Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXIX, págs. 1747-1759 (artículo completo)
Eduardo Dvorkin, Marcela Goldschmit, Mario Storti (Eds.)
Buenos Aires, Argentina, 15-18 Noviembre 2010

# COUPLING HEAVY FLUID RADIATION EFFECTS TO NONLINEAR VIBRATION ANALISYS OF STRUCTURES

**Francisco Ilson da Silva Junior**[a] **and Antoine Chaigne**[b]

[a]*Departamento de Engenharia Mecânica e Produção, Universidade Federal do Ceará, Campus do Pici - Bloco 714 - 60455-760, Brasil, http://www.dem.ufc.br*

[b]*Unité de Mécanique, École Nationale Supérieure de Techniques Avancées, 91761 Palaiseau cedex, France, http://www.ensta.fr*

**Keywords:** Vibroacoustic, Heavy Fluid Radiation, Nonlinear Vibrations.

**Abstract.** This study is motivated by current issues in transportation acoustics, where the goal is to predict and reduce the noise radiated by vibrating structures. Potential applications include passive and active vibroacoustic control of extended sources. In this work, the vibration of structures is described by numerical techniques. Structures like beams and plates can be modeled by finite element techniques. Nonlinearities effects originated by large displacements formulation are use to describe the mechanical system. In this implementation, the dynamic model of the Von-Kármán equations is used and the solutions are projected over the linear modes of the structure. The structural system is coupled to the infinity fluid action. It was used one temporal formulation calculated by a convolution integral of the fluid radiation impedance and the champ of velocities of the mechanical system. The fluid-structure effects are extent to heavy fluid coupling. The goal of this work is put numerical simulation of sound radiated by vibrating structures including sound-structure coupling. The formulation is developed in the extended case including resonance modes and geometrical nonlinearities of the structure. Two geometrical configurations of the plate were used. The results obtained by the numerical method using the present models are achieved and the influence of geometrical nonlinearities on sound radiation is explored. Firstly, the linear model was used to study the acoustic radiation influence on the dynamical responses. One second analysis was performed, using a nonlinear formulation. This way, the heavy fluid radiation effects on the nonlinearities were investigated.

## 1 INTRODUCTION

In the last years, the acoustic propagation radiated by vibrating structures has been studied for the noise reduction application. Several authors showed special interest on perturbations in a linear acoustic field generated by structural non-linearities (Ginsberg, 1975; Nayfeh and Kelly, 1978). One application is the electrodynamic loudspeakers design (Quaegebuer, 2007). It is expected a linear transduction. However, for high levels of vibrations, nonlinear phenomena appear and are responsible for audible distortions (Quaegebeur and Chaigne, 2008). The goal of this paper is to present one formulation for structural-acoustic problems composed by a non-linear elastic plate coupled to the infinity fluid loading. In order to test the methodology, it was proposed several numerical solutions for an elementary model loudspeaker. For this, it is necessary to understand the validity of the proposed models. In this work, the large displacements nonlinear effects are coupled to an impedance fluid model. Two deferents values of impedance was tested. In this work, it was considered light and heavy fluids. It was determinated the radiation effect of structures in high levels of amplitudes on acoustic domains. The model is based in a process of radiation of small amplitude sound waves by an oscillating structure. In this model, fluctuations of physical properties, like as air density and sound speed, are neglected. The mechanical system is composed by a elastic circular plate in baffled conditions. The non-linear geometry results in cubic non-linearity of Von-Kármán equations dynamic analog. It is considered that the sound wave radiation and velocities are governed by the linear wave Helmholtz equation. In fact, sufficiently far from the structural surface, the fluid motion is acoustical, (Kinsler et al., 1995). In order to instigate the coupling among the structural mechanical behavior and the acoustic impedance effect, one temporal formulation calculated by a convolution integral of the fluid radiation impedance and the champ of velocities of the mechanical system was performed. The fluid-structure effects are extent to heavy fluid coupling. Analytical solutions for circular plates was projected on a linear mode base in order to expand the temporal solution of the coupled system. The aims of this paper is to present a formulation of a problem of structural acoustics in heavy fluid loading conditions of a non-linear elastic structure and clarify the nonlinear structural effects on acoustical fluid radiation done by a temporal numerical approach. The outline of the rest of the paper is as follows. In Section 2, the acoustic-structure coupling for a baffled plate modeling a loudspeaker is described. Moreover, the basic assumptions of the model are presented. In the next section, we state the bending plate problem with large displacements. The Von-Kármán dynamic analog model is presented by the governing equations description. In this context, analytical and numerical approaches are applied to describe the system and a modal superposition technique is used to model the coupled dynamic system. In addition, the procedure to decompose the heavy acoustic radiation effect in a time formulation is presented in this section. In Section 4, the numerical results are presented and the performance of the method is illustrated. The conclusions are outlined in Section 5.

## 2 PROBLEM DESCRIPTION

In this work, the dynamical response of a loudspeaker model is performed by numerical methods. The problem consists of a mechanical system under acoustic impedance effects. The structure is placed in baffled conditions. In order to model the dynamical behavior of the system, an acoustic-structure coupling problem among one circular elastic plate and the exterior fluid is evaluated. Therefore, the reaction of the surrounding fluid on the structure is considered as dissipative and it acts against the motion sense. In general, the excitation forces of the loudspeaker system are electromagnetic nature. In this work, the eletro-mechanical coupling analysis is beyond the scope of this paper. The coupling is performed by a convolution integral of the acoustic impedance function and the normal spatial velocity distribution of the plate. The analog equations of Von-Kármán are used to describe the mechanical system. The boundary conditions and the forces of the mechanical system are presented in Fig. (1).

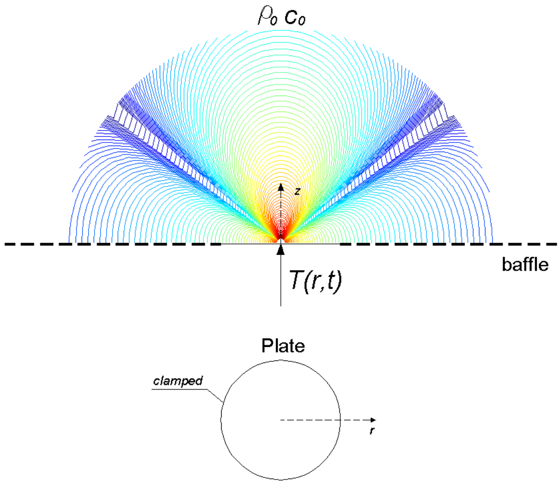



Figure 1: Model for the Acoustic-Plate Coupling.

In this work, two geometrical configurations of the circular plate with acoustic radiation coupling were tested. The first one, it is considered flat and circular. The second configuration is more realistic. Therefore, the loudspeaker is viewed as the combination of an spherical central structure with a conical suspension. More details of the geometries are described in next sections. The assumptions of the model are: the plate is considered thin in according with the Kirchhoff-Love bending theory and the rotatory inertia effect in plane is not considered. In addition, the nonlinear terms are cubic nonlinearities. In the next section, the Governing Equations for the acoustic-structure coupling are presented.

## 3 GOVERNING EQUATIONS

In this approach, the wave propagation theory done by Von-Kármán equations dynamic analog is valid for low frequency range and linear acoustic propagation conditions. For the frequency analysis, all dependent displacements represent small fluctuations around a static reference value and the material behavior is linear and the mechanical properties, like as Young's moduli $E$, density $\rho$ and Poisson's ratio $\nu$, etc, are continues in the elastic domain.We supposed a forced symmetric circular plate model. Therefore, a axisymmetric formulation is developed and depending of the radial position $r$. The transverse displacement is time depending and denoted for $w(r, t)$. Likewise, we have the longitudinal displacement describe by $u(r, t)$. For large displacements, the geometrical nonlinearities are not neglected, taking into account the stretching of the mid-plane of the plate.

### 3.1 The Mechanical Equations

The dynamic equations for the bending circular plate in polar coordinates are written as (Nayfeh and Mook, 1995):

$$\rho h\ddot{w} + D\nabla^4 w = \frac{1}{r}\frac{\partial}{\partial r}\left(F_{,r}w_{,r}\right) - 2\mu\dot{w} + T\left(r,t\right) - T^f\left(r,t\right) \tag{1}$$

where $h$ is the thickness of the plate, $\mu$ is the damping coefficient and $D$ is the flexural rigidity done by:

$$D = \frac{Eh^3}{12\left(1-\nu^2\right)} \tag{2}$$

The terms $T$ and $T^f$ are external pressures on the domain: excitation of system and acoustic fluid coupling, respectively. More details about these terms are presented in next sections. One relation among the nonlinear force $F$ and the displacements is done by:

$$\nabla^4 F = -\frac{Eh}{2r}\frac{\partial}{\partial r}\left(w_{,r}^2\right) \tag{3}$$

For the axisymmetric case, the differential operator $\nabla^4$ is done by:

$$\nabla^4 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2 \tag{4}$$

The longitudinal and transverse displacements and the nonlinear force term $F$ can be related by Eq. (5) and (6).

$$Eh\left(u_{,r} + \frac{1}{2}w_{,r}^2\right) = \frac{1}{r}F_{,r} - \nu F_{,rr} \tag{5}$$

$$Eh\frac{u}{r} = F_{,rr} - \frac{\nu}{r}F_{,r} \tag{6}$$

In this point, one analysis with dimensionless variables is more convenient. The new variables are defined as follows:

$$r = Rr^*, \quad t = R^2\sqrt{\tfrac{\rho h}{D}}t^*, \quad w = \frac{h^2}{R}w^*$$

$$u = \frac{h^4}{R^3}u^*, \quad \mu = \tfrac{24\left(1-\nu^2\right)}{R^4}\sqrt{\rho h^5 D}\mu^*$$

$$T = \frac{Dh^2}{R^5}T^*, \quad F = \tfrac{Eh^5}{R^2}F^*$$

where $R$ is the radius of the circular plate.

By substituting the dimensional relations into Eq. (1), (3), (5) and (6), it results in a dimensionless problem for $r$ with variation [0,1] done by:

$$\ddot{w} + \nabla^4 w = \varepsilon\left[\frac{1}{r}\frac{\partial}{\partial r}\left(F_{,r}w_{,r}\right) - 2\mu\dot{w}\right] + T\left(r,t\right) - T^f\left(r,t\right) \tag{7}$$

$$\nabla^4 F = -\frac{1}{2r}\frac{\partial}{\partial r}\left(w_{,r}^2\right) \tag{8}$$

$$u_{,r} + \frac{1}{2}w_{,r}^2 = \frac{1}{r}F_{,r} - \nu F_{,rr} \tag{9}$$

$$\frac{u}{r} = F_{,rr} - \frac{\nu}{r}F_{,r} \tag{10}$$

where the dimensionless term $\epsilon$ is done by Eq. (11). All dimensionless variables are O(1) as $\epsilon \longrightarrow 1$.

$$\varepsilon = \frac{12\left(1-\nu^2\right)h^2}{R^2} \tag{11}$$

The edges of the plate are considered clamped. Therefore, the boundary conditions for the mechanical system are done by:

$$w(1) = 0, \quad u(1) = 0, \quad w_{,r}(1) = 0 \tag{12}$$

### 3.2 Modal Expansion of the transverse displacement

In order to solve, the transverse displacement is describe by uncoupled form. Therefore, the spatial and temporal problems can be solved. By using, the mathematical properties of the eigenmodes, (Ewins, 1995), an expansion is reached in terms of structural modes, as follows:

$$w\left(r,t\right) = \sum_{m=1}^{\infty} \phi_m\left(r\right)\psi_m\left(t\right) \tag{13}$$

where $\psi_m$ is the generalized temporal variable and $\phi_m$ is the linear mode vector in free-oscillations condition, associated to the natural frequency $\omega_m$. Thus, the modal vector and the natural frequency are obtained by eigenvalue problem solution. In general, the modes $\phi_m$ are normalized to unit, as follows:

$$\int_0^1 r\phi_m\phi_n dr = \delta_{mn} \tag{14}$$

By analytical solution of the circular plate eigenvalue problem, (Meirovitch, 2000), the description of the mode $\phi_m$ is done by:

$$\phi_m = C_m\left[J_0\left(\beta_m r\right)I_0\left(\beta_m\right) - J_0\left(\beta_m\right)I_0\left(\beta_m r\right)\right] \tag{15}$$

where $\beta_m$ is relationship with the natural frequency as $\omega_m = \beta_m^2$. The functions $J_0$ and $I_0$ are the Bessel functions and modified Bessel function of first kind, respectively.

For the circular plates case with the boundary conditions described in Eq. (12), the necessary conditions for the modes functions in $r = 1$ are: $\phi_m\left(1\right) = 0$ and $\phi_m'\left(1\right) = 0$. In addition to exigency that mode function in $r = 0$ is a finite value. It results in a nonlinear roots problem done by Eq. (16).

$$I_0\left(\beta_m\right)J_0'\left(\beta_m\right) - J_0\left(\beta_m\right)I_0'\left(\beta_m\right) = 0 \tag{16}$$

### 3.3 Modal Expansion of the Nonlinear Force Term

The nonlinear force can be expanded in terms of orthogonal modes. In this point, it is more convenient to expand the spatial derivative $G = F_{,r}$ (Nayfeh and Mook, 1995). Thus, the combinations of the Eq. (8) through (10) and Eq. (13) results in the Eq. (17) and (18).

$$G_{,r} - \nu G = 0 \tag{17}$$

$$r^2 G_{,rr} + r G_{,r} - G = -\frac{r}{2} \left( \sum_{m=1}^{\infty} \phi'_m (r) \psi_m (t) \right) \tag{18}$$

Therefore, one possible expansion for $G$ is done by:

$$G (r, t) = \sum_{m=1}^{\infty} J_1 (\zeta_m r) \eta_m (t) \tag{19}$$

where $\zeta_m$ is the argument of the Bessel function $J_1$ and $\eta_m$ is the generalized temporal force function.

The Eq. (17) is a boundary conditions equation for $r = 1$ and substituting the expansion described in Eq. (19), it results in a nonlinear root problem for $\zeta_m$ as follows:

$$\zeta_m J_0 (\zeta_m) - (1 + \nu) J_1 (\zeta_m) = 0 \tag{20}$$

It is noted that problem is material dependent. In general, the employed materials for loudspeaker plates has $\nu = 1/3$.

### 3.4 The Fluid Force Term

For several authors, a problem of non-linear vibrations of a heavy fluid-loaded structure cannot be approached as a standard structural acoustics problem (Sorokin, 2000). In this work, the structural non-linearity is combined with the acoustic radiation effect in a coupled formulation and the structural responses are compared. Actually, several tests to determine the limit of light fluid loading theory in wave propagation analysis of acoustic medium in contact with a structure which presents large displacements can be found in references (Sorokin and Kadyrov, 1999; Sorokin, 2000).

The first hypothesis for the radiation acoustic coupling is the plate edge effects neglecting. It is assumed one fluid pressure force $T^f (r, t)$ acting on a baffled planar piston radiating in free space. The pressure force is done by a convolution among the radiation impedance function $Z_r$ and the plate velocity $\dot{w}$, as follows:

$$Z_r * \dot{w} = \int_0^t Z_r (\tau) \dot{w} (t - \tau) d\tau \tag{21}$$

Following the Rayleigh theory, the radiation impedance of a baffled circular piston of radius $R$ is then given by (Fahy, 2000):

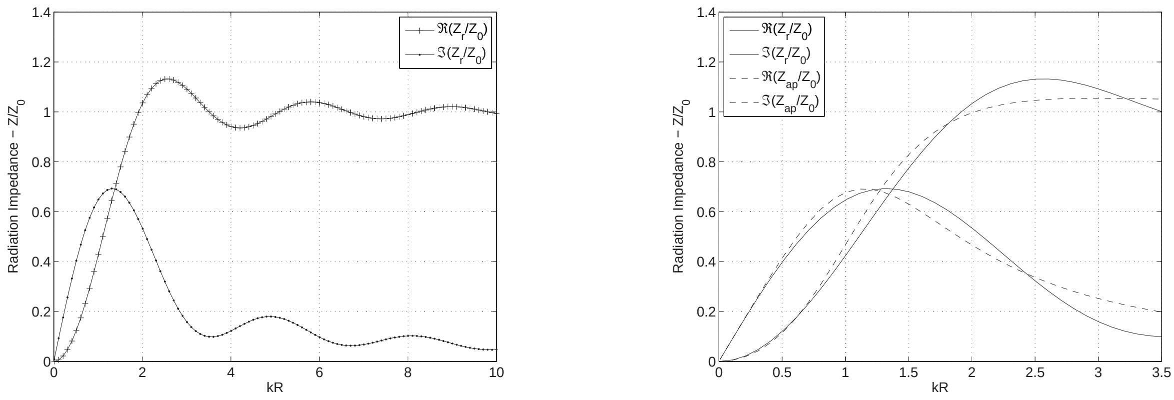
$$Z_r = \rho_0 c_0 [R_1 (2kR) + jX_1 (2kR)] \tag{22}$$

where $\rho_0 c_0$ is the acoustic impedance of the propagating medium and $k$ is the wave number, done by: $k = \omega / c_0$. The terms $R_1$ and $X_1$ are the reactance and inertance of acoustic medium, respectively. These functions are dependent on the frequency and can be expressed as (Kinsler et al., 1995):

$$R_1(2kR) = 1 - 2\frac{J_1(2kR)}{2kR} \quad (23)$$

$$X_1(2kR) = 2\frac{H_1(2kR)}{2kR} \quad (24)$$

where $J_1$ and $H_1$ represent the Bessel and Struve functions of first kind, respectively.



(a) Reactance and Inertance functions

(b) Mean Square Approximation

Figure 2: Radiation Impedance function $Z_r$.

The values of reactance and inertance are normalized to $Z_0 = \rho_0 c_0$. The functions in the frequency domain are presented in Fig. (2a). For temporal analysis, the Eq. (22) must be written in a time formulation. In this work, the radiation impedance function is approximate to an infinity polynomial series (Doutaut et al., 1998). The mathematical description for the temporal transformation is presented in the Eq. (25) and (26):

$$\frac{Z_r}{Z_0} = \frac{\sum_{i=0}^{\infty} \beta_i (j\omega)^i (R/c_0)^i}{\sum_{i=0}^{\infty} \alpha_i (j\omega)^i (R/c_0)^i} \quad (25)$$

where $i$ is the polynomial order, $\beta_i$ and $\alpha_i$ are numerical coefficients obtained by mean square method.

In Fig. (2b), it is presented a mean square approximation for the real and imaginary radiation impedance. In general, a few terms are necessary to reach a good agreement among the curves. In Table 1, it is compared the coefficients values obtained by the Rayleigh second order and Mean Square approximations.

| Coefficients | $\beta_0$ | $\beta_1$ | $\beta_2$ | $\alpha_0$ | $\alpha_1$ | $\alpha_2$ |
|---|---|---|---|---|---|---|
| Rayleigh (second order) | 0.0000 | 0.8488 | 0.4000 | 1.0000 | 1.0186 | 0.4000 |
| Mean Square | 0.0000 | 0.8488 | 0.4887 | 1.0000 | 1.0754 | 0.4887 |

Table 1: Values of $\alpha_i$ and $\beta_i$.

For the convergence of the transformation series by polynomial approximation, is is necessary to satisfy the stability criteria of Kreiss (1968). By Fourier transformation, the Eq. (25) can be rewritten in terms of temporal derivatives, by using the differential operators $A_r$ and $B_r$ as follows:

$$\frac{B_r\left(\frac{R}{c_0}d_t\right)}{A_r\left(\frac{R}{c_0}d_t\right)}=\frac{\sum_{i=0}^{\infty}\beta_i\left(R/c_0\right)^i d^i/dt^i}{\sum_{i=0}^{\infty}\alpha_i\left(R/c_0\right)^i d^i/dt^i} \tag{26}$$

Therefore, the radiation fluid effect introduces differential operators with bigger order in the Mechanical Equations (1).

### 3.5 Summary of the Governing Equations

In order to resume the formulation, one presents the nonlinearly coupled differential equations system in time domain, as follows:

$$\ddot{\psi}_n+\omega_n^2\psi_n=\varepsilon\left[\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\Gamma_{nmpq}\psi_m\psi_p\psi_q-2\mu_n\dot{\psi}_n\right]+T_n\left(t\right)-T_n^f\left(t\right) \tag{27}$$

The Eq. (27) represents a global balance of the mechanical effects. Thus, the quantities and properties are determined by integration over the domain. The Eq. (28) through (30) represent the external forces and the structural damping of the system, respectively.

$$T_n\left(t\right)=\int_0^1 r\phi_n T\left(r,t\right)dr=\int_0^1 r\phi_n\bar{T}(t)\delta\left(r-r_0\right)dr \tag{28}$$

$$T_n^f\left(t\right)=\int_0^1 r\phi_n T^f\left(r,t\right)dr=\int_0^1 r\phi_n Z_r*\dot{w}\left(r,t\right)dr \tag{29}$$

$$\mu_n=\int_0^1 \mu r\phi_n^2 dr \tag{30}$$

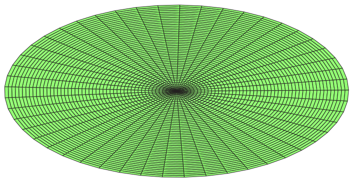
One expression for the nonlinear coefficient $\Gamma_{nmpq}$ is obtained using the orthogonality properties of the modal expansion proposed to the nonlinear force, described in Section (3.3). Finally, the nonlinear coefficient is presented in the Eq. (31) as:

$$\Gamma_{nmpq}=\sum_{k=1}^{\infty}\frac{\int_0^1\phi_p'\phi_q'J_1\left(\zeta_k r\right)dr\int_0^1\phi_n'\phi_m'J_1\left(\zeta_k r\right)dr}{\left(\zeta_k^2-1+\nu^2\right)J_1^2\left(\zeta_k\right)} \tag{31}$$

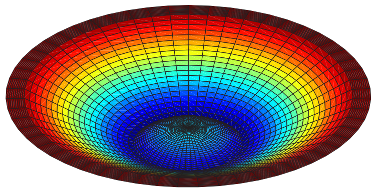
In order to solve the system described in Eq. (27), it was used analytical and numerical methods to determine the modal parameters. In this work, the time solution was obtained by use of mathematical commercial codes, like as Matlab. The analysis of convergence is not presented here. In the next section, several numerical tests are presented.

# 4 NUMERICAL RESULTS

In this section, one presents the numerical results obtained for the dynamical response of coupled plate model with nonlinearity geometrical. In this paper, the numeric tests were performed for two geometrical configurations of the plate. In Fig. (3), it is presented two configurations for the loudspeaker plate.



(a) Flat Plate



(b) Conical Shell

Figure 3: Geometries for the Loudspeaker Plate.

In this analysis, two values for the radius dimension were used in order to observe the influence of the area value on the acoustic radiation damping. The dimensional parameters and mechanical properties of a typical material used to build loudspeakers are listed in Table 2.

| thickness (mm) | Young's moduli (GPa) | Poisson's ratio | density $(kg/m^3)$ |
|---|---|---|---|
| 9 | 1.30 | 0.33 | 1420 |

Table 2: Geometrical and Material Parameters.

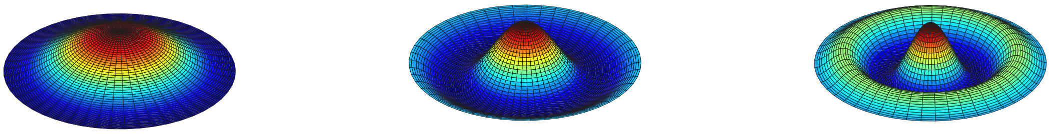
## 4.1 Modal Results

Several parameters described in Governing Equation Section (3) are determined by series solutions and numerical approximations. In this work, a number of five modes were used for the expansion modal. Moreover, the formulation is dimensionless. Thus, the modal results presented are normalized. For the flat plate case, the modal parameters for the three first natural frequencies and nonlinear coefficient are presented in Table 3 and compared with results found in literature.

| **Mode** | $\omega_a$ | $\omega_a$ (Nayfeh and Mook, 1995) | $3\Gamma_{aaaa}$ | $3\Gamma_{aaaa}$ (Nayfeh and Mook, 1995) |
|---|---|---|---|---|
| 1 | 10.2158 | 10.2158 | 1.6222e+002 | 162.22 |
| 2 | 39.7711 | 39.7710 | 5.5521e+003 | 5552.1 |
| 3 | 89.1041 | 89.1040 | 3.4270e+004 | 34401 |

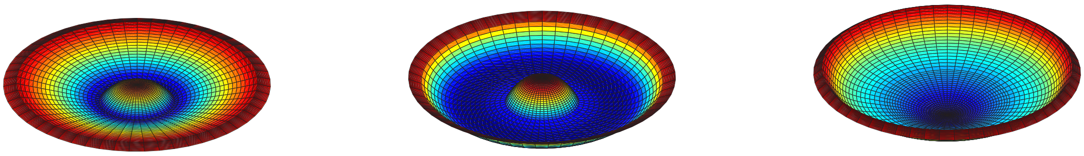
Table 3: Validation of the numerical results obtained by the code implemented.

For the first plate configuration, the structural modes obtained are presented in Fig. (4). For the second plate analysis, the modal parameters were determined by a finite element approach (Zienkiewicz and Taylor, 1991; Reddy, 1984). The first natural frequencies and structural modes are presented in Fig. (5). It can be noted that frequency values are lower than ones obtained in the first analysis.



(a) Mode 1 $\omega_1 = 10.2158$ (b) Mode 2 $\omega_2 = 39.7711$ (c) Mode 3 $\omega_3 = 89.1041$

Figure 4: Axisymmetric structural modes of the flat plate configuration.



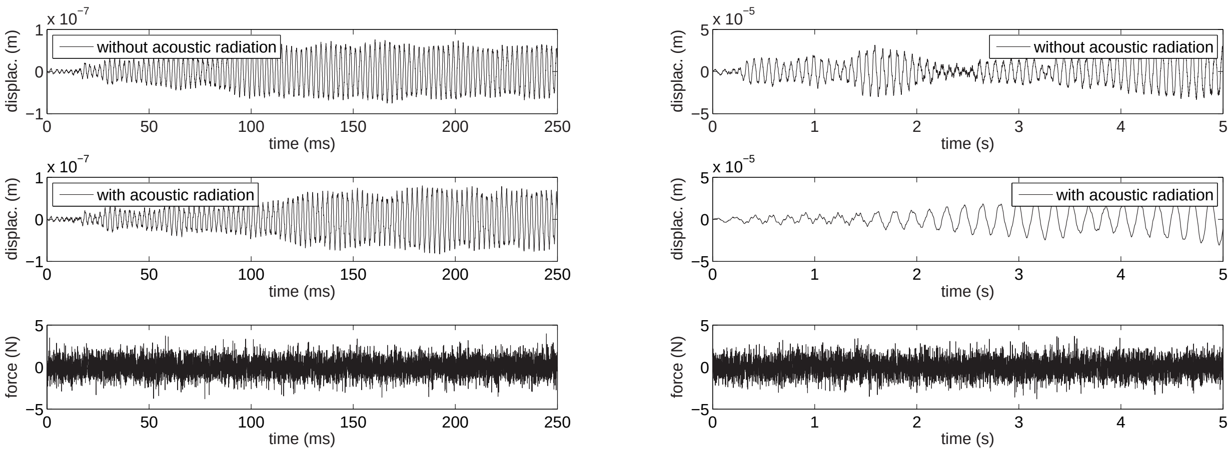
(a) Mode 1 $\omega_1 = 4.1994$ (b) Mode 2 $\omega_2 = 8.9767$ (c) Mode 3 $\omega_3 = 11.3405$

Figure 5: Axisymmetric structural modes of the conical shell configuration.

In general, the first frequency of loudspeaker structure must be as low as possible, otherwise, the others natural frequencies must be as high as possible, in order to extend the bandwidth where the transducer works as a rigid piston (Quaegebeur and Chaigne, 2008).

## 4.2 Time Results

In this section, the numerical results on the time domain for the two plates configurations are presented. The accuracy of the implementation approach was investigated and validated by frequency results convergence.



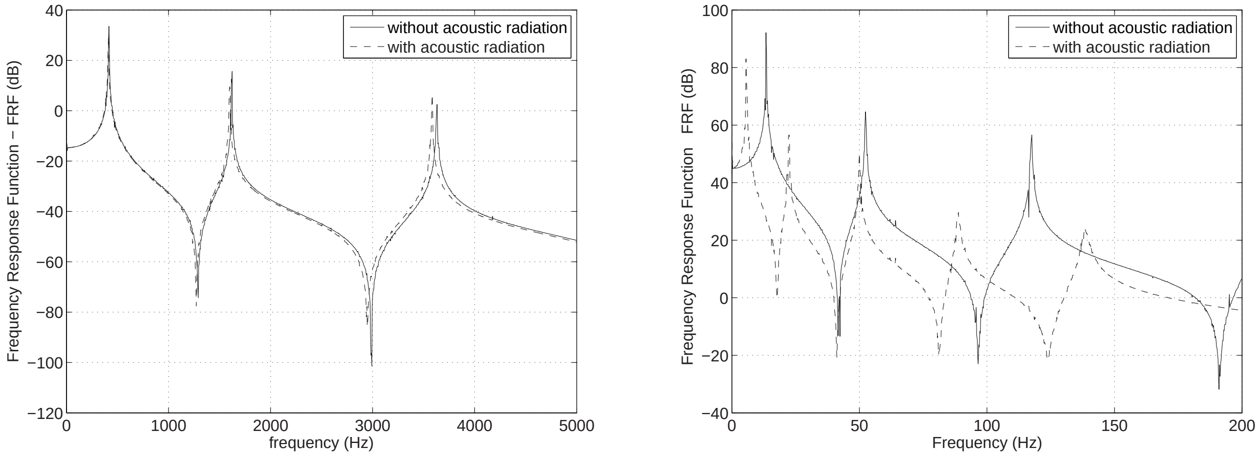

(a) $R = 101.5mm$ (b) $R = 564.2mm$

Figure 6: Analysis of the flat plate with random excitation.

The results are resumed as linear and nonlinear approach. The flat plate was tested for low displacements conditions and for the conical shell, the tests were based in a nonlinear formulation with geometric nonlinearities.

For the flat plate, two excitations forces types were used: random and harmonic natures. In this study case, the influence of the heavy fluid radiation was investigated. Moreover, the dynamical performance of the system was analyzed by spectrum techniques and Frequency Response Functions (FRF). The chosen point for excitation and displacement response was the middle of the domain.

The time results of the flat plate with random excitation can be found in Fig. (6a) and (6b). For the random excitation, the results in frequency domain were obtained and compared for the two radius values. The frequency responses are presented in Fig. (7a) and (7b). It can be noted that in both cases, the natural frequencies changed due to the acoustic radiation damping.




(a) $R = 101.5mm$ (b) $R = 564.2mm$

Figure 7: Frequency Response Functions of the flat plate.

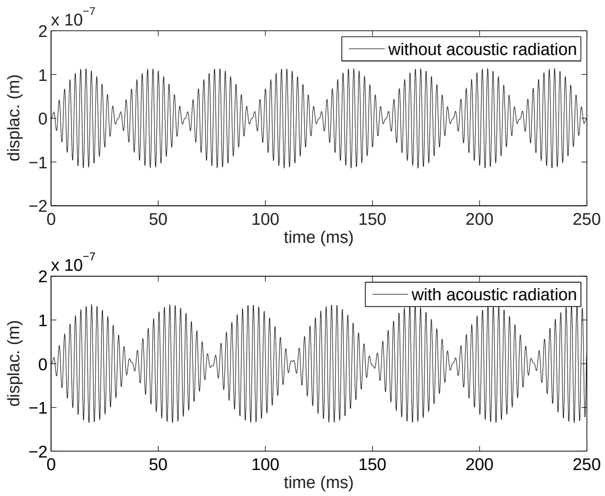
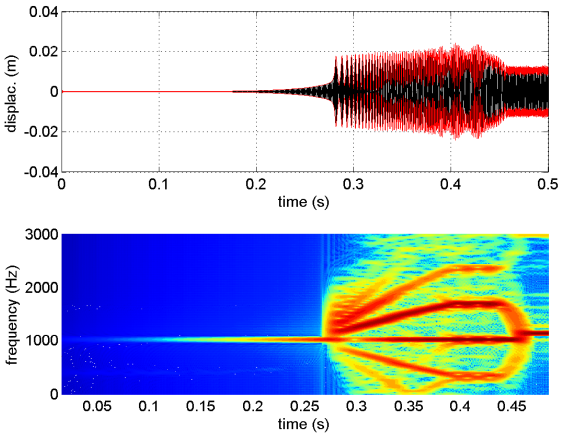



Figure 8: Analysis of the flat plate with periodic excitation.

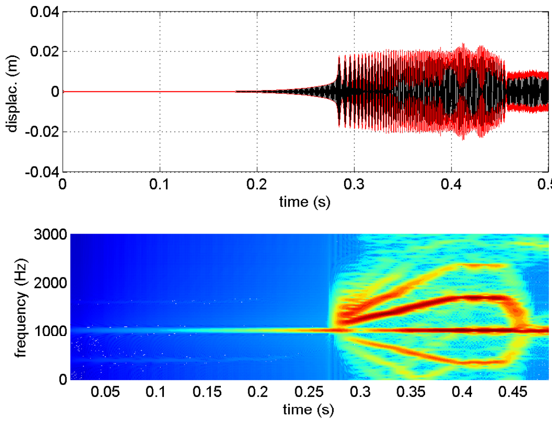
In order to clarify the acoustic radiation effect, the next example aims to provide the temporal response of the system with periodic excitation, as it is described in Fig. (8). The excitation frequency is closer to the first natural frequency. For this reason, the superposition of waves provide a beat vibration. The acoustic radiation modified the beat frequency value.

The proposed nonlinear model was adjusted for the conical shell case and it was admitted only cubic nonlinearities. In fact, the nonlinear effect for conical shell is smaller than flat plate case. The application of this method consists in a first approximation to determine the coupled response of an acoustic-structure system with nonlinearities. This way, the influence of heavy acoustic radiation on the time and spectral results was investigated.

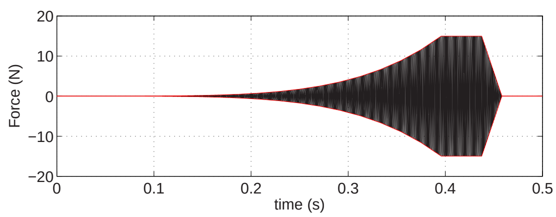
First of all, the conical shell system was excited by the periodic force with increasing amplitude presented in Fig. (9c). The external radius of the conical shell was 101.5 mm. The displacements and spectral results are described in Fig. (9a). Three time ranges can be observed: the first one presents a weak excitation where linear and periodic results are observed, a intermediary range with almost periodic response and the third one where the chaotic phenomenons are evidenced.




(a) Response to excitation with increasing amplitude.




(b) Response to excitation with increasing amplitude and acoustic radiation included.




(c) Excitation with increasing amplitude.

Figure 9: Uncoupled and Coupled Solutions on the Conical Shell Spectrogram.

The fluid coupling effect was introduced in the differential equations. In Fig. (9b), it can be noted that the time solutions of the conical shell excited by periodic force were not modified considerably. The displacement values were slightly affected. Therefore, the damping introduced by heavy fluid radiation did not modify the time response to proposed excitation force. For the spectral diagram obtained, the bifurcations remained the same.

## 5 CONCLUSIONS

In this work, several numerical tests have been performance. Two geometrical configurations for the plate found in typical loudspeakers has been tested. Temporal solutions based in the analog dynamical equations of Von-Kármán for plate bending with large displacements were obtained by using superposition of the linear structural modes. Coupling the system, acoustic radiation effects were introduced. In order to amplify the radiation effects, it was chosen a heavy fluid with large values of impedance. For the flat plate case, the tests provide the influence of the heavy fluid damping effects in the time solutions and the natural frequencies values changing. As expected, increasing the fluid-structure interface, the acoustic radiation effect was amplified and the dynamical responses were modified considerably. It was created a first methodology to detect the heavy fluid radiation effects on the nonlinear vibrations of plates type loudspeaker. Finite elements techniques were used to determine the structural modes of conical shells and adapted for the coupled formulation. Finally, by time results analysis, the heavy fluid radiation effects did not change so evident the nonlinearities of the conical plate proposed.

## 6 ACKNOWLEDGEMENTS

The authors would like to thank CNPq (Conselho Nacional de Pesquisa - Brasil) and DGA (Direction générale de l'armement - France) for financial support for this work .

## REFERENCES

Doutaut V., Matignon D., and Chaigne A. Numerical simulations of xylophones. ii. time-domain modeling of the resonator and of the radiated sound pressure. *Journal of Acoustical Society of America*, 104(3):1633–1647, 1998.

Ewins D.J. *Modal Testing: Theory and Practice*. Reserches Studies Press Ltda, 1995.

Fahy F.J. *Foundations of Engineering Acoustics*. Academic Pres, 2000.

Ginsberg J.H. Multi-dimensional nonlinear acoustic wave propagation, part 2: the non-linear interaction of an acoustic field and plate under harmonic excitation. *Journal of Sound and Vibration*, 40:359–379, 1975.

Kinsler L.E., Frey A.R., Coppens A.B., and Sanders J.V. *Fundamental of Acoustics*. John Wiley and Sons, 1995.

Kreiss H. Stability theory of difference approximations for mixed initial boundary values problems. *Mathematics Computations*, 22:703–714, 1968.

Meirovitch L. *Fundamentals of Vibrations*. McGraw Hill, 2000.

Nayfeh A.H. and Kelly S.G. Non-linear interactions of acoustic fields with plate under harmonic excitations. *Journal of Sound and Vibration*, 60:371–378, 1978.

Nayfeh A.H. and Mook D.T. *Nonlinear Oscillations*. Wiley Classics Library, New York, 1995.

Quaegebeur N. and Chaigne A. Nonlinear vibrations of loudspeaker-like structures. *Journal of Sound and Vibration*, 309:178–196, 2008.

Quaegebuer N. *Vibrations non linéaires et rayonnement acoustique des structures minces de type haut-parleur*. Ph.D. thesis, ENSTA et École doctorale de l'École Polytechnique, 2007.

Reddy J.N. *An Introduction to the Finite Element Method*. MacGraw-Hill, New York, 1984.

Sorokin S.V. Non-linear oscillations of a baffled elastic plate in heavy fluid loading conditions. *Journal of Sound and Vibration*, 232(3):619–643, 2000.

Sorokin S.V. and Kadyrov S.G. Modeling of non-linear oscillations of elastic structures in heavy fluid loading conditions. *Journal of Sound and Vibration*, 222(3):425–451, 1999.

Zienkiewicz O. and Taylor R. *The finite element method*, volume I. McGraw Hill, 1991.