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AN IMPROVED MOVING LAGRANGIAN INTERFACE TECHNIQUE FOR THE ANALYSIS OF TWO-FLUID FLOWS

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Abstract. *In this work an improved moving lagrangian interface technique applied to update the front position between two fluids is presented. The main drawback of classical lagrangian schemes is the progressive distortion in the distribution of the markers used to identify the material front. To avoid this problem a redefinition of markers obtained using a diffuse approximation technique is proposed. The remeshing algorithm imbedded in the computational fluid dynamics code is described. Aspects of the capabilities of the proposed formulation are evaluated in a simple test: the filling of a step mould.*

Resumen. *En este trabajo se presenta una técnica numérica para el seguimiento de una interfaz entre dos líquidos que no se mezclan utilizando para la actualización temporal un método lagrangeano. Con la finalidad de evitar la distorsión de la malla con que se identifica el frente material se propone una técnica de redefinición de dicha malla. La metodología de remallado propuesta se basa en la técnica de aproximación difusa utilizada en los métodos sin malla y particularmente aplicada a la generación de mallas para geometrías complejas. Es requisito de la técnica de remallado que no modifique el volumen y que satisfaga ciertos aspectos relacionados a la física del problema. En este trabajo se propone incorporar algunos criterios para el remallado. Las capacidades del método se analizan en el problema de llenado de un canal escalón.*

1 INTRODUCTION

As it is well-known, many manufacturing processes such as mould filling or hot metal forming requires time-dependent flow analysis with moving two-liquid interfaces or free surfaces. Several difficulties are presented in the numerical simulations of moving interfaces. Mass preserving and discontinuity material properties are one of the most important aspects to properly describe in order to avoid numerical oscillations and to produce accurate results. Several studies have been proposed numerical formulations capable of dealing with solution of this class of problems.

Different numerical techniques have been proposed to analyse flow problems with moving interfaces and free surfaces[1-23]. If the fluid dynamic is computed over a mesh, the techniques can be categorized into two main groups: moving-mesh and fixed-mesh techniques. The Arbitrary Lagrangian-Eulerian (ALE) technique[1,2] and the Deformable-Spatial-Domain/Stabilized Space-Time (DSD/SST) method[3,4] are both moving-mesh formulations. In these methods the interface is tracked with the moving mesh, and the mesh is updated every time step to accommodate the tracking. Such techniques have good numerical accuracy, but the frequency of remeshing may become too high when complex and very unsteady interfaces need to be tracked. A different formulation varying the mesh over time has been recently proposed and successfully used in [5].

In the context of fixed-mesh finite element methods, one of the most common ways of updating the interface has been performed by using an advection equation considered to calculate the time-evolution of a pseudo-concentration function[8-18]. To prevent numerical oscillations that might be generated in solving an advection-dominated problem with a standard Galerkin formulation, methods that stabilize the formulation need to be applied. Among the stabilized methods used for this purpose by other researchers are the Taylor-Galerkin and streamline-upwind/Petrov-Galerkin (SUPG) approaches[10,14-16]. These approaches must be supplemented with techniques that reduce the smearing of the pseudo-concentration profile and yield more accurate representation of the interface. To this end, in the fixed-grid finite difference level set methods[11-13], a distance reinitialization procedure is used to redefine the pseudo-concentration function, assuring that it does not become too flat or too steep near the interface. In this local approach, the interface advection equation is solved only around the interface, with a predetermined width of typically five or six grid subdivisions. Problems involving mass loss were observed and significant improvements were made[11-13]. More recently, a finite element Enhanced-Discretization Interface-Capturing Technique (EDICT)[14,15] has been developed to increase the accuracy in representing the interface.

In the same fixed-mesh finite element context, an alternative methodology to obtain the transient interface location has been proposed where the aim was to track the interface front position in a lagrangian way[19-21]. This technique was improved including a global mass–conservation algorithm and an enhanced element integration of the flow equation in order to capture more properly the discontinuities in the material properties due to material front. This improved technique is called Moving Lagrangean Interface Technique (MLIT)[22,23]. However, the transient Navier-Stokes equations are solved in a traditional eulerian form using a fixed mesh. The stabilized finite element formulation is written in terms of the generalized streamline operator technique. Hence, this proposed methodology retains the advantages of computing the fluid dynamics equations in a unique mesh and provides a good behaviour in unstructured meshes. Moreover, this technique includes the concepts of the interface-sharpening and global mass-conservation. Nevertheless, due to the lagrangian nature of the updating interface algorithm, the mesh used to define the interface usually distorts during the transient analysis. The points that act as markers of the interface need to be re-positioned along the interface. To this end, an interface remeshing technique based on a diffuse approximation technique[24,25] was proposed to redefine the material front at certain times of the analysis at which the front presents a unacceptable distortion[26]. Thus, the flow computation is restarted from these instants using a remeshed front with a predetermined number of new markers.

In this paper, a remeshing technique automatically embedded in the computation of the dynamics of two-fluid flow problems is presented. To this end, a criterion aimed at assessing the distortion degree of the interface mesh is included and, accordingly, a new marker distribution fulfilling the restrictions imposed by boundary conditions and front curvatures is obtained. Some aspects related to the choice of the new number of markers during the analysis are illustrated in the numerical example.

2 THE IMPROVED MOVING LAGRANGIAN INTERFACE TECHNIQUE

The base moving interface formulation defines the material front with a set of points that serve as markers[19]. The main operations involved are: identification of the elements that host the interface points, evaluation of the markers' natural coordinates, calculation of their velocities, updating the front position and redefinition of the material properties in the whole domain. The velocity of the interface points is computed by interpolating the nodal velocity values of the elements where the markers are located. Then, using this velocity, the interface is updated by applying a direct path-particle Lagrangian scheme. Following that, the properties are reassigned at the element integration points according to the updated interface position. Due to the simple algebraic nature of the algorithm, the computer time required to perform a complete interface update is negligible compared to the time required by the fluid dynamics solver. More recently, the Moving Lagrangian Interface Technique (MLIT) was developed[22,23]. This MLIT includes two relevant aspects: a sub-element integration technique, which is applied to the field equations (the Navier-Stokes equations in this case) and a global mass-corrector algorithm.

An assessment of the moving Lagrangian technique with enhanced integration and global mass conservation has been presented in [22-23], including computations with different sets of properties, several finite element meshes and interface discretizations, and how these enhancements improve the solution accuracy. The accuracy of the method was compared to those obtained with Eulerian-Lagrangian techniques, and a very good performance has been observed in representing a water-air interface. Nevertheless, distortions of the marker distribution have been observed during the analysis of some problems.

In this work, the interface mesh distortion is overcome using a remeshing technique embedded in the coupled computation of the unsteady two-fluid flow. When an updated position of the interface is computed using the MLIT, two tasks need to be automatically done. They are detailed below.

The first one is to check if there are markers outside the domain. In such a case, two strategies can be used to correct the marker position. A possible option is to select a new time step that avoids the motion of markers outside the domain. Note that the applicability of this approach is rather limited to the number of points to be re-positioned. An alternative option can be used if the distance from the marker to the boundary domain is small enough (according to a geometric tolerance). Then, the new position of the marker can be adopted as the intersection point between the boundary and the segment determined by the old and updated positions of the marker. The chosen tolerance needs to guarantee mass conservation. This out-of-domain marker controller is active in the implementation we are presenting.

The second task consists in the redefinition of the distribution of markers. To this end, a remeshing technique based on a diffuse approximation that preserves the curvatures is used together with a set of definitions to properly include some requirements of two-fluid flow applications. Figure 1.a illustrates the updated position of an interface front at initial times of the simulation reported in Section 3 that has been taken here as example. After remeshing the full line between the two end markers, the obtained interface front is sketched in Figure 1.b. The well-known fact that diffuse approximation of curvatures smoothes corners is apparent. When the geometry to be approximated is previously known, such fact is easily taken into account by splitting the line between sharp corners. In the case of updated interfaces, the line to be remeshed is not previously known and, therefore, in the present work we propose to split the interface by parts between markers belonging to the boundary domain (remeshing criterion type 1). In addition, to avoid highly distorted interfaces a certain number of new markers are added along the longer interface elements (the interface element is defined between consecutive markers in 2D). In the present work, the remeshing criterion that checks the longer elements is called type 2. As an assumption, the length of a shorter element is set as 0.05 of the length of the part while the length of a longer element is 5 times the shorter one. Moreover, the number of new markers by parts need to be determined in order to avoid unreasonable increasing (or decreasing) number of new markers. This number can be either fixed arbitrarily during the whole analysis or can proportionally vary to the number of longer/shorter elements. A criterion based on this geometric aspect is used in this work to add or remove markers. Finally, Figure 1.c schematically shows the case when a out-of domain marker controller is applied.

The complete algorithm is summarized in the following steps:

1) Update the interface using the MLIT[22,23].
2) Check for markers outside the domain. If it is the case, correct their positions.
3) Apply global mass conservation algorithm[22,23].
4) Repeat step 2).
5) Split the interface by parts considering the remeshing criteria type 1 or 2.

6) Define the number of new markers of each part.
7) Redefine the interface applying the remeshing technique for each part.
8) Repeat step 2).

Some aspects of this proposed formulation are evaluated in the next section.

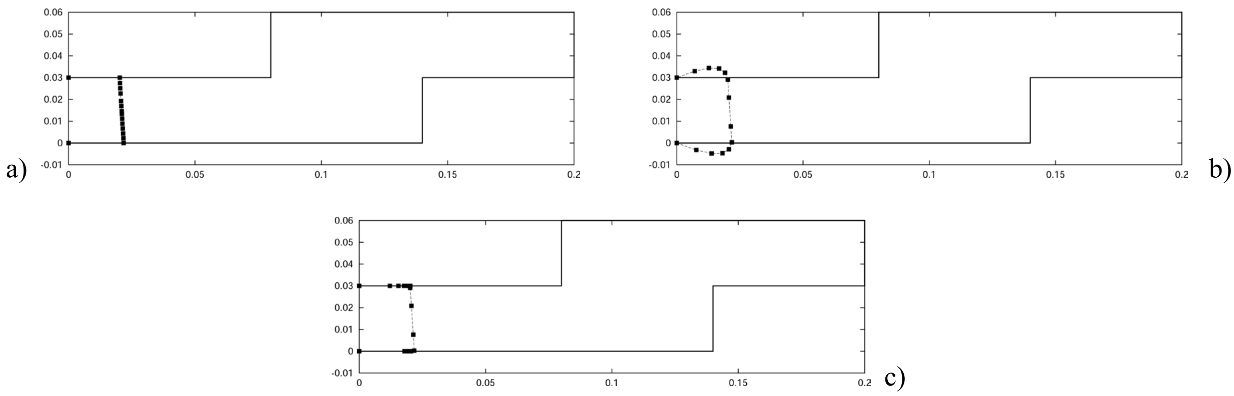

Figure 1: a) Volume preserving updated interface position, b) remeshed interface position without conditions and c) markers position when correction of out-of-domain points is active.

3 FILLING OF A STEP MOULD

Filling of step moulds (lower and upper steps) has been reported in [9]. The problem geometry and fluid properties, shown in Figure 2, are those used in [9,23,26]. Slip boundary conditions are assumed on the walls. The filling material enters through the inflow boundary with a uniform velocity of 0.1m/s, displacing the air that is initially at rest. Traction-free conditions are imposed at the outflow boundary. The inflow velocity (0.1m/s) is low enough to give the interface the chance to spread under the influence of gravity. The mesh is uniform and consists of approximately 700 four-noded elements, and the time step size is 0.01 s.

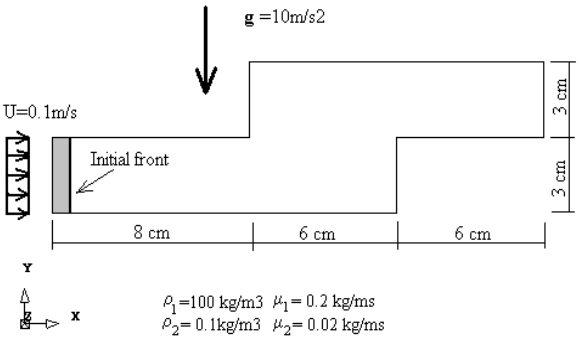

Figure 2: Problem layout.

Time-evolutions of the interface computed with the two remeshing criteria presented above (types 1 and 2) are shown in Figure 3. It is seen that longer elements are not split when remeshing type 1 is used. Although relatively small discrepancies between both predictions are appreciated, the results corresponding to the remeshing type 2 more properly describe the front curvatures.

Figure 4 shows the results obtained using the same initial number of markers but with different spatial distributions. The remeshing type 2 is applied, i.e., the interface is split according to the boundary and the longer elements are subdivided. The number of new markers varies for each time step according to the quality of the interface mesh that can be described in terms of longer/shorter number of elements. Very similar numerical results are obtained in this case.

The results obtained with different number of initial equally distributed markers are presented in Figure 5. The remeshing solution strategy is the same as that described above. The predictions of the interface position using both discretizations practically coincide.

Finally, Figure 6 shows the evolution in time of the number of new markers using different number of initial markers. The initial number of markers is found to practically not influence the automatically set number of markers during the analysis.

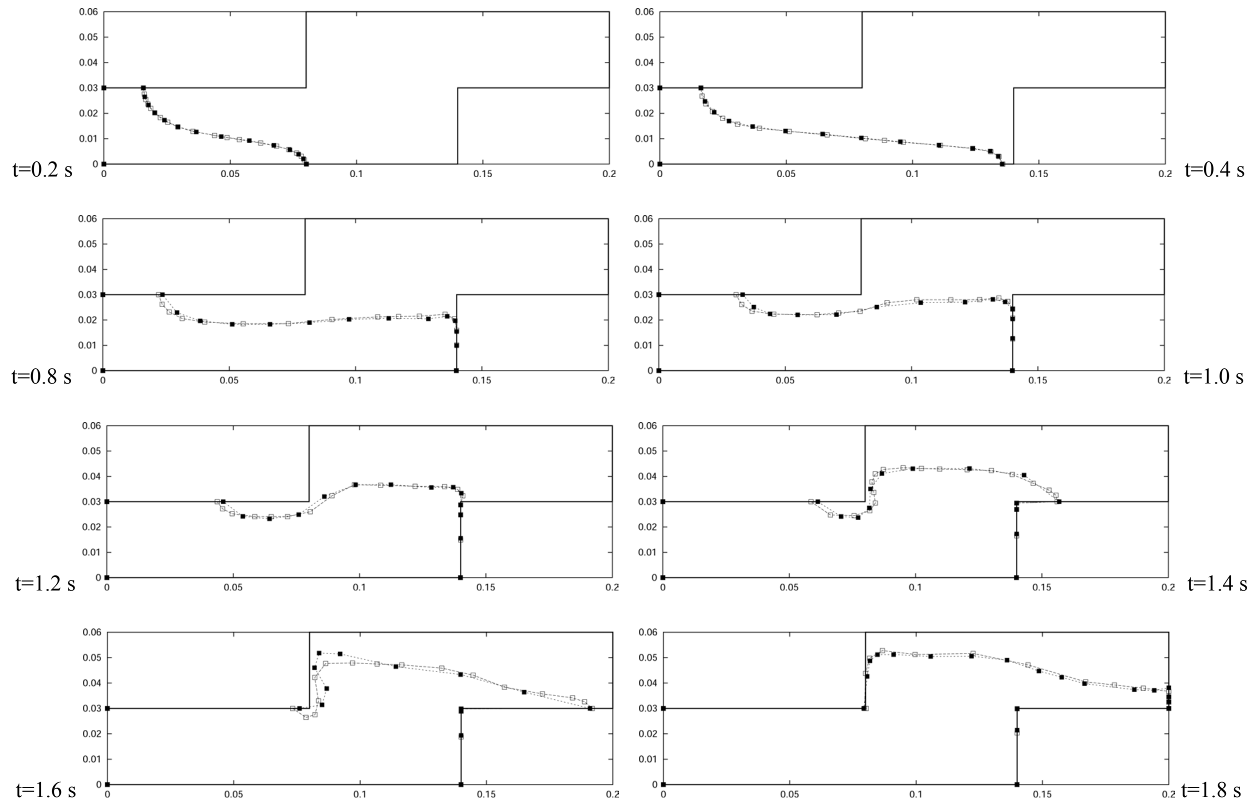

Figure 3: Interface front position at different instants of the analysis (uniform initial distribution with 16 markers): ▪ remeshing considering boundaries only (type 1) and ▫ full remeshing strategy (type 2).

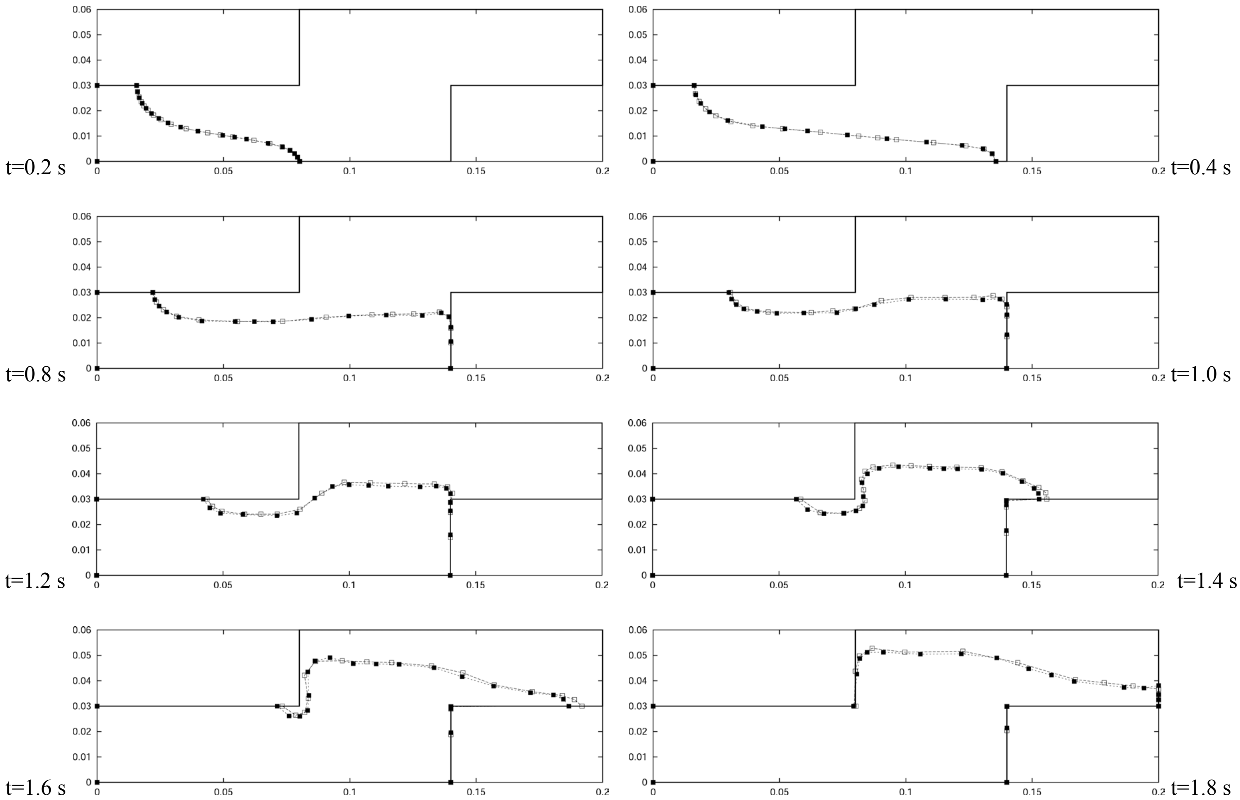

Figure 4: Interface front position at different instants of the analysis (full remeshing strategy, type 2) initial number of markers: 16 with ▫ uniform initial distribution and ▪ non-uniform initial distribution.

4 CONCLUSIONS

In this work a remeshing technique based on diffuse approximation of curvatures is presented to redefine the marker distribution in the Moving Lagrangian Interface Technique. The remeshing scheme is embedded in the two-fluid dynamic computation. The capabilities of the resulting algorithm are the splitting of the interface according to geometrical aspects: restrictions imposed by the boundary domain and quality of the interface mesh measured through the lengths of their elements. The splitting in parts to be remeshed as well as the number of new markers to be adopted has been both automatically determined. The numerical predictions of the interface position during time have been found to be nearly independent of the number of initial markers and distribution. The splitting of longer elements and a selective criterion to determine the number of new markers according to the mesh quality seem to be adequate to describe the proposed interface remeshing problem.

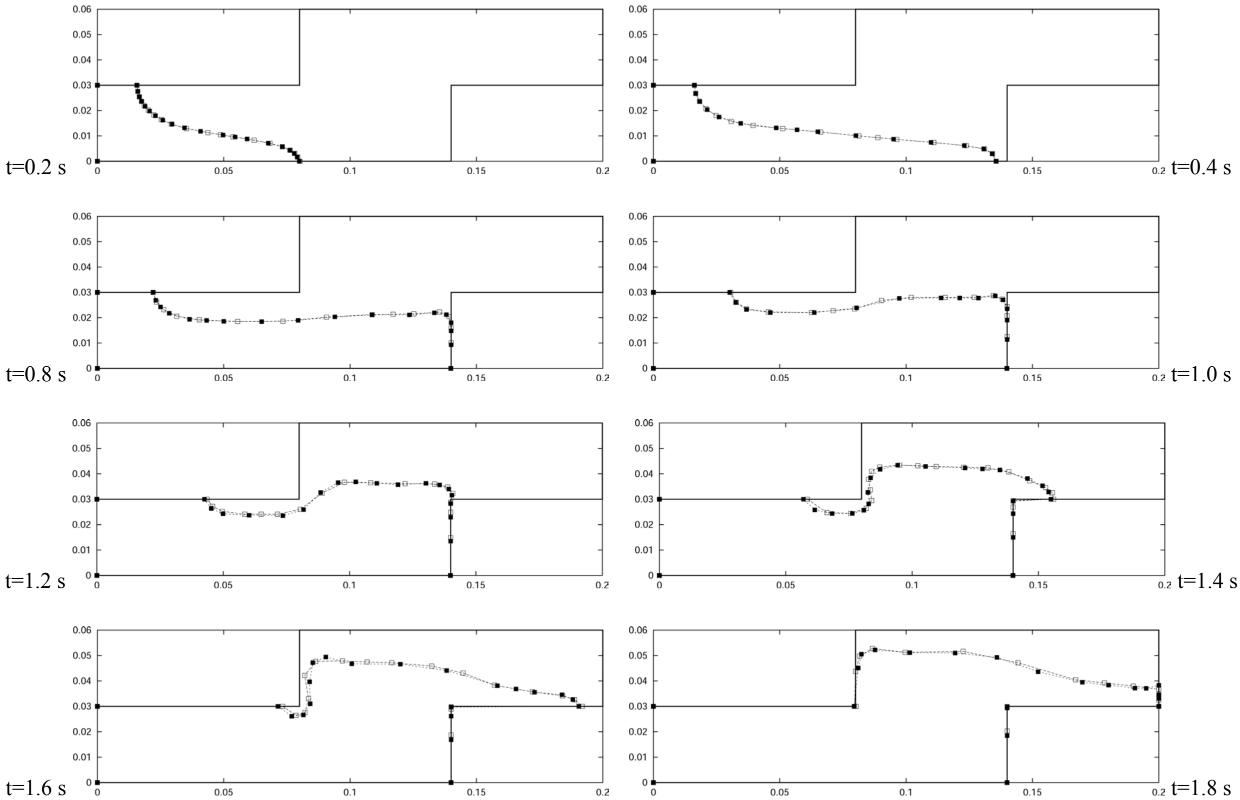

Figure 5: Interface front position at different instants of the analysis (full remeshing strategy, type 2) initial uniform distribution with: ▫ 16 and ▪ 42 initial number of markers.

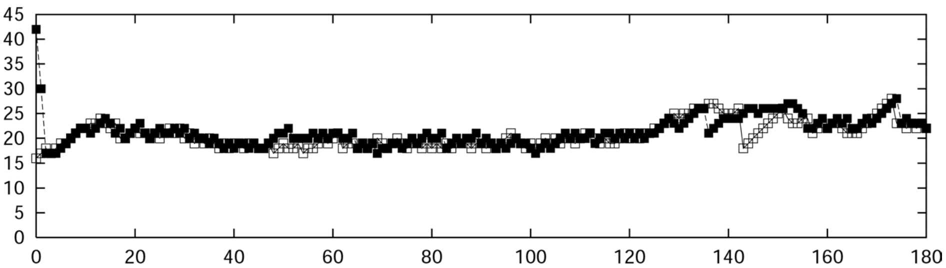

Figure 6: Variation in the number of new markers during the analysis when full remeshing strategy (type 2) is used with uniform distribution of: ▫ 16 and ▪ 42 initial number of markers.

5 ACKNOWLEDGEMENTS

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