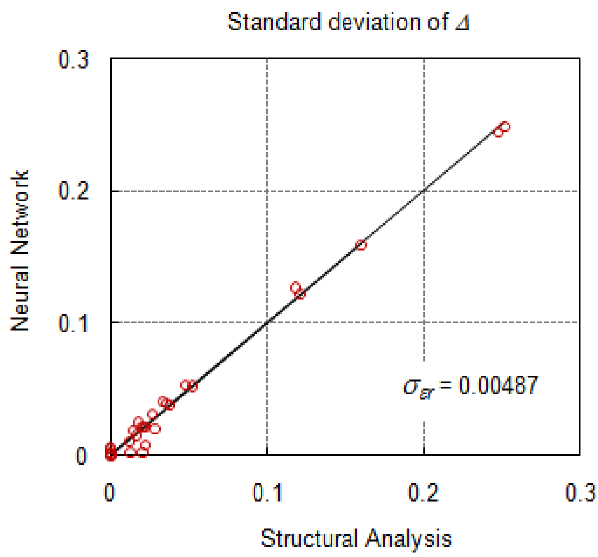

Figure 11. Neural network representation, standard deviation of the response

Variable	Distribution	Mean	Standard Deviation
a_G (m/sec^2)	Lognormal	1.0	0.6
ω_S (rad/sec)	Normal	4π	π
T (sec)	Normal	12	2
M (kN.sec^2/m)	Normal	150	15
D_R	Normal	0.5	0.1

Table 1. Statistical data for the intervening variables

The statistics for the peak ground acceleration a_G correspond the event, and are consistent with a design acceleration (475 years return period, or 10% in 50 years) of 0.31g , assuming that earthquakes have a Poisson arrival rate of 0.2 (on average, one every five years).

Limit definition, factor λ	Reliability index β	
	Hysteresis: Finite Element	Hysteresis: BWBN
0.1	-0.143	0.716
0.2	1.097	1.724
0.4	2.509	2.379
0.6	3.197	2.675
0.8	3.730	2.892
1.0	4.243	3.082

Table 2. Reliability results and comparison between hysteretic models

Table 2 shows that the model used for the hysteretic restoring force has a significant influence on the calculated reliability level and that, in this case, while the approximating BWBN model provides conservative answers for high λ, the reverse occurs at low levels of displacement performance.

Finally, for performance-based design, two performance criteria were chosen: a displacement level associated with moderate damage level, with λ = 0.40, and another associated with more substantial damage, λ = 1.0. For the first criterion, the target reliability was β = 2.5, while for the second the target was β = 4.5. The design parameter was the mean mass M, that is, the problem is to determine the optimized mean value of the applied mass allowing for a coefficient of variation of 0.10. The results are shown in Table 3, again comparing results obtained using either of the two different hysteretic approaches. It is seen from this Table that the permissible mean mass M, for the same set of performance criteria, is substantially influenced by the hysteretic model used.

Hysteresis: Finite element			
Mean Mass (kN.sec^2/m)	Performance criterion λ	Target reliability β	Achieved reliability β
139.83	0.4	2.5	2.589
	1.0	4.5	4.508
Hysteresis: BWBN			
Mean Mass (kN.sec^2/m)	Performance criterion λ	Target reliability β	Achieved reliability β
102.19	0.4	2.5	2.761
	1.0	4.5	4.420

Table 3. Performance-based design, results

4 CONCLUSIONS

A complete study of an engineering problem requires the coupling of analysis tools and reliability analysis, to properly consider the uncertainties in the input variables and the resulting variability in the output. Coupling of these tools is necessary to implement performance-based design, when the system is required to meet different performance criteria with specified minimum reliabilities and at a minimum weight or cost.

Many applications of this coupling exist, particularly for large projects with severe consequences in the case of non-performance. Modern design codes attempt to partially achieve the objectives of a full probabilistic analysis by implementing deterministic design equations which incorporate several factors for the load and for the resistance. In a properly calibrated code, these factors are adjusted to achieve, on average and over a large number of applications, a target reliability which has been chosen for the code.

Not all codes in all countries have been calibrated on a reliability basis. This applies, in particular, to codes for earthquake design. For seismic applications, and because of the many uncertainties associated with the ground motion, a full probabilistic analysis would provide a general approach to seismic reliability estimation and performance-based design. To this end, a straight-forward method has been presented. The method is based on the development of response databases, using different combinations of the intervening random variables and design parameters, combinations which are optimally chosen within the variable bounds. These databases are obtained for the mean and for the standard deviation of the responses over a set of earthquake records, all normalized to have a unit peak acceleration or a unit acceleration response spectrum at a given period. The databases are then used to train corresponding neural networks, a strategy that permits a very efficient evaluation of reliability and to find optimum design parameters satisfying, as best as possible, a set of target reliabilities for the different limit states. Neural networks allow the use of importance sampling simulation (even Monte Carlo simulation) without a heavy computational effort. This simulation task would be very demanding if the responses would need to be calculated, every time, with a new dynamic analysis. At the same time, this efficient implementation of simulation provides an alternative to methods like FORM which, in dynamics problems, sometimes show convergence difficulties.

All analysis and reliability results are, of course, conditional on the models used. An application example has shown that hysteretic modelling can play a substantial role in seismic reliability assessment or performance-based design. It is important that the model used for hysteresis be able to adapt to any input ground motion, since the use of a model fitted to a response from an experimental cyclic displacement history cannot guarantee reliable answers when applied to any other history. This is contrary to the common assumption that hysteresis modelling is not important because any uncertainty associated with it is overwhelmed by the uncertainty in the ground motion.

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