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SIMULTANEOUS DESIGN OF STRUCTURAL TOPOLOGY AND CONTROL FOR VIBRATION REDUCTION USING PIEZOELECTRIC MATERIAL

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Abstract. This article presents a research on the optimal design of actively controlled structures, aiming to suppress vibrations induced by external disturbances. Design is conducted simultaneously for the structural topology and actuator locations. A topology optimization problem is formulated for three material phases (two solid materials and void) with two design variables groups. A non-piezoelectric elastic isotropic material forms the structural only part of the design, while a piezoelectric material composes the active part. Since treating structural and control design variables in the same optimization framework is inconvenient, this work proposes a nested solution approach, where the actuator locations and controller syntheses are considered as sub processes included in the main optimization process dealing with the structural topology. The main optimization loop designs the structural variables, i.e., it is decided where there should be solid material and where there should be voids. The actuators are placed by an inner design loop within the structural problem, by a control law optimization that defines where the material should have piezoelectric properties. Numerical examples show that the approach used in this paper can produce a clear structural topology with a good actuators placement.

1 INTRODUCTION

The increasing demand for lightweight and adaptable structures to important applications spotlights the need of advanced structural optimization and control design methods. Several studies mention the use of smart structures designed by these methods. Such structures, by the distribution of integrated sensors and actuators, have a great capacity for self-monitoring and self-control via a control system. Thereby, the system can detect induced vibration modes and generate control forces to reduce structural vibration. One can mention many applications for such smart structures, especially in the aerospace industry, but also in robotic flexible manipulators and light land vehicles.

The development of efficient methods for design of smart structures is still a very promising field of research, despite great achievements during the past two decades. The wide application field, cited before, supports these researches and developments.

As part of structural design, the topology optimization method contributes effectively in the design of lighter structures, reducing various costs in the aerospace industry, such as high transportation costs. Furthermore, the reduction of raw materials also becomes important for environmental sustainability issues, so common nowadays.

Another argument remembered in the development of smart structures is the fact that these structures usually are light in weight and presents low internal damping (Wang et al. 1999). These characteristics are directly linked to the appearance of large amplitude vibrations, which are especially undesirable in the operation of precision machinery, such as space applications. Therefore, the use of an active vibration control system by automatic modification of its structural response is of great value. This control system requires actuators that generate control forces. Among these actuators one can cite piezoelectric ceramic actuators.

The piezoelectric phenomenon is a form of electro-mechanical coupling in which some crystals and polymers generate an electrical charge when pressed (mechanical strain), or *vice versa*. The use of piezoelectric ceramics as mechanical force generator in control systems and in mechanical actuators is already well known. In general, these applications use pieces of specific size and standard shape; pre-set placement of piezoelectric ceramics in topology optimization is another problem. An alternative to these design constraints is using a methodology that optimally distributes the piezoelectric material to maximize the performance of the ceramics.

The fields of structural optimization and optimal control had significant progress during the last decades. Each discipline has developed several theoretical and computational methods for their own proposals. Overall, the structural design precedes the control design, i.e., structural engineers define a layout in order to withstand the static and dynamic loads, and after that, the control engineers specify the control system using the structure pre-defined. In the field of structural design, topology optimization has emerged as one of the most effective tool, where the goal usually is to minimize the costs under some requirements of resistance and/or dynamic or maximizing the global structure stiffness, both by the optimal distribution of material. In relation to the control project, several theories have been proposed to reduce vibration in structures, among which one can mention the theories of classical, modern and optimum control; the latter being the most widely used.

Since the early 1990s, contrary to common practice, several theoretical works point that simultaneous design of structural optimization and control can reduce costs and increase performance, relative to the project in sequence.

Canfield and Meirovitch (1994) dealt with the design of a structure and its control system as a problem of multi-objective optimization. Pareto optimal designs generated for a simple beam demonstrated the benefits of solving the problem of integrated structural optimization and control. A composite objective function consisting of structural and control parts was developed by Ou and Kikuchi (1996), which is optimized by the method of topology optimization for steady state response. An independent modal space control algorithm (IMSC) is performed on the structure in order to reduce the transient response magnitude and, finally, the ideal placement for actuators, which are considered as punctual forces, is discussed. Wang et al. (1999) treated the structural design variables and the matrix of feedback gains as independent design variables. The mass and the control performance index were chosen as objective functions and constraints in the real part of eigenvalue, imaginary parts of eigenvalues of closed-loop system and actuator control forces are included. Thus the simultaneous optimization has become a multi-objective optimization. Liu and Begg (2000a, 2000b) discussed the optimal control, sensitivity analysis and optimization of an integrated intelligent structure. Five algorithms for simultaneous design based on sequential mathematical programming for and guided random search techniques have been presented and applied to a multidisciplinary optimization in which both the structural layout and the control parameters are involved. However, this study examined only truss structures. Zhu et al. (2002) investigated the simultaneous optimization for structural topology, actuator locations and control parameters of a plate actively controlled by a H_2 control law. Four rectangular piezoelectric actuators were used to suppress bending and torsional vibration in a clamped-free plate. Using a nested solution approach, they could treat the structural and control variables separately. Finally, the control variables were projected on a sub-optimization solving the Ricatti equations. An integrated optimization for smart trusses using genetic algorithms is carried out by Xu et al. (2007). In this work, several constraints were considered: stresses, displacements, natural frequencies and voltage on piezoelectric actuators. In addition, several objective functions were considered, besides the number and location of actuators and the number of controllable modes as design variables. Finally, Raja and Narayanan (2008) considered the optimization of a multidisciplinary *tensegrity* structure using genetic algorithms. A nested strategy was also used, in which the robust control norms H_2 and H_∞ were considered as objective function of the control system. Twist angles and locations of actuators, which are either discreet or continuous, were used as design variables. Moreover, the force generated by the electro-mechanical coupling of the piezoelectric actuator was used in the formulation.

A number of works concern about the placement of actuators and sensors. In general, some use a control performance index as objective function, while others use concepts of observability and controllability. Recently, Kumar and Narayanan (2008) considered the optimal location of sensor-actuator pairs placed on piezoelectric flexible beam using a controller based on a linear quadratic regulator (LQR). The LQR performance index was used as the objective function of the optimization problem, which was solved using genetic algorithms. In a unique work, Carbonari et al. (2007) studied the design, by topology optimization method, of piezoelectric actuators consisting of a flexible structure actuated by piezoelectric ceramics. In this kind of project formulation, the position of piezoelectric ceramic is usually held fixed in the design domain and only the flexible structure is designed by distributing some non-piezoelectric material. However, this work presents a formulation that allowed the simultaneous distribution of non-piezoelectric and piezoelectric material in the design domain in order to obtain some specifics actuation movements, achieving excellent results.

In view of what was presented, this paper aims to develop a new methodology for simultaneous design of structural topology and control for reduction of vibration using piezoelectric actuators. It proposes a procedure that uses the topology optimization method, and as consequence, finds the optimal actuators placement according to the optimal distribution of piezoelectric material, and automatically derives the optimal control. Moreover, unlike Carbonari et al. (2007), considers the three-dimensional structure, what enables the practical application of electrical potential difference across the piezoelectric material.

The works that consider the integrated design of structural optimization and control, in general, work with simple structures such as trusses, beams or even simple *tensegrity* structures. In contrast, this paper considers the topology optimization of thin three-dimensional structures in the simultaneous project. Moreover, proposing a renewal of the work of Ou and Kikuchi (1996), this article considers the use of piezoelectric material, as actuators of the control system instead of punctual forces. The piezoelectric ceramic actuators presents in thin structures considered in this work can be activated on the normal faces of the thinner dimension, which is the direction of polarization of piezoelectric ceramics.

As previously mentioned, the use of piezoelectric material in vibration control or in actuators design is carried out with pieces of standard size and with the positioning pre-defined, restricting the optimal solution. Therefore, based on work of Carbonari et al. (2007), this paper presents a formulation that allows the distribution of non-piezoelectric (elastic isotropic) and piezoelectric material in the project domain. This formulation contributes to a higher power of actuation in vibration control.

2 SIMULTANEOUS DESIGN OF STRUCTURAL TOPOLOGY AND CONTROL

Several authors (Salama et al. 1988, Milman et al. 1991) argue that the structural optimization and control combined can get better results than traditional sequential method. In sequential optimization, one tries to find the structural design variables (ρ_e) in order to minimize the structural objective under certain behavior constraints, given by $g_e(\rho_e) \leq 0$ and $h_e(\rho_e) = 0$. A formulation for this problem is

$$\begin{aligned} &\min_{\rho_e} J_e(\rho_e) \\ &\text{subject to } \begin{cases} g_e(\rho_e) \leq 0 \\ h_e(\rho_e) = 0 \end{cases} \cdot \end{aligned} \tag{1}$$

After having specified the structural design variables, the optimization control, considering ρ_e fixed, can be written as

$$\begin{aligned} &\min_{\rho_c} J_c(\rho_e, \rho_c) \\ &\text{subject to } \begin{cases} g_c(\rho_e, \rho_c) \leq 0 \\ h_c(\rho_e, \rho_c) = 0 \end{cases} , \end{aligned} \tag{2}$$

where J_c is the design criterion for the control and ρ_c are the design variables of control, with their restrictions. In the other hand, simultaneous optimization can be formulated as

$$\begin{aligned} &\min_{\rho_e, \rho_c} \ \alpha J_e(\rho_e, \rho_c) + \beta J_c(\rho_e, \rho_c) \\ &\text{subject to } \begin{cases} g(\rho_e, \rho_c) \leq 0 \\ h(\rho_e, \rho_c) = 0 \end{cases} , \end{aligned} \tag{3}$$

where α and β are the weights for the structural and control function respectively. The benefits of this formulation is that minimizing the sum of two separate objectives is always less than the sum of the minimization of the two individually (Ou and Kikuchi, 1996). This makes the simultaneous optimization attractive to engineers.

It is common to different works that the first paper investigating the integrated design for a structure and control system was conducted by Hale et al. (1985), in which an optimization problem is developed to handle spatial structures where both structural parameters and active control forces are determined in order to minimize a specific cost function. Miller and Shim (1987) worked with objective functions in order to reduce structural mass and strain, kinematics and control energy. Salama et al. (1988) established a precedent followed by many others. They eliminated the variables of structural control considering permanent gains (constants) and selecting specific weight matrices (identity) for the quadratic performance index. Milman et al. (1991) introduced the concept of combined objective function, and provided the necessary conditions for the optimal design of Pareto. Some recent works on simultaneous design has been cited in the introduction.

2.1 Objective function and constraints

Generally, the simultaneous design of structural optimization and control can be put as a nonlinear programming problem in which certain cost function, often multi-objective, is minimized with respect to structural (ρ_e) and control (ρ_c) parameters in the form

$$\min_{\rho_e,\rho_c} f\left(\rho_e,\rho_c\right). \tag{4}$$

If one assume that all minimization are able to find their respective global optimum, i.e., details of the search strategies, initial values of design variables, characteristics of the objective function, etc… are not considered, it is clear that Eq. (4) can be replaced by this nested approach

$$\min_{\rho_e}\min_{\rho_c} f\left(\rho_e,\rho_c\right). \tag{5}$$

Therefore, the original integrated optimization can be placed by a nested structural optimization with the control optimization as a sub-process (Zhu et al. 2002). Thus, one can write:

$$\min_{\rho_e} f_1\left(\rho_e\right), \tag{6}$$

where

$$f_1\left(\rho_e\right)=\min_{\rho_c} f_2\left(\rho_e,\rho_c\right), \tag{7}$$

refers to the control optimization using a certain control law. As shown in the above equations, the first optimization, Eq. (6), is the structural optimization for structural design variables, while the sub-optimization, Eq. (7), is the optimization of control over the control variables. Therefore, when the structural variables are changed, the sub-process is called and new control variables are calculated.

Since the structural and control optimization are treated separately in this nested approach, the various techniques for structural and control design can be combined and implemented without difficulties (Zhu et al. 2002). But it is important to note that the original optimization represented by Eq. (4), and the new optimization represented by Eq. (6) and (7), may converge to different optimal solutions; this happens because obtaining identical solutions depends strongly on the details of the strategies of searching, initial values of design variables, characteristics of the objective function, etc.

In this work the structural optimization can be defined as the minimization of the compliance, depending of structural design variables that define where, on the structure, should be put solid material (elastic isotropic or piezoelectric material) and where should be put voids. The control optimization will be based on minimizing the performance index of an LQR optimal control, depending of design variables that define the control regions where should have elastic isotropic material and where should be taken piezoelectric material (actuators). Furthermore, the control optimization is able to define the gain matrix **B** of LQR optimal control system. In order to solve the optimization problems this work uses the sequential linear programming (SLP).

2.2 Material model for simultaneous design with optimum placement of piezoelectric material

The material model for topology optimization proposed in this paper includes two solid, elastic isotropic material and piezoelectric material, and void. Therefore, the material model that defines the elastic properties, piezoelectric coupling properties and density is given by:

$$\begin{aligned} \mathbf{E} &= \rho_1^{p1}\left(\rho_2^{p2}\mathbf{E_2} + \left(1-\rho_2^{p2}\right)\mathbf{E_1}\right), \\ \gamma &= \rho_1^{p1}\left(\rho_2^{p2}\gamma_2 + \left(1-\rho_2^{p2}\right)\gamma_1\right), \\ \mathbf{d} &= \rho_1^{p1}\rho_2^{p3}\mathbf{d_2}, \end{aligned} \tag{8}$$

where **E**, γ and **d** define the effective properties of the interpolated material. $\mathbf{E_1}$ and $\mathbf{E_2}$ are the elastic properties of non-piezoelectric and piezoelectric material respectively, as well as, γ refers to densities; $\mathbf{d_2}$ defines the properties of electro-mechanical coupling for the piezoelectric material, being null for non-piezoelectric material. ρ_1 and ρ_2 are the pseudo-density defined in each finite element and will be directly related to the design variables of structural and control respectively. It is observed by examining Eq. (8) that: common elastic isotropic material is obtained when $\rho_1 = 1$ and $\rho_2 = 0$, piezoelectric material is obtained when $\rho_1 = 1$ and $\rho_2 = 1$, and void is obtained when $\rho_1 = \rho_2 = 0$. $p1$, $p2$ and $p3$ are the penalty coefficients that try to recover the presence or absence of material, piezoelectric or non-piezoelectric material and finally, the coupling properties, respectively.

This material model distributes elastic isotropic material and piezoelectric material optimally. As consequence, automatically sets the placement of piezoelectric actuators contributing to the optimization of the control system. A material model similar to that, have already been presented in Bendsøe and Sigmund (2003) and also in Carbonari et al. (2007), but was not used in designing control system.

3 STRUCTURAL DESIGN

As previously mentioned, the structural objective function that it is used in this work is the minimization of mechanical compliance (maximizing the global stiffness), which can be written as

$$f_1\left(\rho_1,\rho_2\right) = J_e = \mathbf{f}^{\mathbf{T}}\mathbf{u}\,, \tag{9}$$

where $\mathbf{f}^{\mathbf{T}}$ is a vector of static loads and **u** is the vector of global displacements of the structure generated by this loading. Although the compliance function depends on the design variable ρ_2 which defines where should have piezoelectric material, the design of structural optimization provides only the minimization of compliance on the variable ρ_1, which defines the presence or absence of solid material (piezoelectric or non-piezoelectric.) Thus, one can write

$$\min_{\rho_1} J_e(\rho_1,\rho_2)$$
$$\text{subject to} \begin{cases} 0<\rho_{1i}\le 1 \quad (i=1,2,3,\ldots,n_1) \\ V_1=\dfrac{\int_\Omega \rho_{1i}\mathrm{d}\Omega}{\int_\Omega \mathrm{d}\Omega}\le V_1^{\max} \end{cases} \tag{10}$$

where ρ_1 is a vector of structural design variables, ρ_{1i} is the i^{th} element component and n_1 is the number of structural design variables (equal to the number of finite elements.) The second restriction limits the total volume of material (piezoelectric or common) to a volume fraction $V_1^{\max}$ pre-established.

4 SYSTEM CONTROL DESIGN

The objective function for the control system used in this work is the minimizing of the performance index of an LQR optimal control, given by:

$$f_2(\rho_1,\rho_2)=J_c=\frac{1}{2}\int_0^\infty\left(\mathbf{x}^\mathrm{T}\mathbf{Q}\mathbf{x}+\mathbf{j}^\mathrm{T}\mathbf{R}\mathbf{j}\right)\mathrm{d}t,, \tag{11}$$

where $\mathbf{x}$ is a vector of state variables, the vector of modal coordinates in this case, $\mathbf{j}$ is a vector of control forces, $\mathbf{Q}$ is a semi-positive definite matrix of weights for the state variables and $\mathbf{R}$ is a positive definite matrix of weights for the control forces. Therefore, the optimization of control can be written as:

$$\min_{\rho_2} J_c(\rho_1,\rho_2)$$
$$\text{subject to} \begin{cases} 0<\rho_{2i}\le 1 \quad (i=1,2,3,\ldots,n_2) \\ V_2=\dfrac{\int_\Omega \rho_{2i}\mathrm{d}\Omega}{\int_\Omega \mathrm{d}\Omega}\le V_2^{\max} \end{cases}, \tag{12}$$

where ρ_2 is the vector of design variables of control, ρ_{2i} is the i^{th} component of the vector referring to the element i, and n_2 is the number of design variables for control (equal to the number of finite elements). The second restriction limits the total volume of piezoelectric material to a volume fraction $V_2^{\max}$ pre-established.

5 SENSITIVIY ANALYSIS

First order optimization algorithms, such as the sequential linear programming, require sensitivities (derivatives) of objective and constraints with respect to design variables. In this case it was possible to derive analytical sensitivities.

5.1 Sensitivity of the material model

The sensitivities of the material model, described in the section 2.2, in relation to the design variables ρ_1 and ρ_2 are easily calculated, as follows:

$$\frac{\partial \mathbf{E}}{\partial \rho_1} = p1\rho_1^{p1-1}\left(\rho_2^{p2}\mathbf{E_2} + \left(1-\rho_2^{p2}\right)\mathbf{E_1}\right), \tag{13}$$

$$\frac{\partial \gamma}{\partial \rho_1} = p1\rho_1^{p1-1}\left(\rho_2^{p2}\gamma_2 + \left(1-\rho_2^{p2}\right)\gamma_1\right), \tag{14}$$

$$\frac{\partial \mathbf{d}}{\partial \rho_1} = p1\rho_1^{p1-1}\rho_2^{p3}\mathbf{d_2}, \tag{15}$$

and

$$\frac{\partial \mathbf{E}}{\partial \rho_2} = p2\rho_1^{p1}\rho_2^{p2-1}\left(\mathbf{E_2} - \mathbf{E_1}\right), \tag{16}$$

$$\frac{\partial \gamma}{\partial \rho_2} = p2\rho_1^{p1}\rho_2^{p2-1}\left(\gamma_2 - \gamma_1\right), \tag{17}$$

$$\frac{\partial \mathbf{d}}{\partial \rho_2} = p3\rho_1^{p1}\rho_2^{p3-1}\mathbf{d_2}, \tag{18}$$

where all the terms have already been presented.

5.2 Sensitivity of compliance

Compliance is defined as the work done by the external forces, given by Eq. (9). In order of simplicity the indices 1 and 2 will be neglected in the pseudo-density ρ_1 and ρ_2. By the adjoint method can be concluded that the compliance sensitivity in relation to a pseudo-density ρ can be written as:

$$\frac{\partial W}{\partial \rho} = -\tilde{\mathbf{u}}^{\mathbf{T}}\frac{\partial \mathbf{K}}{\partial \rho}\mathbf{u}, \tag{19}$$

where $\tilde{\mathbf{u}} \in \Re^n$ is any arbitrary constant real vector, $\mathbf{K}$ is the stiffness matrix and $\mathbf{u}$ is the displacement vector. This equation is in the form of an equilibrium equation and for flexibility one can get directly $\tilde{\mathbf{u}} = \mathbf{u}$. Remembering that the stiffness matrix of a piezoelectric material can be written as

$$\mathbf{K} = \begin{bmatrix} \mathbf{K_{uu}}(\mathbf{c}) & \mathbf{K_{uj}}(\mathbf{e}) \\ \mathbf{K_{ju}}(\mathbf{e}) & \mathbf{-K_{jj}}(\boldsymbol{\kappa}) \end{bmatrix}, \tag{20}$$

and that the compliance is related only to mechanical degrees of freedom, Eq. (19) can be rewritten as

$$\frac{\partial W}{\partial \rho} = -\mathbf{u}^{\mathbf{T}}\frac{\partial \mathbf{K_{uu}}(\mathbf{c})}{\partial \rho}\mathbf{u}. \tag{21}$$

Therefore, the compliance derivatives are easily calculated. It is important to note that these derivatives are "localized", i.e., only requires information at the element level, however, there is a hidden global effect because the displacement vector depends on the pseudo-densities of all elements (Bendsøe and Sigmund, 2003).

Finally, observing Eq. (13) and (16) when applied to Eq. (21), one can note the sensitivity of compliance with respect to design variables ρ_1 will always be negative, i.e., the addition of material, piezoelectric or not, decreases the compliance (increases stiffness). Moreover, the sign of sensitivity to design variables ρ_2 depends on the difference between the values of the matrices of elastic properties of both materials (isotropic elastic and piezoelectric).

5.3 Sensitivity of performance index of the control system

Since the behavior of the control system depends heavily on the structural design variables and the feedback logic, a systematic sensitivity analysis is essential for the development of a well-behaved algorithm for solving a problem of this complexity. The sensitivities of the first order, for the control optimization part are given below.

According to Liu and Beeg (1998), the first order sensitivity of the quadratic performance index J with respect to a design variable ρ is given by

$$\frac{\partial J}{\partial \rho} = \frac{\partial J}{\partial \mathbf{A}} \otimes \frac{\partial \mathbf{A}}{\partial \rho} + \frac{\partial J}{\partial \mathbf{B}} \otimes \frac{\partial \mathbf{B}}{\partial \rho} + \frac{\partial J}{\partial \mathbf{S}} \otimes \frac{\partial \mathbf{S}}{\partial \rho}, \tag{22}$$

where "$\otimes$" means the sum of all products of two entries for each matrix and

$$\frac{\partial J}{\partial \mathbf{A}} = \mathbf{PL}, \tag{23}$$

$$\frac{\partial J}{\partial \mathbf{B}} = -\mathbf{PLS}^{\mathrm{T}}\mathbf{G}^{\mathrm{T}}, \tag{24}$$

$$\frac{\partial J}{\partial \mathbf{S}} = -\mathbf{G}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\mathbf{PL} + \mathbf{G}^{\mathrm{T}}\mathbf{RGSL}. \tag{25}$$

The sensitivities in relation to design variables can be obtained after by the Leibnitz's chain rule. Thus, one can obtain:

$$\frac{\partial \mathbf{A}}{\partial \rho} = \begin{bmatrix} 0 & 0 \\ -\mathrm{diag}\left[\dfrac{\partial \omega_i^2}{\partial \rho}\right] & -2\mathrm{diag}\left[\xi_i \dfrac{\partial \omega_i}{\partial \rho}\right] \end{bmatrix}, \tag{26}$$

$$\frac{\partial \mathbf{B}}{\partial \rho} = \begin{bmatrix} 0 \\ \dfrac{\mathbf{\Phi}^{\mathrm{T}}}{\partial \rho}\mathbf{B}_0 + \mathbf{\Phi}^{\mathrm{T}}\dfrac{\partial \mathbf{B}_0}{\partial \rho} \end{bmatrix}. \tag{27}$$

as **S** was defined as a unitary matrix, it can be written

$$\frac{\partial \mathbf{S}}{\partial \rho} = 0, \tag{28}$$

And so the third term in Eq. (22) can be neglected. Finally, since the matrix $\mathbf{B_0}$ is directly related to region of the electro-mechanical ($\mathbf{K_{ju}}$) in the stiffness matrix, the sensitivity of $\mathbf{B_0}$ can be written as:

$$\frac{\partial \mathbf{K_{uj}}(\mathbf{d})}{\partial \rho} = \int_{\Omega_e} \mathbf{B_u^T} \ \frac{\partial \mathbf{d}}{\partial \rho} \ \mathbf{B_j} \ d\Omega_e \,, \tag{29}$$

where $\partial \mathbf{d}/\partial \rho$ is given by Eq. (15) and (18).

6 SUMMARY OF THE PROPOSED METHODOLOGY

For clarity sake, the methodology proposed in this paper can be summarized in 11 steps:

1. Data entry (optimization and FEM);
2. Assemble elements neighborhood table for the sensitivity filters;
3. Start SLP loop;
4. Solve a static problem for the external loads;
5. Calculate the sensitivities of the structural problem and apply the filter to this data;
6. Solve the compliance minimization problem for the structural pseudo-density;
7. Solve a modal problem for the desired number of modes;
8. Calculate the sensitivities of the control problem and apply the filter to this data;
9. Solve the problem of minimizing LQR control system index;
10. Check the convergence:
    If converged, go to step 11;
    Else, recalculates the move limits and return to step 3 with the new pseudo-densities;
11. Exit and plot.

7 RESULTS

In order to verify the performance of the proposed simultaneous design presented in this paper, this section shows some results. The material properties of the elastic isotropic material (aluminum) and piezoelectric material (PZT5A) are presented in Table 1. A single cantilever beam and a beam fixed at both ends are analyzed (Figure 1). In both cases, the shaped beam structures, with measures 800 x 100 x 10 [mm^3] are discretized into 3200 finite element (160 x 20 x 1).

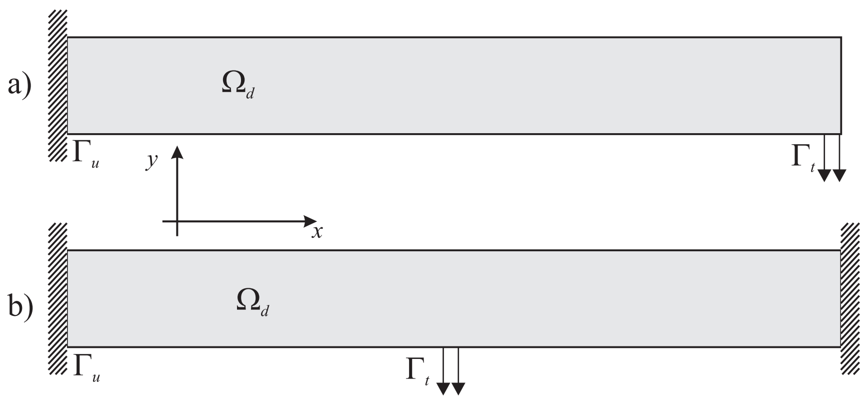

Figure 1: Cantilever beam and beam fixed at both ends

The parameters used in design optimization are the same for all simulation cases analyzed. The amount of volume constraints are equal to 50% and 5% for the total volume restriction and piezoelectric volume restriction, respectively; the initial values of the pseudo-densities ρ_1

and ρ_2 are uniform for all elements and equal to 0.40 and 0.04, respectively, in this way, the optimization problem starts just below the feasible limit. The weighting matrix of the quadratic performance index of the control system **Q** and **R** are set equal to the identity matrix. Finally, it was tried to control the first six vibration modes of each case. The results are shown below.

PZT5A	
elastic constants	(10^{10} N.m^{-2})
c_{11}^{E}	12.1
c_{12}^{E}	7.54
c_{13}^{E}	7.52
c_{33}^{E}	11.1
c_{44}^{E}	2.11
c_{66}^{E}	2.26
piezoelectric constants	(C.m^{-2})
e_{31}	-5.4
e_{33}	15.8
e_{51}	12.3
dielectric constants	
$\varepsilon_{11}^{S}/\varepsilon_0$	916
$\varepsilon_{33}^{S}/\varepsilon_0$	830
Density	7750 kg.m^{-3}

Aluminum	
Young's modulus	71x10^9 N.m^{-2}
density	2700 kg.m^{-3}
Poisson's ratio	0.33

Table 1: Material properties.

7.1 Cantilever beam

This sample is divided in two sub-cases. In the first one, the vibration modes are calculated normally, i.e., considering only the cantilever constraint. In the second, besides fixing the base of the beam, it is considered that the degrees of freedom in *z*-direction are restricted, allowing only movements in the *xy* plane. The value of the static vertical load on the tip of the beam is set equal to 1000 [N], which is applied in four nodes.

The first six vibration modes for the case considering free the degrees of freedom in *z*-direction can be seen in Figure 2.

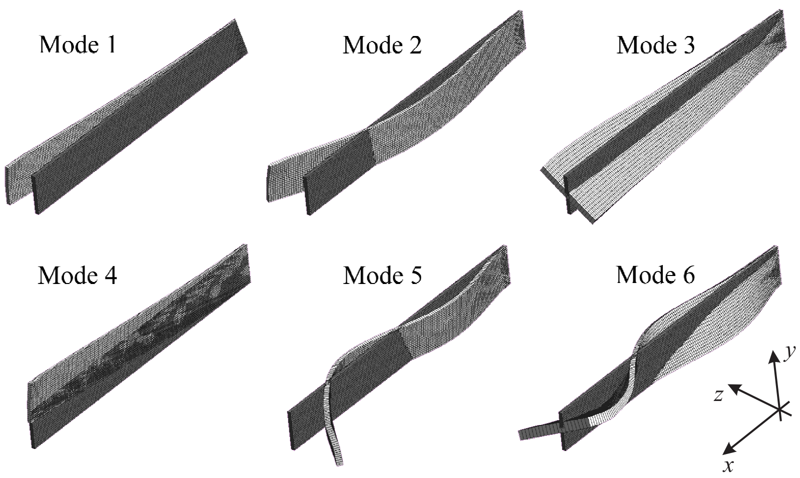

Figure 2: Structure and six modal shapes (*z* free).

The optimal topologies for the distribution of solid material (elastic isotropic or piezoelectric) and the distribution of piezoelectric material (actuators) considering the first sub-case can be seen in Figure 3.

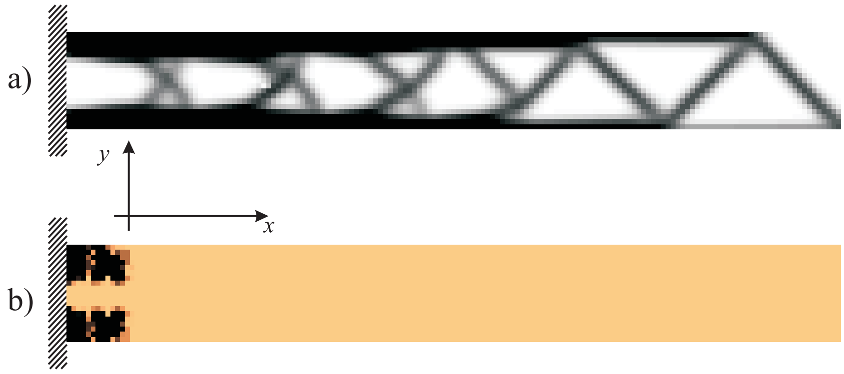

Figure 3: Optimal topologies (z free). a) ρ_1 b) ρ_2

The first six vibration modes for the case considering restricted the degrees of freedom in z-direction can be seen in Figure 4.

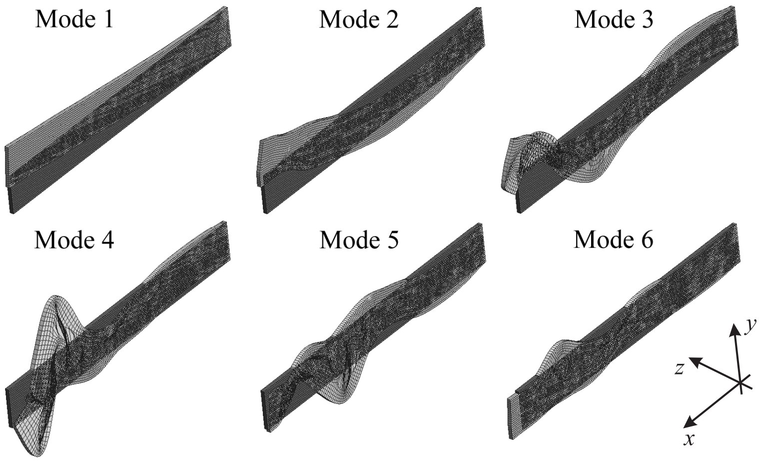

Figure 4: Structure and six modal shapes (z fixed)

The optimal topologies for the distribution of solid material (elastic isotropic or piezoelectric) and the distribution of piezoelectric material (actuators) considering the second sub-case can be seen in Figure 5.

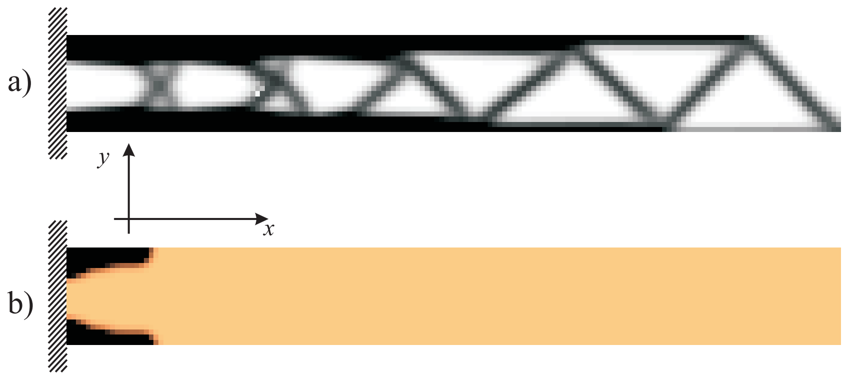

Figure 5: Optimal topologies (z fixed). a) ρ_1 b) ρ_2

It can be seen there is a little difference among the topology results presented in Figure 3 and 5. The distribution of piezoelectric material is better defined in the second sub-case. Moreover, there isn't a topology change in z-direction since the structure presents just one layer of finite elements in this direction. Consequently, it seems make no sense consider the lateral movement, and this will be done in the next examples.

7.2 Beam fixed at both ends

All degrees of freedom at the two ends of the beam are restricted in the three directions in this example. The degrees of freedom in z-direction are restricted in all nodes, also. A vertical force of 1000 [N] is applied in the central span of the beam. This case tries to verify the influence of the sensitivity filter used in topology optimization. Two sub-cases are analyzed. The first case considers the same minimum radius of filtering (r_{min} = 0.012 [m]) for both pseudo-densities. In the second case, it was defined r_{min}^{1} = 0.011 [m] for the filter sensitivities regarding the structural analysis, which is related to the pseudo-density ρ_1 and it is set up a second r_{min}^{2} = 0.014 [m] for the filter of sensitivities regarding the control system analysis, which is directly related to the pseudo-density ρ_2.

The optimal topologies for the distribution of solid material (elastic isotropic or piezoelectric) and the distribution of piezoelectric material (actuators) considering the same r_{min} for the two filters are showed in Figure 6.

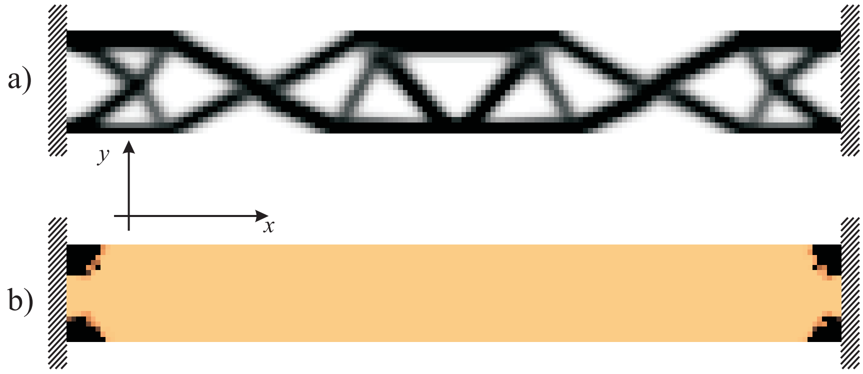

Figure 6: Optimal topologies ($r_{min}^{1} = r_{min}^{2}$). a) ρ_1 b) ρ_2

The optimal topologies for the distribution of solid material (elastic isotropic or piezoelectric) and the distribution of piezoelectric material (actuators) considering different r_{min} for the two filters are showed in Figure 7.

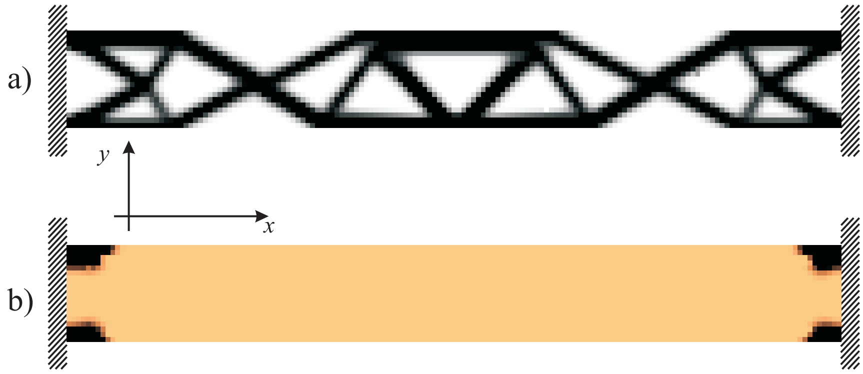

Figure 7: Optimal topologies ($r_{min}^{1} \neq r_{min}^{2}$). a) ρ_1 b) ρ_2

Despite the change in the structure (Figure 7a), which is already expected, due to decrease in the filtering radius, there is strong resemblance of the distribution of piezoelectric material (Figure 7b) in both cases. However it is already expected since the change in radius is small. More studies need to be done to infer the influence of the radius of sensitivities.

8 CONCLUSION

This work presents a new methodology of simultaneous structural and control design. The optimization is based in a nested approach that separates the two design procedures inside a SLP loop. Some preliminary results were presented. Comparative results show the influence of changes in boundary conditions and filtering radius. Using this methodology, the structural topology, actuators placement and the control system are completely defined. Results concerning the structural vibration response will be investigated in future work. This approach has great potential for application in the design of smart structures, and deserves further study. It can easily be applied to complicated geometries.

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