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NATURAL CONVECTION AND ENTROPY GENERATION IN A SMALL ASPECT RATIO CAVITY.

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Abstract. In this work, the heat and mass transfer process in a vertical cavity of small aspect ratio, $AR = 1/10$, heated from two vertical plates localized in the side walls of the cavity near the bottom, is studied. The equations of motion are written in non-dimensional form, depending on two non-dimensional parameters (the Rayleigh and Prandtl numbers) and are solved numerically by the use of the SIMPLE algorithm. Calculations are performed for a fixed value of the Prandtl number and three different Rayleigh numbers. The non-dimensional heat flux on the plates, given by the average Nusselt number, and the non-dimensional entropy production of the process are calculated.

1 INTRODUCTION

Natural convection has been widely studied due to its presence and determinant role in many of the flows around us. It is found in atmospheric currents, heat exchangers, solar collectors and even in nuclear reactors.

In particular, it is of great practical interest to study natural convection in square cavities, and a great effort has been devoted to this particular system along decades. Recent studies are given by Aminossadati and Ghasemi (2009), where the behaviour of a nanofluid in a rectangular cavity heated from a section of the bottom wall is studied. AlAmiri et al. (2009), analyzed the flow in rectangular cavities with partial divisions. A similar investigation is presented by Wu and Ching (2010), where the natural convection of air inside a cavity with partitions on its top wall is studied. Xu et al. (2009) treated the case of a rectangular cavity with a thin fin on one of its side walls. In addition, numerical stability analyses of flows in rectangular cavities driven by natural convection are presented by Prasad and Das (2007) and Xiaohua et al. (2009).

Due to the practical importance that systems like the presented above have, many optimization studies of the transport processes have been developed, like those given by del Río et al. (1998) and Lambert et al. (2009). In recent years, the heat transfer studies for the design of thermal devices have inclined by the analyses of second law (of thermodynamics), and for the minimal entropy production, studies on this subject are presented by Baytas (2000), who analyzed the entropy production in an inclined square cavity. Narusawa (2001), who studied theoretically and numerically the entropy production of a flow in a rectangular duct heated from below, for forced and mixed convection cases. Varol et al. (2008), investigated the entropy generation for natural convection in a square cavity limited for solid walls of different finite depths. Later, Varol et al. (2009) studied the entropy generation of a buoyancy driven flow in a trapezoidal cavity filled with a porous medium and with a solid wall of finite depth.

2 PROBLEM FORMULATION.

A square cavity as depicted in Fig. 1 is considered, the aspect ratio is chosen to be $L/H = 12$, where L is the length of the cavity and H its width.

The walls of the cavity are taken to be adiabatic insulators, except by two portions of length H located symmetrically on the side walls, a distance $2H$ from the bottom of the cavity, which are held at constant temperature T_1 higher than the initial temperature of the fluid T_0.

With the use of the length scale H, the buoyancy induced charateristic velocity $\sqrt{gd\Delta\rho/\rho_0}$ and the overall temperature difference $T_1 - T_0$, the following non-dimensional variables arise

$$x_i = \frac{x_i^*}{H},\ u_i = \frac{u_i^*}{\sqrt{gH\Delta\rho/\rho_0}},\ t = \frac{t^*}{H/\sqrt{gH\Delta\rho/\rho_0}},\ p = \frac{p^*}{gH\Delta\rho},\ \theta = \frac{T - T_0}{T_1 - T_0}, \tag{1}$$

where x_i represents the i-th component of the position vector, and u_i represents the i-th component of the velocity vector. p stands for the pressure, t represents the time and θ is the non-dimensional normalized temperature, with T being the temperature in dimensional form. ρ is the fluid density, ρ_0 is the fluid density at the reference temperature T_0, and $\Delta\rho$ is the density variation due to the temperature change $T_1 - T_0$. g is the gravitational acceleration. The $*$

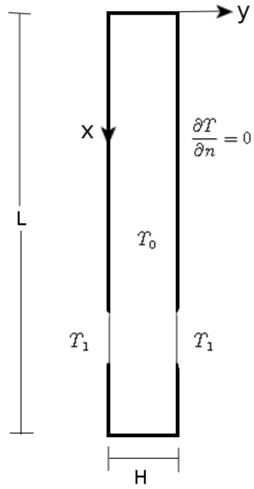

Figure 1: Schematic representation of the cavity.

superscript stands for dimensional quantities.

With these non-dimensional variables, the governing equations take the following form, where the Boussinesq approximation is used to describe the fluid under consideration,

$$\frac{\partial u_j}{\partial x_j} = 0, \tag{2}$$

$$\frac{Du_i}{Dt} = -\frac{\partial P}{\partial x_i} + \theta\,\delta_{i1} + \sqrt{\frac{Pr}{Ra}}\frac{\partial^2 u_i}{\partial x_j \partial x_j}, \tag{3}$$

$$\frac{D\theta}{Dt} = \frac{1}{\sqrt{RaPr}}\frac{\partial^2 \theta}{\partial x_j \partial x_j}. \tag{4}$$

Here $Pr = \nu/\kappa$ is the Prandtl number, $Ra = \beta g H^3(T_1 - T_0)/(\nu\kappa)$ is the Rayleigh number, $P = p + x/[\beta(T_1 - T_0)]$ with x as vertical coordinate (see Fig. 1), β as the volumetric expansion coefficient, ν as the kinematic viscosity, κ as the thermal diffusivity and δ_{ij} is the Kronecker delta ($i, j = 1, 2$), with $\delta_{ij} = 0$ if $i \neq j$ and $\delta_{ii} = 1$.

The two dimensional form of Eqs. (2)-(4) are solved with the following boundary conditions

$$\begin{aligned} u_i &= 0, \text{ on the walls of the cavity,} \\ \frac{\partial \theta}{\partial n} &= 0, \text{ on the walls of the cavity, except in} \\ & \quad x = 0,\ y \in [2,3] \text{ and in } x = 1,\ y \in [2,3] \\ \theta &= 1, \text{ in } x = 0,\ y \in [2,3] \text{ and in } x = 1,\ y \in [2,3]. \end{aligned} \tag{5}$$

and initial conditions at $t = 0$: $u_i = \theta = 0$.

2.1 Entropy Generation for Heat and Mass Transfer.

In a macroscopic system, entropy variations ds are due to the entropy exchanged with the surroundings in form of heat and mass transfer $d_e s$, and to the internal production of entropy in irreversible processes $d_i s$

$$ds = d_e s + d_i s,$$

where

$$d_i s \geq 0.$$

In the Linear Irreversible Thermodynamics formulation, an explicit expression for the entropy balance is obtained in terms of the velocity and temperature fields, and for a fluid as the considered in the present work, takes the following form

$$\rho \frac{DS^*}{Dt} = -\frac{\partial}{\partial x_j^*}\left(\frac{q_j^*}{T}\right) + \frac{k^*}{T^2}\left(\frac{\partial T}{\partial x_j^*}\right)^2 + \frac{\lambda^*}{T}\left(\frac{\partial u_k^*}{\partial x_k^*}\right)^2 + \frac{\mu^*}{2T}\left(\frac{\partial u_i^*}{\partial x_j^*} + \frac{\partial u_j^*}{\partial x_i^*}\right)^2. \quad (6)$$

In the above expression, S is the entropy per unit mass, q_j is the heat flux vector, k is the thermal conductivity, λ is the second viscosity coefficient, and μ is the dynamic viscosity. It is noted that this balance equation has the form

$$\rho \frac{DS^*}{Dt} = -\frac{\partial}{\partial x_j^*} J_{Sj}^* + \sigma^*,$$

where $J_{Sj}^* = q_j^*/T$ is the entropy flux, and σ^* is the internal entropy production given by

$$\sigma^* = \frac{k^*}{T^2}\left(\frac{\partial T}{\partial x_j^*}\right)^2 + \frac{\lambda^*}{T}\left(\frac{\partial u_k^*}{\partial x_k^*}\right)^2 + \frac{\mu^*}{2T}\left(\frac{\partial u_i^*}{\partial x_j^*} + \frac{\partial u_j^*}{\partial x_i^*}\right)^2, \quad (7)$$

from this equation, it is clear that entropy is produced by two different means, the first one is the heat transfer in the system, and the second one is due to the viscous effects in the fluid. For most of the cases, the contribution of the former is several orders of magnitude greater than the later.

Using the eqs. (1), the internal entropy production can be written in non-dimensional form as

$$\sigma_q = \frac{1}{(\theta + a)^2}\left(\frac{\partial \theta}{\partial x_j}\right)^2 \quad (8)$$

$$\sigma_v = \frac{\lambda_s}{2(\theta + a)}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2, \quad (9)$$

where σ_q is the entropy produced by heat transfer, σ_v is entropy produced by viscous effects, $a = T_0/(T_1 - T_0)$, and $\lambda_s = gH\beta\mu/k$.

3 NUMERICAL SOLUTION.

The equations of motion were discretized using the power-law scheme, described by Patankar (1980), and solved with numerical codes developed in Fortran 90 language by use of the SIMPLE algorithm Patankar (1980). A non-uniform grid, with 76 nodes in the horizontal direction and with 151 nodes in the vertical direction was used, the grid spacing in the horizontal direction was smaller near the cavity walls, as well as in the near zone of the plates location in the vertical direction. The solutions obtained with this grid proved to be mesh independent. Convergence on each time step was declared when the residual of the discretized equation cas less than $1\times^{-10}$.

4 RESULTS.

Fig. 2 shows the temperature field for 4 different times and three values of Rayleigh number $Ra = 1 \times 10^4$, $Ra = 1 \times 10^5$ and $Ra = 1 \times 10^6$.

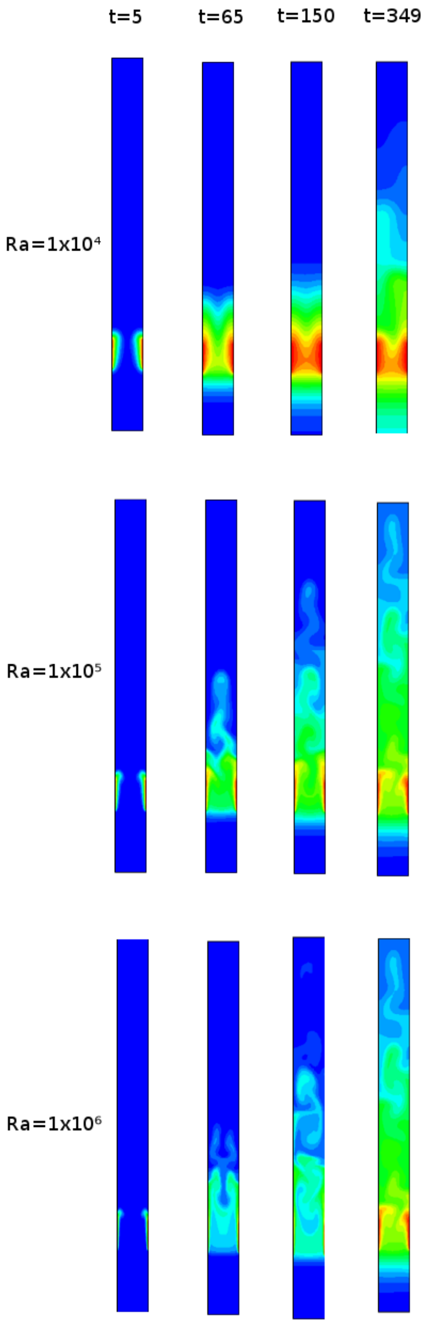

Figure 2: Temperature field.

With the numerical solution for temperature, the average Nusselt number, $\bar{N}u$, was calculated,

$$\bar{N}u = \int_{\Omega} -\frac{\partial \theta}{\partial x_j} n_j d\Omega, \tag{10}$$

here, Ω represents the surface of the cavity. The results for $Ra = 1 \times 10^4$, $Ra = 1 \times 10^5$ and $Ra = 1 \times 10^6$ are shown in Fig. (3)

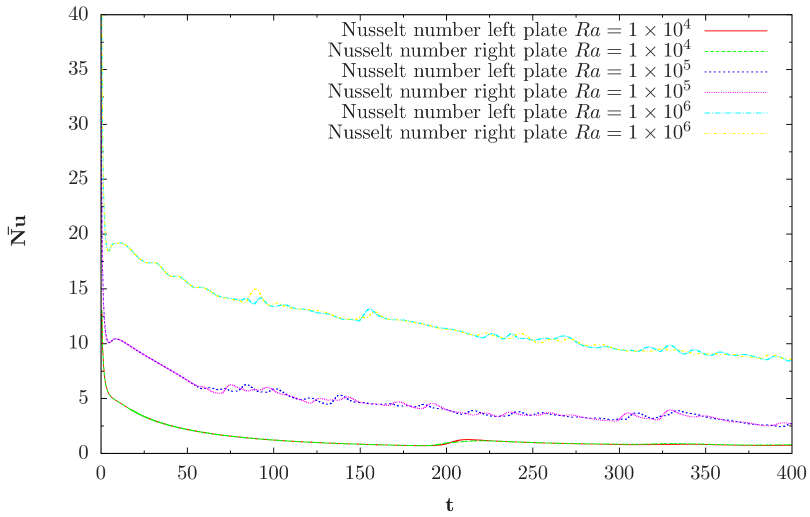

Figure 3: Average Nusselt number.

In the three cases, for $t < 5$ the effects of heat conduction are more important than those of heat convection, when $t > 5$ convection starts to dominate and two symmetric recirculation regions start to develop, this development continues until the symmetry of the solution is lost, and eruptions of hot fluid start to come from the zone near the plates. The symmetry break occurs in $t = 175$ for $Ra = 1 \times 10^4$, in $t = 55$ for $Ra = 1 \times 10^5$, and in $t = 63$ for $Ra = 1 \times 10^6$.

Fig. 4 shows the average temperature in the cavity $\bar{\theta}$,

$$\bar{\theta} = \frac{\int_\Omega \theta dxdy}{\int_\Omega dxdy}, \tag{11}$$

It is noted that average temperature is higher for $Ra = 1 \times 10^4$, this happens because the fluid in the near zone of the plates remains in that zone for long periods of time, consequently, the temperature in this small zone increases to values near the wall temperature, creating a zone with very high temperature and small area. Still, the heat flux in the plates remains small, given that temperature is very similar in the sourrounding fluid, and accordingly the temperature gradient is also small.

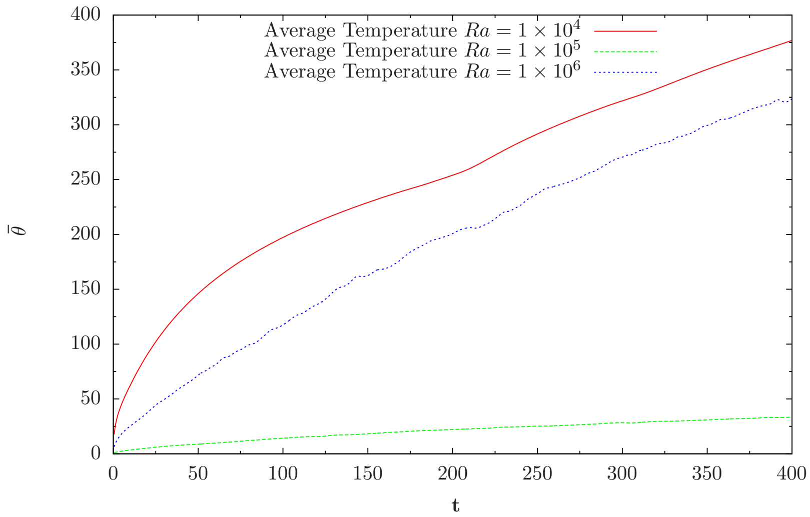

Figure 4: Average temperature in the plate.

Information about the occurrence of the eruptions in the fluid can be obtained by looking at the temperature in the point $(0.5, 9.5)$, which is located between the plates. It is noted, that an ascention of fluid with high temperature, makes the fluid with lower temperature go down in the cavity. This creates fluctuations in the temperature at the proposed point, and this fluctuations give information about the frequency of the eruptions.

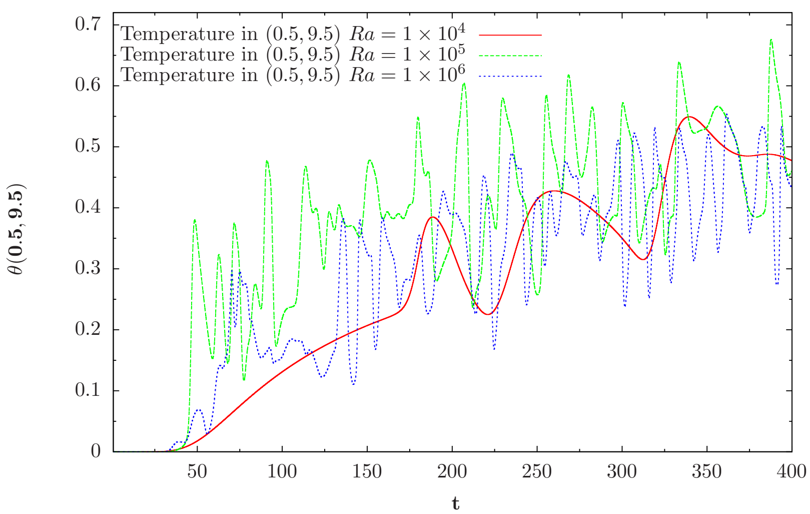

Figure 5: Temperature in $(0.5, 9.5)$.

Fig. 6 shows the total entropy produced in the cavity $\bar{\sigma}$,

$$\bar{\sigma} = \int_{\Omega} \sigma d\Omega. \tag{12}$$

Entropy production, follows a similar trend as the heat flux, this was expected given the relation between the two quantities.

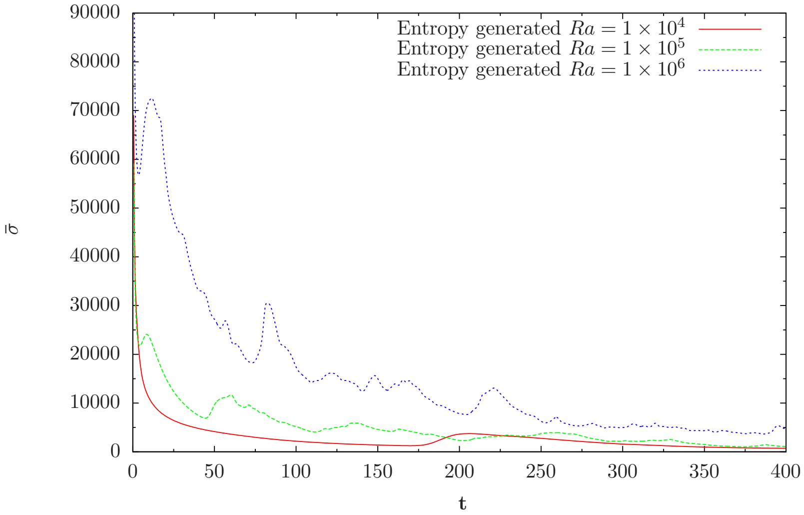

Figure 6: Total Entropy generated.

5 CONCLUSIONS.

Heat transfer for a square cavity with a symmetrical heating was investigated numerically by use of the SIMPLE algorithm. The flow was characterized by quantifying the non-dimensional heat flux, the average temperature in the cavity and the total entropy production. In the present work, results for three different values of Rayleigh number are presented, nonetheless, more values of the Rayleigh number are being investigated at the present time.

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