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MECHANICAL BEHAVIOUR OF WOOD BEAMS WITH GRAIN ORIENTATION

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Abstract. Wood is a material with a mechanical behavior that is dependent on the direction of its fibers. Due to its internal structure, wood is generally treated as linear orthotropic, with the material directions (longitudinal, radial and tangential) ideally coincident with the longitudinal and transverse directions of a coordinate system associated with a structural member. Such coincidence does not occur in the field of lumber production. In wood beams, the fiber orientations are typically not aligned with the longitudinal direction from 5° to 15°. This affects the wood mechanical properties and requires a transformation of coordinates of the elastic coefficients to adjust them to the adopted model for structural analysis. Thus, additional terms appear in the constitutive relationship transforming wood as an anisotropic material. By introducing these coefficients on the elastic model, a study of the stresses in wood beams, constituted by a Brazilian species named Goupia glabra, was developed by applying an analytical method and a program of finite elements. Numerical examples confirm that the fiber orientation, even for small angles, has a large influence on the mechanical behavior of wood beams.

1 INTRODUCTION

Among construction materials, wood, due to its internal structure with axes of elastic symmetry longitudinal, tangential and radial, presents an orthotropic pattern. In terms of engineering elastic models, wood is ideally treated as linear orthotropic, where the principal material axes (longitudinal, radial and tangential) are coincident with the longitudinal and transverse directions of a Cartesian coordinate system associated with a structure, called body coordinates.

Focusing particularly on wood beams, the fiber orientations are not aligned with the longitudinal direction in and consequently, the coincidence between material and body axes no longer exists.

In the field of the wood construction (or the lumber production) it is usual wood beams and also the wood laminated beams, especially of hardwood species, to contain the angle of the fiber orientations laminas in the range of 5º to 15º .

Bodig and Jayne (1981) shown an example in which a lumber of Sugar Maple with 7º of fiber inclination has a reduction of 19.25% of the elastic modulus. Hermanson et al. (1997) comments that it is typical in the USA sawn lumber with fibers misaligned of 0º to 15º. Yet Wood Handbooks presents some information on variation of strength for different grain slopes showing ,for example, that for an angle of 11.3º the modulus of rupture in bending decreases around 55 % comparing to a straight-grained piece. In same direction, the Brazilian code NBR 7190 allows the use of wood pieces having fibers with the maximum 6º of grain inclination to avoid this reduction in strength. Since the strength and elastic properties have a strong relationship, it can be expected that the elastic modulus will have a significant reduction as well.

Thus, in body coordinates, even for an orthotropic material, the structure appears to be anisotropic. Obviously, wood beams maintain their orthotropic features since the principal elastic axes are distinguished. This is not verified in an anisotropic material.

In this context, a deep study of elastic coefficients applied to wood also implies in the knowledge of elastic behavior of anisotropic. If the structure of an anisotropic body expresses some type of symmetry, its properties of elasticity also indicate such. Thus, the elastic symmetry expresses the fact that in each point of the solid exists equivalent symmetrical directions with respect to the elastic properties. If the symmetry of the elastic properties of an anisotropic body exists, the equations of Hooke´s Law can be simplified, also occurring many simplifications in the constitutive matrix.

In general, an anisotropic material does not exhibit elastic symmetry and it is necessary to take into account the difference in mechanical properties for different directions. Consequently, the study of these solids becomes more complex than other solids that possess other types of elastic symmetry, as for example, isotropic solids. Some research studies that evidence this complexity can be cited, as for example, the work of Carrier and Ithaca (1943), of Green and Zerna (1954), of Nair and Reissner (1976), Bodig and Jayne (1982), Kilic et al. (2001) and of Mascia and Vanalli (2002).

In fact, the study of the anisotropy requires in the knowledge of the constituent law that consequently governs the elastic behavior of the material and in determining the components of the constituent tensor (or matrix), S_{ij}. Thus, an orthotropic elastic model needs the determination of the following parameters: the elastic moduli, the shear moduli and the Poisson´s ratios, whereas to an anisotropic elastic model other parameters among them, the coefficients of mutual influence are required.

As a matter of fact, when the influence of the fiber orientation on the constitutive relations and on the stress and strain fields of a wood beam is analyzed, obviously, other deformations appear besides of ones presented in the orthotropic beams. These deformations are associated to the coefficients of mutual influence that quantify the participation of normal stresses in shear strains or shear stresses in normal strains. So, the variation of grain angle constitutes the fundamental cause of wood anisotropy. It is responsible for the greatest changes in the values of the constitutive tensor components, i.e., in the wood´s elastic coefficients.

In this context, the aim of this paper is to analyze the effect of the orientation of the fibers on the mechanical behavior of wood beams the fiber inclined taking into account the theory of elasticity applied to anisotropic body. For practical reasons, it is proposed the range of this orientation from 5° to 15°. It was used the method established by Hashin (1967) for analysis of stresses in plane beams under the uniform distributed loads analyze in cantilever beam. The obtained solutions are compared with solutions for isotropic beams considering or not the shear effect, and also with numerical solutions for orthotropic beams derived of a commercial program of finite elements. On the whole, these analyses are addressed to evaluate the effect of the wood´s fiber orientation on the mechanical behavior of wood beams.

2 DESCRIPTION OF PROBLEM

The most general elastic constitutive model formulated to describe the mechanical behavior of material is the anisotropic model. This kind of model implies that there is no material symmetry, and mechanical properties in certain directions are different. On the other hand, if there is material symmetry, the material can be denominated, for example, orthotropic or isotropic. In this context, the adequacy of a determined material for a certain elastic model is based on the existence of elastic symmetry axes. In these axes, denominated elastic principal axes, there is invariance of the constitutive relations under a group of transformations of coordinate axes.

In fact, the study of anisotropy implies knowing the constitutive law that governs the elastic behavior of the material and consequently, determining the constitutive tensor and its components. In a completely elastic and anisotropic model this tensor has 21 unknown constants.

This way, the constitutive relationship can be written in the following indicial form:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{1}$$

in which i,j,k,j are: 1,2 and 3, and $\sigma_{ij}, \varepsilon_{kl}$ are components of second-order tensors , representing stresses and strains, and C_{ijkl} are components of a fourth-order tensor, known as a stiffness tensor.

The Eq. (1) can be expressed as:

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl} \tag{2}$$

where S_{ijkl} is the compliance tensor.

The tensor C_{ijkl} has 81 constants to be determined and must be symmetrical due to Cauchy's second law of motion. In addition, since both σ_{ij} and ε_{kl} are symmetrical, the number of elastic constants is reduced to 21, with 18 independent ones (Lekhnitskii et al., 1968). This implies that in an anisotropic material with the principal stress directions do not coincide with the principal strain directions. The constitutive laws may also be written in matrix form, a 6x6 symmetric matrix as follows:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1112} & C_{1123} & C_{1131} \\ & C_{2222} & C_{2233} & C_{2212} & C_{2223} & C_{2231} \\ & & C_{3333} & C_{3312} & C_{3323} & C_{3331} \\ & & & C_{1212} & C_{1223} & C_{1231} \\ & & & & C_{2323} & C_{2331} \\ Sym & & & & & C_{3131} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix} \tag{3}$$

Similarly for S_{ijkl}:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{Bmatrix} = \begin{bmatrix} S_{1111} & S_{1122} & S_{1133} & 2S_{1112} & 2S_{1123} & 2S_{1131} \\ & S_{2222} & S_{2233} & 2S_{2212} & 2S_{2223} & 2S_{2231} \\ & & S_{3333} & 2S_{3312} & 2S_{3323} & 2S_{3331} \\ & & & 4S_{1212} & 4S_{1223} & 4S_{1231} \\ & & & & 4S_{2323} & 4S_{2331} \\ Sym & & & & & 4S_{3131} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} \tag{4}$$

where: ε_{ij} are normal strain components for i=j and shear strain components for i ≠j , σ_{ij} is normal stress component for i=j, is shear stress component for i≠j .

In contracted notation for stresses, strains and, consequently, for the constitutive tensor, Eq. (4) can be given by (Ting, 1996):

$$\begin{Bmatrix} \varepsilon_{1} \\ \varepsilon_{2} \\ \varepsilon_{3} \\ \varepsilon_{4} \\ \varepsilon_{5} \\ \varepsilon_{6} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ & & S_{33} & S_{34} & S_{35} & S_{36} \\ & & & S_{44} & S_{45} & S_{46} \\ & & & & S_{55} & S_{56} \\ Sym & & & & & S_{66} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_{1} \\ \sigma_{2} \\ \sigma_{3} \\ \sigma_{4} \\ \sigma_{5} \\ \sigma_{6} \end{Bmatrix} \tag{5}$$

Considering, now, a material with elastic symmetry axes, in which the elastic properties remain constant. If the elastic properties of the material are the same in certain directions, then the material exhibits symmetry with respect to these directions.

Thus when a body presents certain kinds of symmetry, the constitutive relations are simplified. These simplifications can be done in different ways just as those used by Lekhnitskii (1981), that used two different coordinate systems, symmetrical one to other and compared the obtained constitutive relations and identified the existence of the elastic symmetry.

A material with elastic symmetry under the linear transformation $\bar{x}_i = t_{ij} x_j$, with t_{ij} being the coordinate transformation tensor, requires that the constitutive tensor, either C_{rspq} or S_{rspq}, comply with the following condition:

$$\bar{S}_{rspq} = t_{ri} t_{sj} t_{pk} t_{ql} S_{ijkl} \qquad (6)$$

In this context, there are four cases of elastic symmetry, which are considered most important. They are: one plane of elastic symmetry, three planes of elastic symmetry (orthotropic material), transversely isotropy material and isotropic material. Since the purpose of this paper is to adopt wood as an orthotropic body, only this kind of elastic symmetry is analyzed.

Thus, a body referred to a coordinate system xi is defined as orthotropic material if through each point there are three mutually perpendicular axes of elastic symmetry. Then, using the coordinate system x_1, x_2 and x_3 (or x, y, and z), perpendicular to the three planes of material symmetry and considering the elastic properties to be invariant under counterclockwise rotation 180° of about three axes, and using one at time as showed in Figure 1, it is possible to determine the constitutive tensor for orthotropic materials.

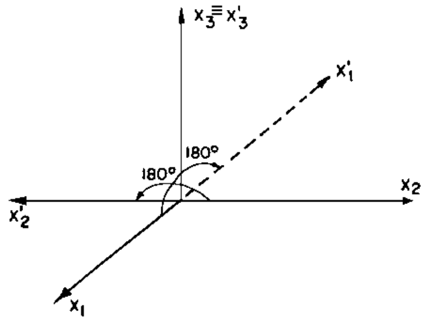

Figure 1: 180°- Rotation about x_3.

Consequently, one obtains that:

$$t_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad (6)$$

And find, either C_{rspq} or S_{rspq} can be written in matrix form by:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{22} & S_{23} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{55} & 0 \\ Sym & & & & & S_{66} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} \quad (7)$$

As commented, wood containing an internal structure with three axes of elastic symmetry has an orthotropic pattern. Thus, there are 9 constants to be determined. The three mutually axes of elastic symmetry are longitudinal, tangential and radial, denoted by L, T and R.

Dealing with wood beams, considered a plane structure, with inclined fibers, as illustrated in Figure 1, the coincidence or not between the geometric axes of the beam, i.e.: x,y, and the principal axes of elasticity of the beam,i.e.:1,2, may become more or less complex the analyses of both stresses and strains.

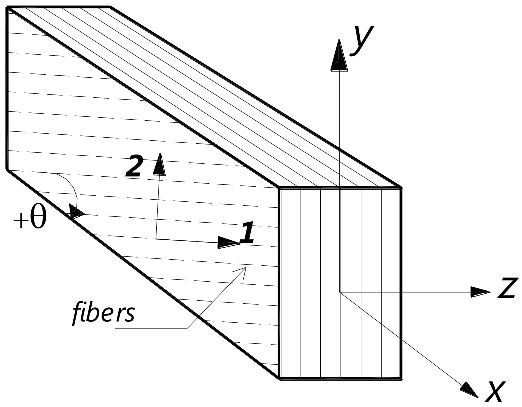

Figure 2: Wood Laminated Cantilever Beam

Thus, this variation of grain angle constitutes the fundamental cause of wood anisotropy. It is responsible for the greatest changes in the values of the constitutive tensor components, i.e., in these wood elastic constants. Taking into account the plane case and the coincidence of the axes, the constitutive matrix is simplified, becoming orthotropic with four independent terms and different of zero. So, it has that:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} \quad (8)$$

and the stiffness coefficients S_{ij}, considering the constitutive matrix symmetry, are $S_{11} = \frac{1}{E_1}$, $S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2} = S_{21}$, $S_{22} = \frac{1}{E_2}$ and $S_{66} = \frac{1}{G_{12}}$ with E_i are called Young´s modulus, G_{ij} shear modulus and ν_{ij} Poisson´ ratio.

However, the simplifications presented in the orthotropic matrix disappear since the coincidence between the geometric axes of the beam and the principal axes of the lamina were not considered, appearing then other elements in the constitutive tensor, which becomes anisotropic. Consequently, the constitutive relations for a plane case assume the form:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{61} & \bar{S}_{62} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{9}$$

Hence, it is evident that beams constituted by orthotropic materials can become anisotropic since the zero elements $S_{ij} : S_{16}, S_{26}$ in original matrix may no longer be zero in the new matrix $\bar{S}_{ij}$. Focusing on Eq.(9), the shear stress will produce now normal strains and on the other hand, normal stress will produce shear strain. These effects were not present in orthotropic material stressed along its principal elastic directions. This way, it is obvious that there is no difference between the solutions of problems for orthotropic beams and anisotropic ones since the material direction of the orthotropic laminas are not aligned with the body directions. The only advantage associated with the orthotropic material in relation to anisotropic one is that it is easier to characterize experimentally. However, if the principal material axes are not located, then the orthotropic material is indistinguishable from an anisotropic one, as comment by both Jones (1975) and Lekhnitskii (1981).

By returning to Eq.(9), it is noted that:

$$\left[\bar{S}\right] = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & \frac{\eta_{xy,x}}{E_x} \\ -\frac{\nu_{xy}}{E_y} & \frac{1}{E_y} & \frac{\eta_{xy,y}}{E_y} \\ \frac{\eta_{x,xy}}{G_{xy}} & \frac{\eta_{y,xy}}{G_{xy}} & \frac{1}{G_{xy}} \end{bmatrix} \tag{10}$$

where $\eta_{i,ij}$ and $\eta_{ij,j}$ are defined by Jones(1975) as coefficients of mutual influence of first and second kind, or still as coupling coefficients by Tsai and Hahn (1980).

So, the coefficients of mutual influence of the first kind $\eta_{i,ij}$ characterize stretching in i-direction caused by shear in ij-plane that is $\eta_{i,ij} = \frac{\varepsilon_i}{\gamma_{ij}}$ for $\sigma_i = \sigma$ and all other stresses are zero.

The coefficients of mutual influence of the second kind $\eta_{ij,i}$ characterize shearing in ij-plane caused by a normal stress in the i-direction that is $\eta_{ij,i} = \frac{\gamma_{ij}}{\varepsilon_i}$ for $\tau_{ij}=\tau$ and all other stresses are zero.

From a general point of view, the angles that relate the grain orientation and the geometric axes of the analyzed structure establish the anisotropic pattern of wood beams. Thus, considering the angle θ from Figure 1, the constitutive matrix is transformed by using the matrix of transformation [T] and its transpose [T]T according to:

$$\begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{61} & \bar{S}_{62} & \bar{S}_{66} \end{bmatrix} = [T]^T . \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} . [T]^T \tag{11}$$

The matrix of transformation, presented in Eq. (11), is given by:

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\cdot\sin\theta\cdot\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\cdot\sin\theta\cdot\cos\theta \\ -\sin\theta\cdot\cos\theta & \sin\theta\cdot\cos\theta & \cos^2\theta-\sin^2\theta \end{bmatrix} \tag{12}$$

By focusing Eq.(5), $\bar{S}_{16}$ and $\bar{S}_{26}$ for orthotropic material stressed in non-principal material directions depend on four independents coefficients, E_x , E_y , G_{xy} and ν_{xy}. It is obvious that for anisotropic material this is not happened since by definition, it has no principal material directions for this kind of material. Thus, with the use of Eq.(11) or (12) serves to find all the constants that are necessary for the development of the anisotropic constitutive relations, and they are written by:

$$1/E_x = \frac{1}{E_1}\cos^4\theta + (\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1})\sin^2\theta\cos^2\theta + \frac{1}{E_2}\sin^4\theta \tag{13}$$

$$1/E_y = \frac{1}{E_2}\cos^4\theta + (\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1})\sin^2\theta\cos^2\theta + \frac{1}{E_1}\sin^4\theta \tag{14}$$

$$1/G_{xy} = 2(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}})\sin^2\theta\cos^2\theta + \frac{1}{G_{12}}(\sin^4\theta + \cos^4\theta) \tag{15}$$

$$\nu_{xy} = E_x\left[\left(\frac{\nu_{12}}{E_1}\right)(\sin^4\theta + \cos^4\theta) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}}\right)\sin^2\theta\cos^2\theta\right] \tag{16}$$

$$\eta_{xy,x} = E_x\left[\left(\frac{2}{E_1} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)\sin\theta\cos^3\theta - \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)\sin^3\theta\cos\theta\right] \tag{17}$$

$$\eta_{xy,y} = E_y\left[\left(\frac{2}{E_1} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)\sin\theta\cos^3\theta - \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)\sin^3\theta\cos\theta\right] \tag{18}$$

Although the coupling coefficients are acknowledged in studies of the anisotropic models, very little information is available in relation to wood structures. For example, Mascia (1991) analyzed the coefficients of mutual influence of the first and the second kind for the Brazilian wood species: Angico (*Parapiptadenia)*, Ipê (*Tabebuia sp*) Pinus Caribaea (*Bahamensis*) and Guapuruvú (*Schizolobium parahyba*), by beams with inclined fibers around X degrees.

Finally, one does observe that Eqs. (13) to Eq. (18) provides all the elastic coefficients for the considered model (anisotropic model) but it is fundamental for researches an evaluation of the coefficients of mutual influence to take into account their level of magnitude and their influence on the mechanical behaviour of wood beams.

As a theoretical application, the stresses in wood laminated beams, subjected to uniformly distributed load, are analyzed by focusing on an analytical method developed by Hashin (1967). Furthermore, a numerical procedure from Ansys[1] software based on element finite method to compare with the analytical results is used.

3 STRESSES IN WOOD LAMINATED CANTILEVER BEAMS – HASHIN´S ANALYTICAL METHOD

In this section, the distributions of normal and shear stresses in cantilever beams are determined by means of an analytical method, based on the stress function of Airy (ϕ), developed by Hashin (1967) for the analysis of plane anisotropic beams. As an application, it was considered a beam subjected to the uniformly distributed loads, as Figure 3:

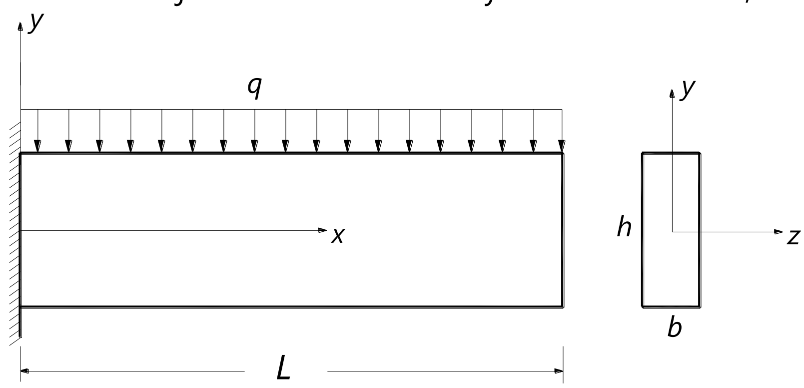

Figure 3: Wood Cantilever Beam

In summary, these methods can be formulated by using: the constitutive relations, the equilibrium equations, the compatibility equation and, naturally, the Airy stress function (ϕ) defined by:

$$\sigma_{11} = \sigma_1 = \phi,_{22} \tag{19}$$

$$\sigma_{22} = \sigma_2 = \phi,_{11} \tag{20}$$

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[1] ANSYS Manual and Software Version 5.5, Swanson Analysis Systems, Inc. Canonsburg.

$$\sigma_{12} = \tau_{12} = -\phi_{,12} \tag{21}$$

On the basis of these relations and equations, we obtain the following differential equation:

$$S_{22}\phi_{,1111} - 2S_{26}\phi_{,1112} + (2S_{12} + S_{66})\phi_{,1122} - 2S_{16}\phi_{,1222} + S_{11}\phi_{,2222} = 0 \tag{22}$$

which models the anisotropic plane problem.

Now considering the stress function of Airy (ϕ), Hashin (1967) presented an analytical method that allows the construction of the following stress function polynomial:

$$\phi(x, y) = \sum_{m=0}^{m=M} \sum_{n=0}^{n=N} C_{mn} x^m y^n \tag{23}$$

where C_{mn} are constants to be determined by the resolution of a linear system of equations.

Introducing Eq.(23) in Eq. (22) and equating the coefficients $x^m y^n$ of equal powers, Hashin (1967) developed relations, among the C_{mn} coefficients and obtained the normal and shear stresses:

$$\sigma_x = \frac{\partial^2 \phi(x, y)}{\partial y^2} = \frac{q\left[\left(3E_x y + 4G_{xy}\begin{pmatrix} -12y \cdot \eta_{xy,x}^2 + 5l \cdot \eta_{xy,x} - 10x \cdot \eta_{xy,x} - \\ -3 \cdot \nu_{xy} \cdot \eta_{xy,x} \cdot y \end{pmatrix}\right)h^2\right]}{40h^3 G_{xy}} + \\ + \frac{q\left[5y\left(2G_{xy}\begin{pmatrix} 3l^2 - 6xl - 6 \cdot \eta_{xy,x} \cdot yl + 3x^2 + 8 \cdot \eta_{xy,x}^2 \cdot y^2 + \\ + 2 \cdot \nu_{xy} \cdot y^2 + 12 \cdot \eta_{xy,x} \cdot x \cdot y \end{pmatrix} - E_x y^2\right)\right]}{40h^3 G_{xy}} \tag{24}$$

$$\tau_{xy} = \frac{\partial^2 \phi(x, y)}{\partial y \partial x} = \frac{q(3l - 3x - 4 \cdot \eta_{xy,x} \cdot y)(y^2 - h^2)}{4h^3} \tag{25}$$

4 RESULTS

In this section, the normal and shear stresses of wood cantilever beams (Brazilian species named Goupia glabra), showed in Figure 3, are analyzed, considering: a-anisotropic case using the Theory of Hashin (HAS); b- orthotropic case using a numerical method based on a finite element method (Software Ansys-COM SOFT); c- isotropic case focusing the influence of the shear (Theory of Timoshenko - ISO1) and d- isotropic case without the influence of the shear (Theory of Euler-Bernoulli - ISO2).

The dates for analysis are showed in Tables 1 and 2:

Constants	E_1	E_2	G_{12}	ν_{12}
Values	10180 MPa	992 MPa	650 MPa	0,438

Table 1: Elastic Constant Values (Brazilian species named *Goupia glabra*).

Ratio (h/l)	Beam Span (m)	Section Height (m)
1/10	3,00	0,30
1/5	3,00	0,60

Table 2: Height of the cross section – span of beam (*h / l*) relations.

For the plated beam, considering also b = 1 m; q = 10 kN/m, whose relation h / L = 1/10, it has the following diagrams:

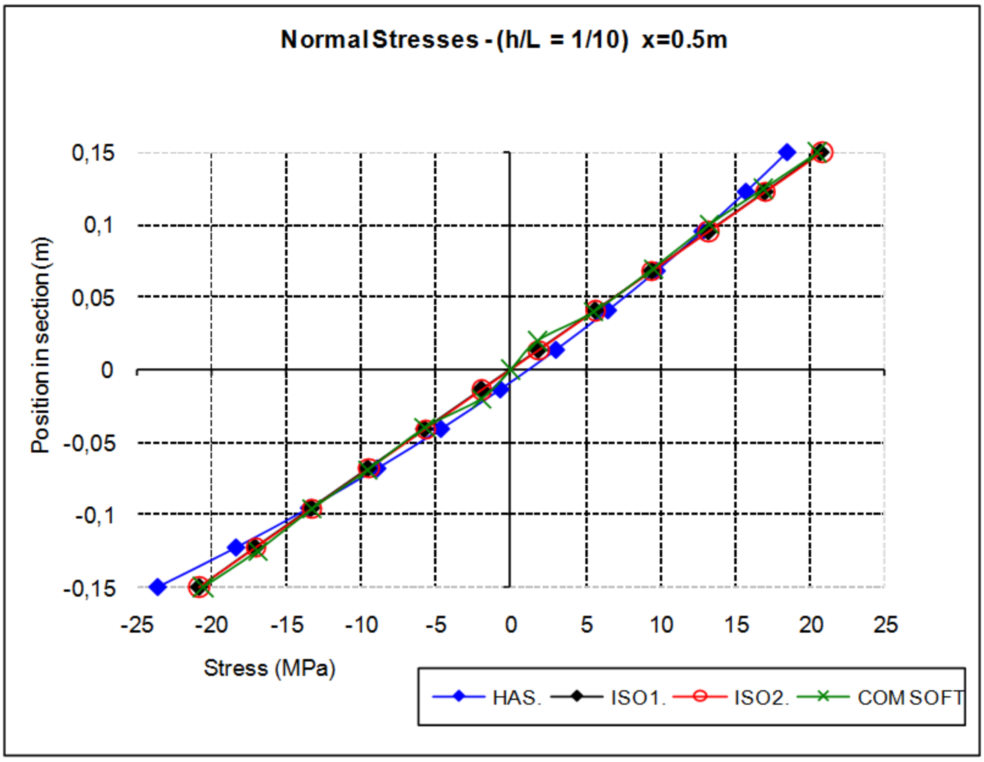

Figure 4: Normal Stresses – h/L=1/10, x=0.5m

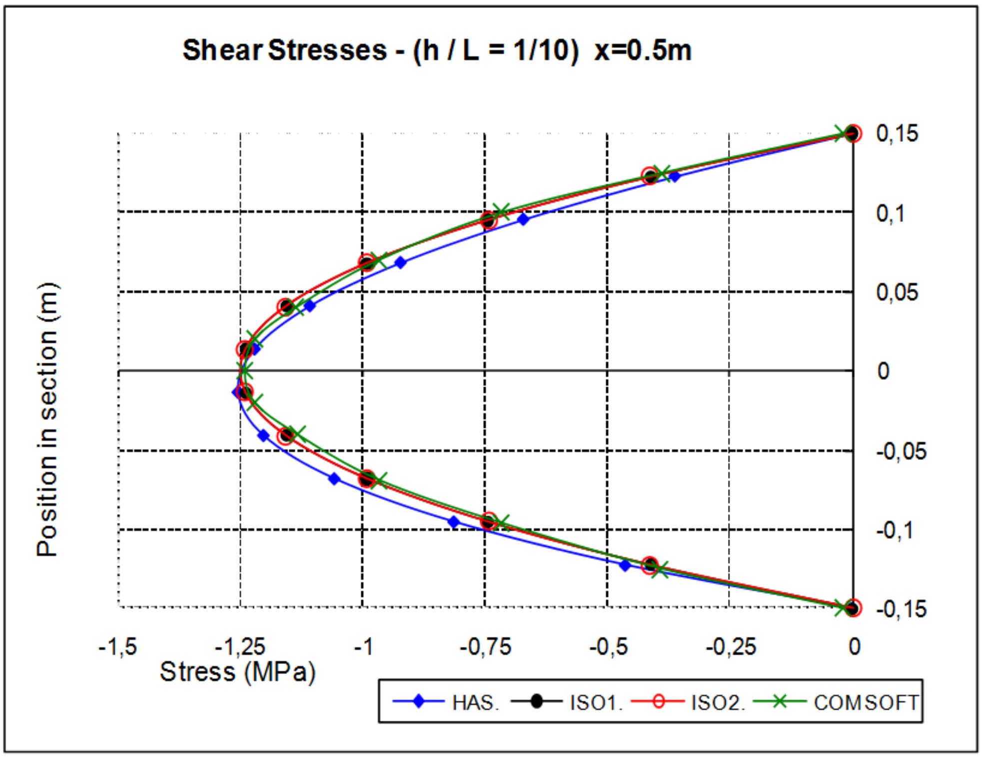

Figure 5: Shear Stresses – h/L=1/10, x=0.5m

For the plated beam, considering also b = 1 m; q = 10 kN/m, whose relation h / l = 1/5, it has the following diagrams:

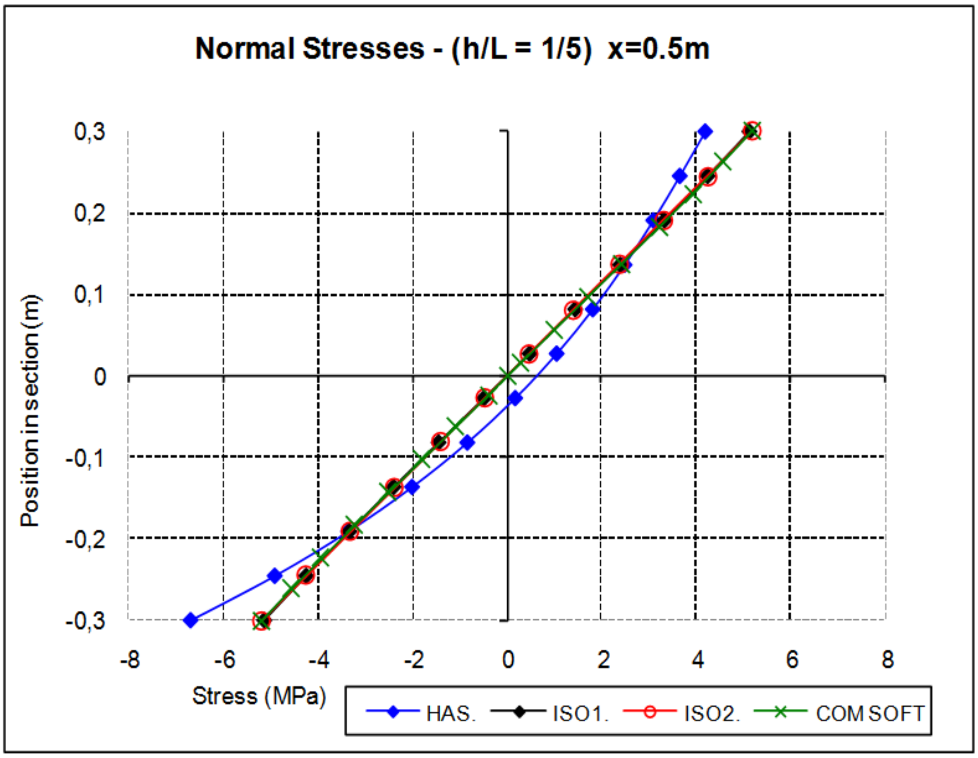

Figure 6: Normal Stresses – h/L=1/5, x=0.5m

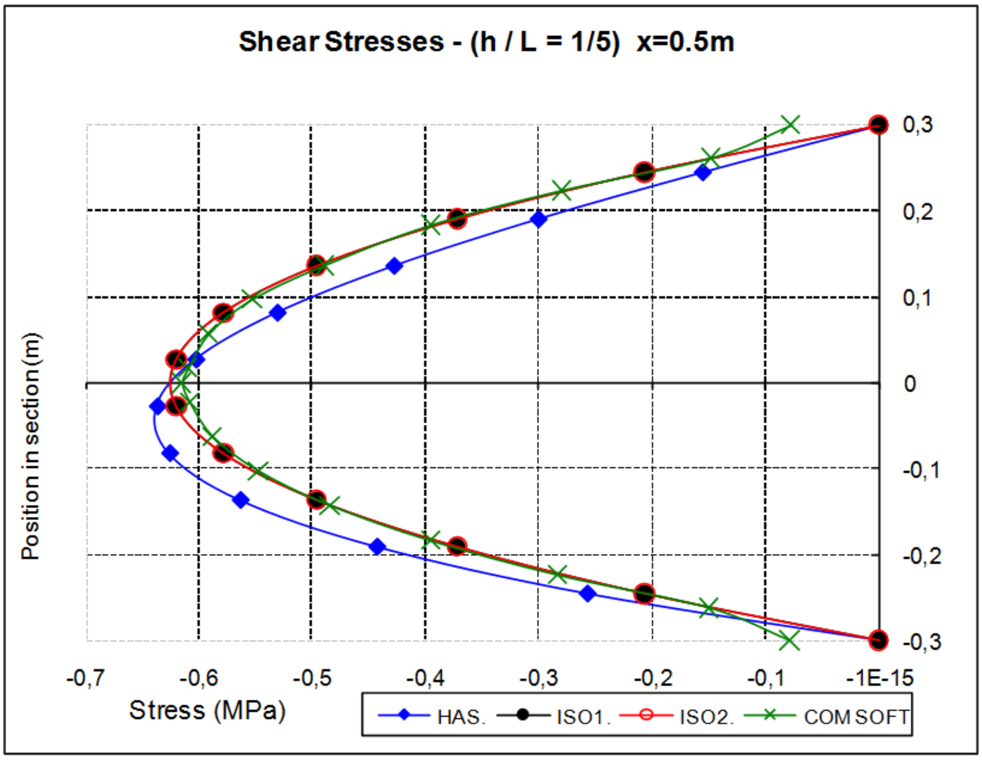

Figure 7: Shear Stresses – h/L=1/5, x=0.5m

Notice that an increase of the relation h/L results in an increase in the differences of the stresses in the isotropic and the anisotropic beams. On the other hand, the isotropic and the orthotropic beams had a similar stress behavior. Clearly, the normal stress is significantly affected by the anisotropic elastic coefficients from 12% to 26% approximately. Finally, it also observed that the distribution of the stresses in the anisotropic beams is not symmetric.

Using the Eq. (13) to Eq. (18), is possible also to find the following diagrams with the values for the variations of the elastic constants in relation to the angles of carbon fibers can be obtained:

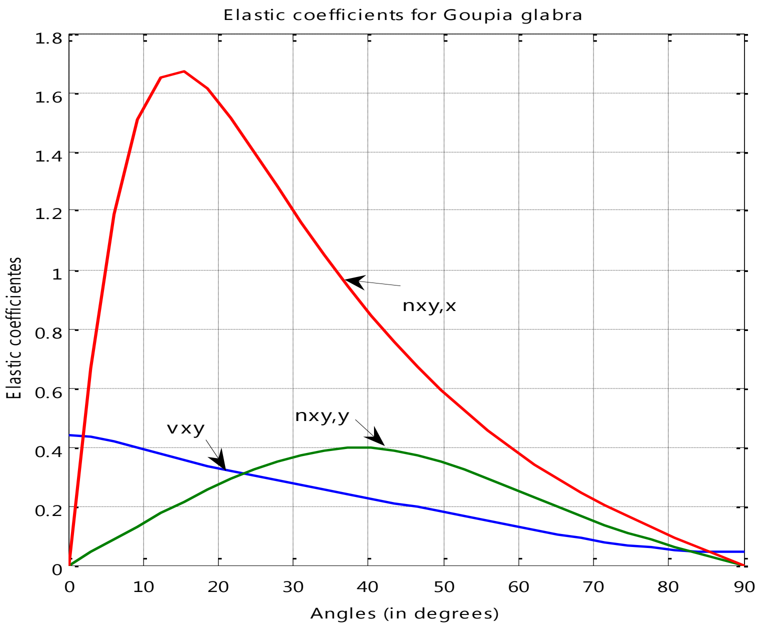

Figure 8: Elastic Coefficients for *Goupia glaba* – $n_{xy,x}$, $n_{xy,y}$, v_{xy}.

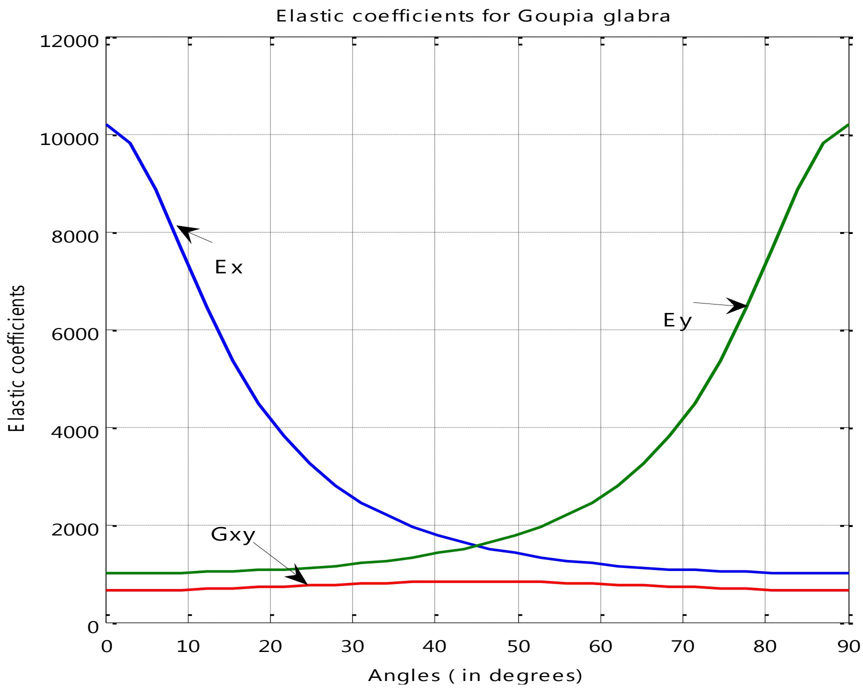

Figure 9: Elastic Coefficients for *Goupia glaba* – E_x , E_y , G_{xy}.

5 CONCLUSIONS

This paper focused on an analysis of the distribution of stress field on wood beams based on concepts of anisotropic elasticity. From this a theoretical application in wood cantilever beams was carried out. Although this research is restricted to a unique wood species and specifically to the LR plane several interesting conclusions can be drawn from the current study as follows:

- The stress distribution, especially the normal stress, and the displacement fields on wood beams are affected by the coupling coefficients evidencing the wood anisotropy;
- The shear influence varied with the height-span beam relation and increased when this relation decreased. Depending on the orientation of fibers in the wood beam, the determination of the stresses, as usually admitted by the theory of Euler-Bernoulli, can present results very different from which will actually occur. The beam theory of Timoshenko can be accepted as more accurate;
- The Airy stress function demonstrated to be efficient as an analytical method in structural mechanics and the numerical results confirmed this. However, it is necessary to emphasize that the methods of solutions based on the stress functions are approached (stress function is polynomial).

Finally, this research demonstrated that it is relevant to take into consideration the influence of the coefficients of mutual influence on the mechanical behavior of wood laminated beam and this may be useful to those especially engaged in investigating wood as an anisotropic material applied to structures.

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