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INVESTIGATIONS IN A CRACKED PLATE UNDER BENDING

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Abstract. This work focuses on the study of bending effects on the closure of crack faces and the resulting effects on the stress intensity factors. A plate with a pre-existing through-the-thickness crack is considered under the action of a remote bending moment. This bending induces contact of the crack faces, which introduces additional load along the crack extension and strongly influences the stress intensity factor, the stress and the displacement fields at the crack tip. To the knowledge of the authors, there are few if any experimental studies of fracture in plates subjected to bending, and in particular, cyclic loading of propagating cracks in such plates. Steel sheets were used as specimens and the loading was cyclic pure bending. Three-dimensional finite element analyses using FRANC3D and ABAQUS are compared with the experimental results. Contact was modeled in the numerical analyses. The influence of the contact on the crack tip is investigated for three thicknesses and three geometries. It was found that the edge contact at the crack tip varies with different thicknesses. However, for the same thickness but varied geometry, the edge contact is the same. The numerical results agree with the experimental results.

1 INTRODUCTION

Progress has been made in the development of bending theory for cracked thin plates since the first work (Williams, 1961). His equations for the elastic stresses local to the crack tip contained two unspecified constants, which were defined by Sih, Paris and Erdogan (1962).

Additionally, other theories have also described the local stress near the tips of through cracks in plates, the surfaces of the crack were taken to be stress free in either the Kirchhoff or the Reissner sense, and the crack tip was taken to be straight through the thickness of the plate (Williams, 1961; Knowles and Wang, 1960; Hartranft and Sih, 1968). However, there is ample experimental evidence that the restrictions of the mathematical model are violated in reality (Erdogan, Tuncel and Paris, 1962).

Out-of-plane bending will produce tension on one surface of the plate and compression on the other. This compression induces contact of the crack faces. In this circumstance, the behavior of the front crack during growth is not so simple.

To the knowledge of the authors, there are few studies that address the numerical simulation of crack propagation in plates under cyclic bending, other than that by Roy *et al.* (2005). However, Roy *et al.* combined out-of plane bending with a tension load.

Furthermore, there are no experimental studies of crack propagation under cyclic bending loads. For example, Erdogan *et al.* (1962) and Yan *et al.* (2010) did experiments using out-of-plane bending, but in these works the loading is applied statically. Potyondy considered cyclic loading in fracturing analysis of shells, but the crack face contact was not taken into account because a membrane loading and a bulge-out effect were considered. (Potyondy, Wawrzynek and Ingraffea, 1995).

In 1969, the behavior of pre-catastrophic crack extension in a plate in combined extension and approximately cylindrical bending was studied by Wynn and Smith (1969). They compared the experimental stress with Sih-Hartranft bending theory. The results are similar to experiment results in regions where the crack remained open at fracture, but appeared to provide a lower bound in the region where crack closure occurred.

Smith and Smith (1970) first studied this problem experimentally using frozen stress photoelasticity and the data were also compared with Hartranft-Sih theory. They concluded that crack face contact during bending increased the crack tip stress over the no contact case. More frozen stress photoelasticity experiments were presented in Mullinix and Smith (1974), but an extension load was applied simultaneously with the bending load to ensure that the crack did not close. Those experimental results agree with the Sih theory only for thin to moderately thick cracked plate geometries ($t/2a<1$).

In 1992, a theoretical work was performed using a line contact analysis for Kirchhoff theory (Young and Sun, 1992). This study considered closure at the compressive edges for an infinite plate containing a center crack under bending. It was found that the closure at the compressive edge tends to reduce the crack opening displacement at the tension side and as a result, reduce the stress intensity factors.

In addition, Heming (1980) used finites elements with Reissner theory kinematics, and also assuming a line contact during bending. He also found that the opening displacements on the crack are reduced. Alwar and Ramachandran (1983) considered a three dimensional finite element analysis for this problem. By iteration they were able to accurately determine the actual area of contact. As Young and Heming, they concluded that closure reduces the crack tip stress intensity. Later, analytical works were developed for an infinite plate that solves the area contact using Reissner theory (Slepyan, Dempsey and Shekhtman, 1995; Dempsey, Shekhtman and Slepyan, 1998). They determined the shape of the closure region and its dependence on the remote loading, as well as length to plate thickness ratio for a pre-existing through crack. Zehnder and Viz (2005) provide some reviews on this subject.

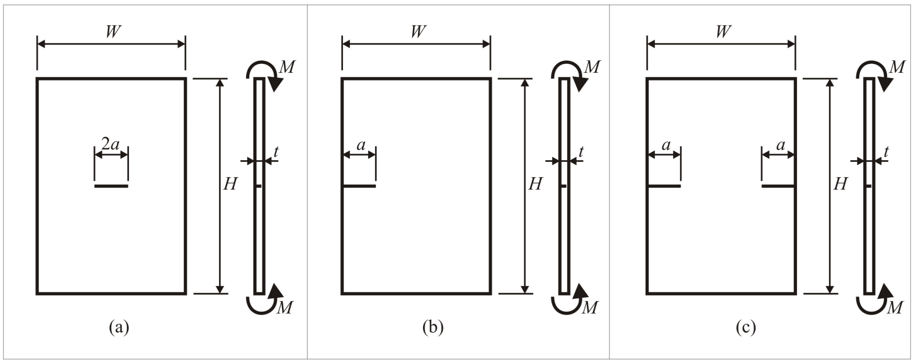


Figure 1: Geometry (a) Central cracked plate. (b) A single edge-cracked plate. (c) Double edge cracked plate.

This article presents an initial investigation of the stress intensity factor under out-of-plane bending using three geometries of a pre-existing through cracked plate. Additionally, the crack opening in three different thicknesses was studied. Three dimensional finite element analyses (FEA) were conducted, simulating the partial crack-face contact for elastic-linear material using FRANC3D and ABAQUS. The FEA and experimental results are compared for a single edge-cracked plate. The preliminary findings here should help to further work on this subject.

2 THREE DIMENSIONAL ANALYSIS

The finite element method is suitable for solving problems involving more complex plate geometries and loading. In this study, three dimensional elements are used to solve the partial crack closure problem in the plate under bending. The idealizations of the configurations of interest are shown in the Figure 1.

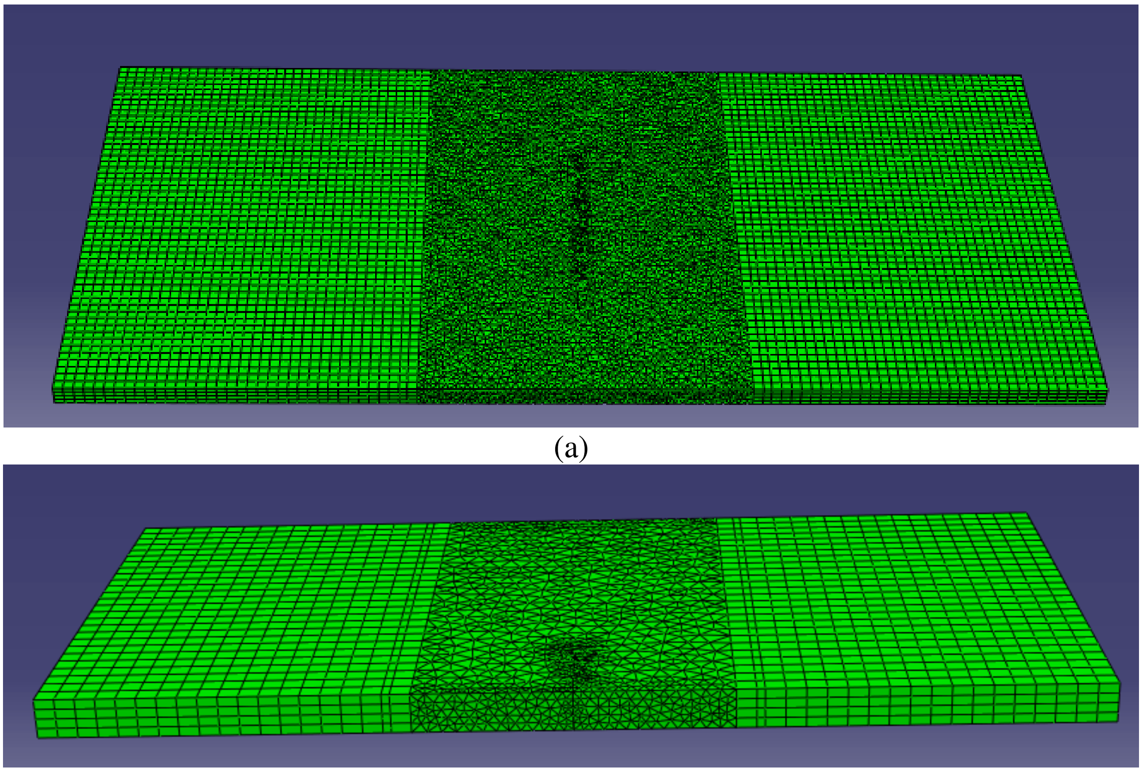
The width W of the plate is 130 mm; its height H is 90 mm, the thickness t should be much less than W and the crack's length a is 25 mm. The Young's modulus and Poisson's ratio were set at 205 GPa and 0.3, respectively. The geometries illustrated in Figure 1 were simulated for $t = 5$ mm, 10 and 20 mm (in this case, W / t = 26, 13 and 6.5, respectively). Additionally, the model with a single edge-cracked plate was analyzed for thicknesses t 5, 10 and 20 mm.

The model was separated into three regions, Figure 2 shows two example meshes for the center (a) and edge (b) crack plates. The elements in the region of the crack are C3D20, C3D10 and C3D15. And the elements in the uniform meshes at either end are C3D20R.

The number of the elements used in the crack tip for each thickness is shown in Table 1. Displacement correlation was used to determine stress intensity factor. The orientations of the local coordinates at the crack tip are shown in Figure 3.

$t\,[\text{mm}]$	Elements number on the front crack
5	8
10	18
20	26

Table 1: Elements Number on the front crack.



(a)

(b)

Figure 2: Mesh (a) Central cracked plate, t = 5 mm. (b) A single edge-cracked plate, t = 10 mm.

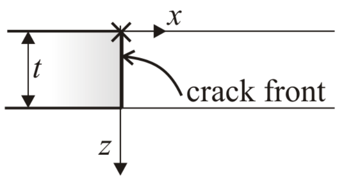


Figure 3: Coordinate around the crack tip.

The stress intensity factors are normalized as

$$Y = \frac{K}{\sigma\sqrt{\pi a}} \quad (1),$$

where σ is the characteristic stress and a is crack length.

The stress intensity factor solutions to the geometries illustrated in Figure 1 are calculated for a plate under extension loading; see Jansen, Zuidema and Wanhill (2006). The non-dimensional solution to centre cracked plate for $a/W \le 0.35$ is

$$Y^0_{(a)} = 1 + 0.256\left(\frac{a}{W}\right) - 1.152\left(\frac{a}{W}\right)^2 + 12.20\left(\frac{a}{W}\right)^3 \quad (2),$$

wich has an accuracy of 0.5%. The result for a single edge cracked plate is

$$Y_{(b)}^{0}=1.122-0.231\left(\frac{a}{W}\right)+10.550\left(\frac{a}{W}\right)^{2}-21.710\left(\frac{a}{W}\right)^{3}+30.382\left(\frac{a}{W}\right)^{4} \quad (3),$$

wich is accurate to 0.5% for $a/W \leq 0.6$. Lastly, the normalized stress intensity factor for the double edge cracked plate is

$$Y_{(c)}^{0}=\frac{1.122-1.122\left(\frac{a}{W}\right)-0.820\left(\frac{a}{W}\right)^{2}+3.768\left(\frac{a}{W}\right)^{3}-3.040\left(\frac{a}{W}\right)^{4}}{\sqrt{1-\frac{2a}{W}}} \quad (4),$$

with accuracy of 0.5% for any a/W. For these plates, the normalized stress intensity factors are presented in Table 2.

	Geometry	Y^0
(a)	Central cracked	1.09
(b)	Single edge cracked	1.35
(c)	Double edge cracked	1.15

Table 2: Normalized stress intensity factor for extension loaded, Y^0 .

The comparison of the stress intensity factors for various geometries with $t = 5$ mm is shown in Figure 4. When the crack faces are allowed to overlap (no contact considered), the stress intensity factor across the plate thickness is linear and is skew-symmetric about the mid-plane, as expected. When this effect is considered, the stress intensity factor in the crack tip shows a gradient through the thickness of the plate in the tension region and then is null where the surfaces are in contact, the compression region.

Due to closure, the maximum stress intensity factor for a cracked plate under bending is around 45 percent of stress intensity factor in the cracked plate under an extension loaded, see Table 3. Therefore, the crack-face contact has significant effect on the stress intensity factors.

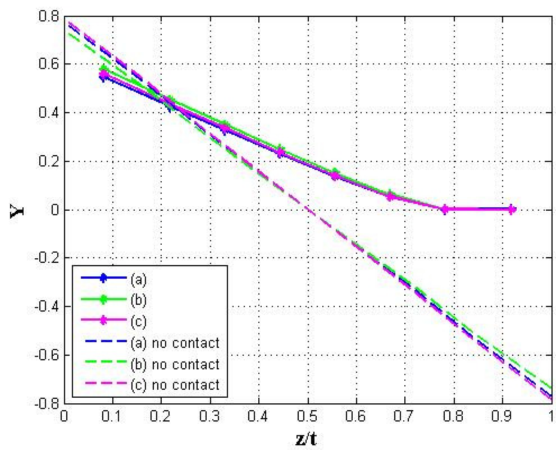


Figure 4: Normalized stress intensity factor along the crack front (t = 5 mm): (a) Central cracked plate. (b) A single edge cracked plate. (c) Double edge cracked plate.

For the same plate thickness, the contact region in the crack tip is coincident for all geometries and it corresponds to about 20% of the crack front length, this result is in agreement with numerical results by Alwar and Ramachandran (1983). There is a region of non-linear stress intensity factor in the transition between the opening crack and closed crack, due to non-linear behavior of the contact.

	Geometry	Y_{max}	Y_{max} / Y^0
(a)	Central cracked	0.55	0.50
(b)	Single edge cracked	0.58	0.43
(c)	Double edge cracked	0.56	0.49

Table 3: Ratio between Y_{max} and Y^0.

The comparison of the stress intensity factors for various geometries with $t = 10$ mm and $t = 20$ mm are shown in Figure 5 and Figure 6, respectively. These results were obtained for the contact and no contact case. As for the 5 mm plate, the stress intensity factors for the three geometries are close, and the contact region along the crack front is also coincident. For these two thicknesses, the contact region is around 30%.

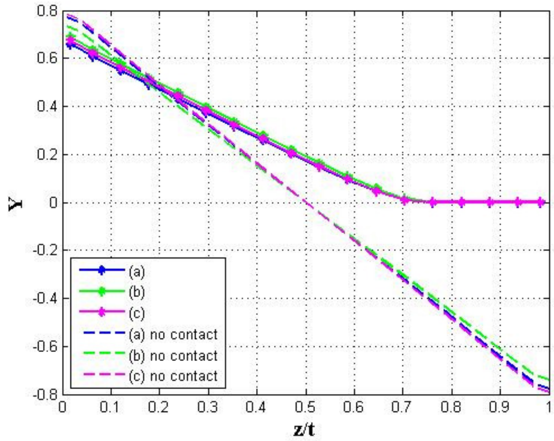


Figure 5: Normalized stress intensity factor along the crack front (t = 10 mm): (a) Central cracked plate. (b) A single edge cracked plate. (c) Double edge cracked plate.

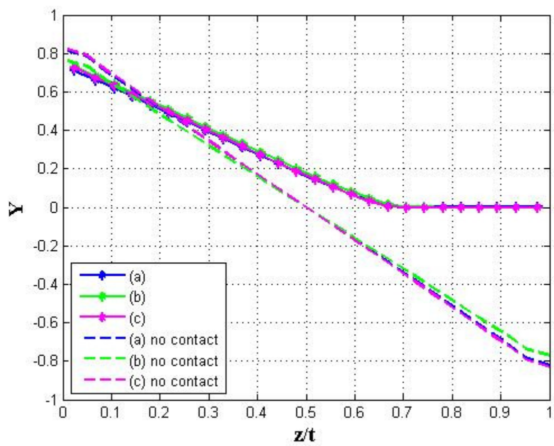


Figure 6: Normalized stress intensity factor along the crack front (t = 20 mm): (a) Central cracked plate. (b) A single edge cracked plate. (c) Double edge cracked plate

The comparison of the stress intensity factors for a single edge cracked plate with various thicknesses is presented in Figure 7. As noted in this Figure, the crack opening depends on the thickness. The opening of the crack front for $t = 20$ mm (68%) is lower than the opening for $t = 10$ mm (71%). Also, the opening for $t = 10$ mm is lower than the opening for $t = 5$ mm (80%). Due to closure, the stress intensity factor for a cracked plate under bending for 10 mm and 20 mm is reduced approximately 50 percent.

The magnitude of maximum stress intensity factor for $t = 5$ mm, $t = 10$ mm and $t = 20$ mm are close. Additional analyses should be performed to study the effects of thickness for thin plates.

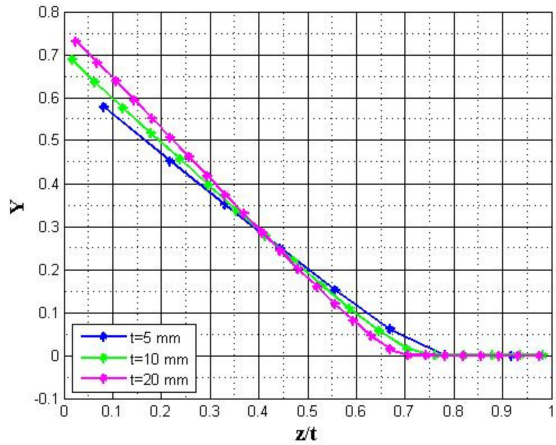


Figure 7: Normalized stress intensity factor for various thicknesses along the crack front of a single edge cracked plate.

3 THE EXPERIMENT

An experiment was designed to simulate pure bending of a plate. To that end, a steel plate with thickness of 5 mm was tested. A servo-hydraulic machine was used to do this experiment. The load applied is cyclic and sinusoidal with constant amplitude. In this experiment, a frequency of 5 Hz was used initially, which was then increased to 10 Hz. The rollers have lengths near the width of plate. The detailed sketch is illustrated in Figure 8.

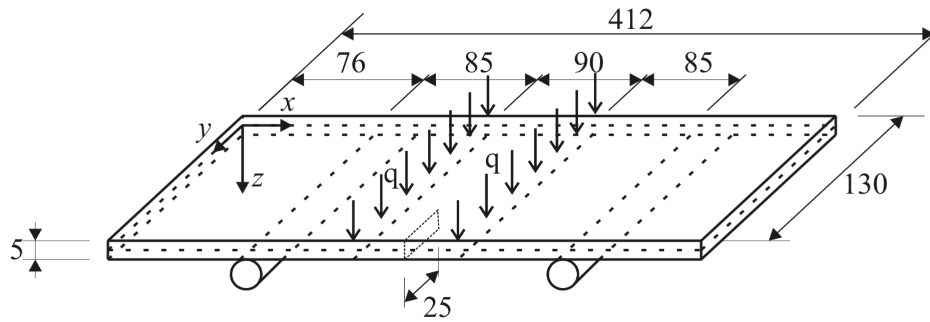


Figure 8: Detailed sketch of plate dimensions.

Assuming linear elastic fracture mechanics (LEFM), the characteristic stress is very low, so that the stress intensity factor should be near 10 MPa$\sqrt{\text{m}}$. As a consequence, the average load is 1.9 kN and the amplitude is 1.2 kN. The experimental setup is shown in the Figure 9.

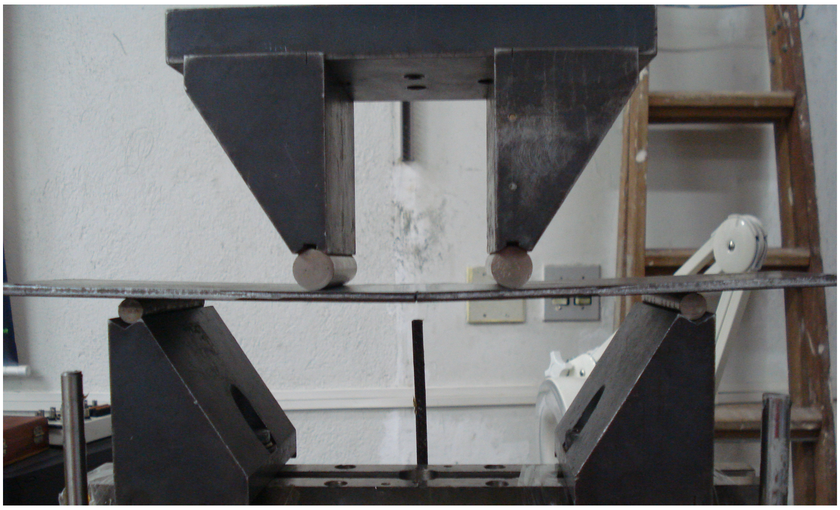


Figure 9: Experimental setup.

As found in the numerical simulations, the highest stress intensity factor K is at the tension surface. Consequently, the crack grows faster in the region with highest K, as shown in Figure 10. The through-the-thickness crack has become a two-dimensional crack.

In crack initiation, the crack opening is 70% of the thickness. Thus, the error between numerical and experimental result is 18%.

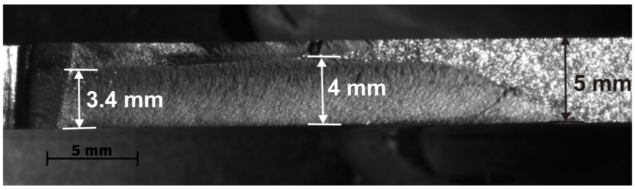


Figure 10: View of the growth crack.

4 SUMMARY AND CONCLUSIONS

Plates with a pre-existing through-the-thickness crack were studied under the action of a remote bending moment. The dependence of the contact region in the crack tip was investigated, as well as the bending effect on the stress intensity factor.

Two sets of numerical analyses were performed. One set uses different crack geometries for a constant plate thickness, and the second set uses a single crack geometry for three different plate thicknesses. The contact region length in the crack tip is smaller for thickness 5 mm. The contact region length is around 30% of the thickness for 10 mm and 20 mm.

The influence of the thickness in magnitude of the stress intensity factor was investigated. When this thickness is 5 mm, the maximum stress intensity factor is 45% of the extension stress intensity factor. However, for thickness 10 and 20 mm, the maximum stress intensity factor is 50% of extension stress intensity factor.

Comparisons were made also with experiments, the results agree. More experiments to study the influence of the thicknesses in this problem should be performed.

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