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# PROGRESS IN MODELING SCALAR TRANSPORT IN 'COMPLEX' TURBULENT SHEAR FLOWS

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**Abstract.** *We are concerned with the development of a mathematical model for the turbulent scalar fluxes that can be used reliably in the prediction of 'complex' shear flows. These flows are defined as those whose turbulence structure is distorted by the imposition of rates of strain additional to the usual mean shear, or by body forces arising from buoyancy, streamline curvature or system rotation. The effects of the extra strain rates or body forces on the turbulent mixing processes are far greater than what might be expected from inspection of the relevant terms in the governing conservation equations. Eddy-viscosity models fail badly in predicting momentum transport unless modified in some ad-hoc way. Similarly, models based on Fick's or Fourier's law have proved inadequate for the case of scalar transport. This paper presents an alternative to these models; namely an explicit and algebraic model for the turbulent scalar fluxes that allows them to properly depend on the details of the turbulence and the imposed strain field. The model is applied to flows involving streamline curvature and is compared with predictions from other non-linear algebraic closures and with experimental findings.*

## 1 INTRODUCTION

'Complex' flows, by which it is meant turbulent shear flows whose structure is distorted by extra rates of strain or by interactions with an externally-imposed body-force field, are often encountered in Nature and in engineering practice. Examples include stratified flows in the atmospheric and aquatic environments, flows over curved surfaces such as aircraft wings and turbine blades and flows with system rotation such as those found in turbomachinery. Complex flows are also defined as 'flows that cannot be predicted using methods developed for simple shear layers' (Bradshaw, 1973). Specifically, the use of closure models based on the familiar eddy-viscosity hypothesis fails to capture the changes to the turbulence structure and their consequences on the mean-flow parameters. A classic example here is the case of a turbulent boundary layer developing over a curved surface. Rayleigh (1917) showed that the motion of a fluid element following a curved trajectory is stable if the radial gradient of angular momentum is positive away from the center of curvature, and unstable if the gradient is negative. The effects when the flow is turbulent are to diminish the rates of turbulent transport of heat, mass and momentum in the former case and enhance these in the latter case relative to plane flow values. In strong stabilizing curvature, turbulence is suppressed altogether and the flow becomes laminar-like in many of its features (Bradshaw, 1973). Used in the standard form developed with reference to plane-flow data, eddy-viscosity models fail to predict the response of the Reynolds stresses to streamline curvature unless modified in some ad–hoc way. Far better representation of the changes in the turbulence structure due to streamline curvature is obtained by solving modeled equations describing the conservation of each non-zero component of the Reynolds stresses (Gibson, et al. 1981). The success of these models stems in large part to their ability to account, with no recourse to closure assumptions, for the rate of generation of the individual stresses through the interaction of turbulence with the modified strain field. Algebraic stress models, obtained by simplification of the complete differential equations by local–equilibrium assumptions, also retain these terms and have thus proved adequate in some applications.

In contrast to the Reynolds stresses, comparatively little has been done on predicting the effects of streamline curvature on the turbulent scalar fluxes $\overline{u_i\theta}$. The conventional approaches (Fick's law for mass flux, Fourier's law for heat flux) have been to assume these fluxes to be proportional to the gradients of the scalar quantity being diffused:

$$-\overline{u_i\theta} = \frac{\nu_t}{\sigma_t}\frac{\partial\Theta}{\partial x_i} \tag{1}$$

where $\Gamma_t$ is the eddy diffusivity and $\sigma_t$ is the turbulent Prandtl or Schmidt number. This quantity is defined as the ratio of momentum to scalar diffusivities and is invariably assigned a constant value depending on whether the flow develops remotely from, or in close proximity to a solid wall. The assignment of a constant value to $\sigma_t$ implies that the effects of streamline curvature impact the Reynolds stresses and the scalar fluxes to the same extent. For the case of heat transfer, this outcome is not supported by the experimental evidence (Simon & Moffat, 1979, Gibson et al., 1982) which suggest that the heat fluxes are affected to a greater extent than the Reynolds stresses. In the experiments of Gibson et al., for example, the Stanton number on a mildly-curved convex surface fell by 18% of the flat-plate value while the drop in wall skin-friction coefficient amounted to only 10% of the corresponding plane value.

The problem with the simple gradient-transport model must stem, in part, from the absence from this model of an explicit dependence of the turbulent scalar fluxes on the mean strain field altogether, and, especially, the additional strain rates associated with streamline curvature. Such a dependence is required by the exact equations that govern the evolution of the turbulent scalar fluxes (§2). It would thus appear that the correct prediction of curvature effects would result simply by abandoning gradient-transport closures in favor of models derived from the evolution equations for the scalar fluxes. This approach was first explored by Gibson (1978) using a scalar flux model that has given satisfactory results for the analogous case of density stratified flows. The outcome of this study was most surprising in that it showed that the predicted Prandtl number actually *decreased* with increasing stability.

This paper presents a progress report on current research aimed at improving the prediction of turbulent scalar transport in 'complex' flows. Specifically, it addresses the question of whether adoption of explicit and non-linear models for the turbulent scalar fluxes would yield a response to stabilizing curvature effects that is more in line with the experimental findings. The models chosen were formulated following entirely different approaches to that of Gibson (1978). They are briefly described in the next section.

## 2 MODELS FOR $\overline{U_I \theta}$

The need for alternative models to the simple gradient-transport hypothesis becomes clear upon inspection of the exact equations governing the evolution of these quantities. For steady flows, these equations take the form:

$$
\begin{aligned}
\overbrace{U_k \frac{\partial \overline{u_i \theta}}{\partial x_k}}^{convection} = & - \overbrace{\frac{\partial}{\partial x_k}\left(\overline{u_k u_i \theta} + \frac{\overline{p' \theta}}{\rho}\delta_{ik} - \gamma \overline{u_i \frac{\partial \theta}{\partial x_k}} - \nu \overline{\theta \frac{\partial u_i}{\partial x_k}}\right)}^{diffusion} \\
& - \overbrace{\overline{u_k \theta}\frac{\partial U_i}{\partial x_k} - \overline{u_k u_i}\frac{\partial \Theta}{\partial x_k}}^{production} \\
& - \overbrace{(\gamma + \nu)\overline{\frac{\partial \theta}{\partial x_k}\frac{\partial u_i}{\partial x_k}}}^{dissipation} \\
& - \overbrace{\overline{\frac{p'}{\rho}\frac{\partial \theta}{\partial x_i}}}^{correlations} \qquad (2)
\end{aligned}
$$

where $\gamma$ is the molecular diffusivity, $\rho$ is the fluid density, and $p'$ is the fluctuating pressure.

The term on the left–hand–side represents the rate of transport of the turbulent scalar fluxes by the mean flow. The terms on the first line on the right of the equality sign represent the rate at which the scalar fluxes are transported by the turbulence. Retention of this two groups of terms in a model allows for the non–alignment of the fluxes with the local gradients of the scalar quantity being transported. It also allows for non–local effects in determining the level of the turbulent scalar fluxes and thus for the possibility of these being finite in regions of the flow where the transported scalar is uniformly distributed. However, retention of convection and diffusion yields a model which would require the solution of a set of differential transport equations which may be quite demanding of computer resources. Gibson (1978) related the transport terms of the scalar fluxes to those of the scalar variance and then invoked local-equilibrium assumptions to render the differential equations into equivalent algebraic expressions for the scalar fluxes. These expressions were implicit in the scalar fluxes and depended for their success on the validity of closure approximations used for the last term in Eq. (2). This term, which represents the correlation between the fluctuating pressure field and the gradients of the fluctuating scalars, is particularly difficult to model due to the absence of guiding experimental measurements. Existing models for this term have thus invariably been constructed not so much by rigorous analysis but, rather, by analogy to models for the equivalent terms in the Reynolds-stress equations. It would thus seem appropriate to lay the blame for the incorrect prediction of the effects of streamline curvature on the model for this term and to seek improvements through the adoption of an altogether different approach.

A guide in the development of a better alternative to algebraic stress modeling lies in the two terms in Eq. 2 which represent the rates at which the scalar fluxes are generated by the turbulent interaction with mean velocity and the scalar fields. These terms make substantial contributions to the balances of $\overline{u_i\theta}$, and thus for a model for the turbulent scalar fluxes to correctly capture the effects of streamline curvature it must at the very least contain an explicit dependence on the mean strain field.

Several proposals have been made to include an explicit dependence on the gradients of mean velocity in the model for turbulent scalar fluxes (e.g. Yoshizawa 1985, 1988). Here, we shall focus on three representative models that have been obtained from very different approaches.

The first of the models considered here is that of Rogers et al. (1989). These authors replaced the terms representing the time–change of the scalar fluxes and the fluctuating pressure–scalar–gradient correlations by a multiple of the scalar-flux vector. Then, by limiting consideration to homogeneous flows and by neglecting dissipation of $\overline{u_i\theta}$ obtained the following expression:

$$0 = -\overline{u_iu_j}\frac{\partial\Theta}{\partial x_j} - \overline{u_j\theta}\frac{\partial U_i}{\partial x_j} - C_D\frac{1}{\tau}\overline{u_i\theta} \tag{3}$$

where $C_D$ is a model coefficient and $\tau$ is the turbulent time scale. To obtain an explicit relation for the scalar fluxes, Eq. 3 is solved for $\overline{u_i\theta}$ resulting in the expression:

$$-\overline{u_i\theta} = O_{in}^{-1}\overline{u_nu_j}\frac{\partial\Theta}{\partial x_j} \tag{4}$$

where $O_{ij}^{-1}$ is the reciprocal of the determinant of the tensor $O_{ij}$, defined as:

$$O_{ij} = \frac{C_D}{\tau}\delta_{ij} + \frac{\partial U_i}{\partial x_j} \tag{5}$$

The coefficient $C_D$ was determined by curve-fitting the model expression to results from Direct Numerical Simulations obtained for a homogeneous shear flow with a passive scalar field. This resulted in the following formulation:

$$C_D = 18\left(1 + \frac{130}{PrRe_t}\right)^{0.25}\left(1 + \frac{12.5}{Re_t^{0.48}}\right)^{-2.08} \tag{6}$$

where $Re_t$ is the turbulent Reynolds number and $Pr$ is the molecular Prandtl number. No other model for the turbulent scalar fluxes contains dependence on this parameter whose influence on the turbulent transport processes becomes important only at low values of the local turbulent Reynolds number.

Rubinstein and Barton (1991) applied renormalization group analysis to compute anisotropic corrections to the gradient-transport model. This approach allows for the effects of the universal small scales of turbulence on the large scales to be systematically accounted for. The results are corrections to the simple gradient-transport expression that are linear in the mean velocity gradients. The final model reads:

$$-\overline{u_i\theta} = \left(C_1\frac{k^2}{\epsilon}\delta_{ij} + C_2\frac{k^3}{\epsilon^2}\frac{\partial U_j}{\partial x_i} + C_3\frac{k^3}{\epsilon^2}\frac{\partial U_i}{\partial x_j}\right)\frac{\partial \Theta}{\partial x_j} \tag{7}$$

where ($C_1, C_2$,$C_3$)=(0.118,−0.166,−0.076). The first term in this model is immediately recognizable as the gradient-transport model given by Eq. 1. The remaining terms introduce the dependence on the local rate of strain required by Eq. 2. The linkage between the scalar fluxes and the turbulence field is provided primarily by the scalar quantity $k$ rather than by the individual components of the Reynolds-stress tensor, as suggested by Eq. 2.

The Younis, Speziale & Clark (2004) model was developed by postulating a functional relationship based on the terms that appear in the exact equation and then using tensor representation theory to derive an explicit, algebraic model for $\overline{u_i\theta}$. For non-buoyant flows, and after the application of constraints, the resulting model reads:

$$\begin{aligned} -\overline{u_i\theta} &= C_1\frac{k^2}{\epsilon}\frac{\partial \Theta}{\partial x_i} + C_2\frac{k}{\epsilon}\overline{u_iu_j}\frac{\partial \Theta}{\partial x_j} \\ &+ C_3\frac{k^3}{\epsilon^2}\frac{\partial U_i}{\partial x_j}\frac{\partial \Theta}{\partial x_j} + C_4\frac{k^2}{\epsilon^2}\left(\overline{u_iu_k}\frac{\partial U_j}{\partial x_k} + \overline{u_ju_k}\frac{\partial U_i}{\partial x_k}\right)\frac{\partial \Theta}{\partial x_j} \end{aligned} \tag{8}$$

where the C's are dimensionless coefficients that were assumed to be constant. They were assigned the values ($C_1$, $C_2$, $C_3$, $C_4$) = (−0.045, 0.37, −0.0037, −0.023) obtained from the LES and DNS results of Kaltenbach, Gerz and Schumann (1994) for scalar diffusion in homogeneous turbulence with uniform shear and constant scalar gradients. Here, too, a term equivalent to the gradient–transport hypothesis is present in addition to terms that introduce the explicit dependence on the Reynolds stresses and the mean velocity gradients. There is an additional term, not present in either one of the two previous models, which contains the products of the gradients of mean velocity and the gradients of the scalar quantity. The need for such term is suggested by the rigorous analysis of Dakos & Gibson (1987) of the fluctuating pressure-scalar-gradient correlations in Eq. 2.

## 3 ANALYSIS

We first consider the case of plane homogeneous grid turbulence with a uniform cross–stream gradient of a mean scalar. The local level of the turbulent scalar flux is determined by both transport from upstream (something which cannot be captured by any algebraic model), and by the interactions between the decaying turbulence field and the scalar gradients at that particular point. The models ability to account for the latter effect can be seen from Fig. 1 where the predicted and measured streamwise evolution of turbulent eddy diffusivity are compared. This quantity is defined as:

$$D_{ij} \equiv -\overline{u_i\theta}/\frac{\partial\Theta}{\partial x_j} \tag{9}$$

The symbols are the measurements of Sirivat & Warhaft (1981) for two values of mean velocity and for several values of the uniform mean temperature gradient generated by two alternative methods. For completeness, the results obtained with the conventional gradient–transport model and with the model of Rogers et al. (1989) are also shown in Fig. 1. There is clearly little that distinguishes the models of Gibson (1978), Rogers et al. (1989) and Younis et al. (2004) from each other. All succeed to reproduce approximately the correct rate of decay of eddy diffusivity with streamwise distance. The trends are also captured by the model of Rubinstein & Barton (1991) but the absolute levels are underpredicted by this and by the gradient–transport model.

We turn next to consideration of the effects of streamline curvature on the turbulence structure. The response of the turbulence structure to curvature effects is obtained from the solution of the equations governing the conservation of the Reynolds stresses. These can be written as:

$$\begin{aligned}
\overbrace{U_k\,\frac{\partial\overline{u_iu_j}}{\partial x_k}}^{convection} = & -\overbrace{\left(\overline{u_iu_k}\frac{\partial U_j}{\partial x_k}+\overline{u_ju_k}\frac{\partial U_i}{\partial x_k}\right)}^{production}\\
& -\overbrace{\frac{\partial}{\partial x_k}\left[\overline{u_iu_ju_k}+\frac{1}{\rho}(\overline{p'u_i}\delta_{jk}+\overline{p'u_j}\delta_{ik})-\nu\frac{\partial\overline{u_iu_j}}{\partial x_k}\right]}^{diffusion}\\
& -\underbrace{2\nu\overline{\left(\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}\right)}}_{dissipation}+\underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}}_{redistribution}
\end{aligned} \tag{10}$$

We focus on flows developing away from rigid walls and with turbulence Reynolds numbers high enough for viscous dissipation to be assumed isotropic. To simplify the analysis, the net (convective plus diffusive) transport of each component of $\overline{u_i u_j}$ is assumed to be proportional to the net transport of $k$ multiplied by the factor $\overline{u_i u_j}/k$ (Rodi, 1976). This, coupled with the assumption of local equilibrium, reduces the differential transport equations for $\overline{u_i u_j}$ into a set of algebraic equations for the relative stress levels. Of primary interest here are the normal and shear stress components in the plane of mean-flow curvature. For a flow with mean circumferential velocity U, following a curved path with radius of curvature r, these equations are obtained as:

$$\begin{aligned} \frac{\overline{u^2}}{k} &= \frac{2}{3}(1+2\Phi) + \frac{4S}{1-S}\Phi \\ \frac{\overline{v^2}}{k} &= \frac{2}{3}(1-\Phi) - \frac{4S}{1-S}\Phi \\ \frac{-\overline{uv}}{k} &= \sqrt{\frac{\beta}{1-S}\frac{\overline{v^2}}{k}} \end{aligned} \qquad (11)$$

The coefficients $\Phi$ and $\beta$ are defined by:

$$\begin{aligned} \Phi &= \frac{1-C_2}{C_1} \\ \beta &= \Phi\left(1-(\frac{2\overline{u^2}}{\overline{v^2}}-1)S\right) \end{aligned} \qquad (12)$$

and $S$ is the curvature parameter:

$$S = \frac{U/r}{\partial U/\partial y} \qquad (13)$$

The curvature parameter assumes the value of zero for a plane shear layer, negative values for a curved shear layer where the sense of curvature is destabilizing and positive values for a shear flow with stabilizing streamline curvature. The constants for the slow and rapid parts of the pressure–strain correlations model are assigned the values proposed by Gibson & Younis (1986) viz. $C_1$ = 3.0 and $C_2$ = 0.3.

It is expected that for strong stabilizing curvature turbulence would be extinguished entirely and the shear stress fall to zero (So & Mellor, 1973) at the critical value of the curvature parameter S. This behavior is reproduced by the present model as can be seen in Fig. 2 which shows the variation of the relative stress levels with S. The critical value of S is predicted as being 0.17 compared to a value of 0.15 implied by the So & Mellor data. Also plotted in Fig. 2 are the measurements of Champagne et al. (1970) obtained in a plane homogeneous free shear layer. These measurements were originally used to calibrate the coefficients of the model for the pressure-strain correlations. The sensitivity of the relative-stress levels to streamline curvature is mainly due to the curvature-related production terms that appear in the equations for the shear stress and the normal-stress components $\overline{u^2}$ and $\overline{v^2}$:

$$\begin{aligned} P_{\overline{uv}} &= -\overline{v^2}\frac{\partial U}{\partial y}\left(1-(2\overline{u^2}/\overline{v^2}-1)S\right) \\ P_{\overline{u^2}} &= -2\overline{uv}\frac{\partial U}{\partial y}(1+S) \\ P_{\overline{v^2}} &= 4\overline{uv}U/r \end{aligned} \tag{14}$$

From the above, the effects of positive S are clearly to diminish the shear stress and, consequently, the normal stress component $\overline{v^2}$. The effects on $\overline{u^2}$ are less clear in that the reduction in $\overline{uv}$ is counteracted by the contribution associated with S leading to values of this normal-stress component that are either unchanged from, or even somewhat higher than plane-flow values. Similar response to stabilizing body force was observed in the results of Speziale et al. (2000) for flow in a pipe rotated about its longitudinal axis.

An indication of the response of the thermal field to changes in S can be deduced by recasting the model of Younis et al. (2004) thus:

$$\begin{aligned} -\overline{u\theta} &= C_2\frac{k}{\epsilon}\overline{uv}\frac{\partial\Theta}{\partial y} + C_3\frac{k^3}{\epsilon^2}\frac{\partial U}{\partial y}\frac{\partial\Theta}{\partial y} + C_4\frac{k^2}{\epsilon^2}\frac{\partial\Theta}{\partial y}P_{\overline{uv}} \\ -\overline{v\theta} &= C_1\frac{k^2}{\epsilon}\frac{\partial\Theta}{\partial y} + C_2\frac{k}{\epsilon}\overline{v^2}\frac{\partial\Theta}{\partial y} + C_4\frac{k^2}{\epsilon^2}\frac{\partial\Theta}{\partial y}P_{\overline{v^2}} \end{aligned} \tag{15}$$

The explicit appearance of Reynolds–stress production terms in these expressions would suggest that the present model formulation is capable of directly reproducing the effects of streamline curvature on the turbulent scalar fluxes without recourse to ad–hoc modifications. In contrast, the gradient–transport model would have obtained the $\overline{u\theta}$ component erroneously to be zero while the vertical flux component $\overline{v\theta}$ would have been obtained from an expression equivalent to only the $C_1$ term above.

Of particular interest is the ratio of cross-stream to streamwise fluxes, and the turbulent Prandtl number implied by the various models. These are obtained by the Younis et al. model as:

$$\frac{\overline{v\theta}}{\overline{u\theta}} = \frac{C_1 + C_2\overline{v^2}/k}{C_2\overline{uv}/k - C_3\frac{1}{\overline{uv}/k(1-S)} - C_4\frac{\overline{v^2}/k}{\overline{uv}/k(1-S)}} \tag{16}$$

$$\sigma_t = \frac{(-\overline{uv}/k)^2(1-S)}{C_1 + C_2\overline{v^2}/k} \tag{17}$$

Both parameters exhibit an explicit dependence on the curvature parameter S. For the case of zero curvature, and after substituting for the model results for the relative stress levels, $\sigma_t$ is predicted by this model to be 0.69 which is in close accord with the consensus of experimental data from a number of inhomogeneous free shear flows (Launder, 1976).

The model of Rubinstein and Barton (1991) yields the following:

$$\frac{\overline{v\theta}}{\overline{u\theta}} = \frac{C_1}{C_2}\overline{uv}/k(1-S) \tag{18}$$

$$\sigma_t = \frac{1}{C_1}(-\overline{uv}/k)^2(1-S) \tag{19}$$

For S=0, the turbulent Prandtl number is obtained as 0.98.

The model of Rogers et al. (1989) yields:

$$\frac{\overline{v\theta}}{\overline{u\theta}} = \frac{\overline{v^2}/k}{\overline{uv}/k + \frac{2}{C_D}\frac{\overline{v^2}/k}{\overline{uv}/k(1-S)}} \tag{20}$$

$$\sigma_t = C_D\frac{1}{2}\frac{\overline{uv}/k}{\overline{v^2}/\overline{uv}}(1-S) \tag{21}$$

With the assumption of very large turbulent Reynolds number, this model predicts $\sigma_t$=2.0.

Gibson (1978), from a term-by-term modeling of Eq. 2 and by invoking local-equilibrium assumptions, obtained the following expressions for the ratio of heat fluxes and the turbulent Prandtl number:

$$\frac{\overline{v\theta}}{\overline{u\theta}} = \frac{\overline{v^2}}{\overline{uv}}\frac{\beta}{\phi_\theta\sigma_t + \phi'_\theta} \tag{22}$$

$$\sigma_t = \frac{\beta + a\phi'_\theta\phi'_{\theta 1}}{\phi_{\theta 1} - a\phi_\theta\phi'_{\theta 1}} \tag{23}$$

where a, $\phi_\theta$, $\phi'_\theta$ and $\phi'_{\theta 1}$ are functions of the model constants taken here as defined in the original reference.

The results obtained from the various models are plotted in Fig. 3. Shown there are the variation of the relative flux levels and the turbulent Prandtl number with S. All the models are seen to exhibit similar trends: the relative flux levels increase with increased stability. At the critical value of S, the ratio of $\overline{v\theta}/\overline{u\theta}$ becomes zero. This behavior is consistent with the experimental results of Gibson & Verriopulos (1984) obtained in a boundary layer developing over a convex-curved surface. The results for the turbulent Prandtl number are also similar for the various models – all of which show $\sigma_t$ as *decreasing* with increased stability. Thus the predicted behavior is directly opposite to the experimental observations irrespective of the approach adopted for modeling the scalar fluxes. There are, of course, limitations to the assumption of local turbulence equilibrium especially at high values of S (Girimaji, 1997). This would be equally true for the Reynolds stresses and the scalar fluxes. While the Reynolds stresses were obtained here by invoking local equilibrium (i.e. by setting $P_k = \epsilon$, where $P_k$ is the rate of production of $k$), none but one of the models for the turbulent scalar fluxes (that of Gibson, 1978) were derived by invoking the additional constraint of local equilibrium of the turbulent thermal field (i.e. by setting $P_{\overline{\theta^2}} = \epsilon_{\overline{\theta^2}}$, where $\overline{\theta^2}$ is the temperature variance). Departures from the local-equilibrium approximation cannot therefore provide a convincing explanation for the discrepancy between the predictions and the observed behavior. This inevitably leads to the conclusion that the effects of transport (both by mean flow and turbulence fluctuations) on the heat transfer rates in the experiments of Simon & Moffat (1979) and Gibson et al. (1982) are very substantial and are largely responsible for the decrease in $\sigma_t$ with increased stability. This does not necessarily mean that the non-linear explicit algebraic models would be unsuited for shear flows subjected to strong stabilizing streamline curvature but, rather, that assessment of these models can only be undertaken by actual computation of inhomogeneous curved shear flows in which the effects of transport are fully accounted for.

## 4 CONCLUDING REMARKS

A number of alternative, explicit, algebraic non–linear models for the turbulent scalar fluxes were used to assess the response of the turbulent Prandtl number to the effects of longitudinal streamline curvature. All the models tested predicted the turbulent Prandtl number to decrease with increasing stability implying that the heat-transfer rates are more sensitive to stabilizing curvature effects than the momentum transfer rates. This is opposite to what is indicated by experiments. That a number of different scalar flux models, derived through very different approaches, should perform so badly would suggest that the discrepancy may well be due more to the importance of transport effects in the experiments, rather than to fundamental defects in the models' examined. This hypothesis can be tested only by actual computations of curved inhomogeneous shear flows - an urgent task in view of the frequent occurrence of these flows in nature and in engineering practice.

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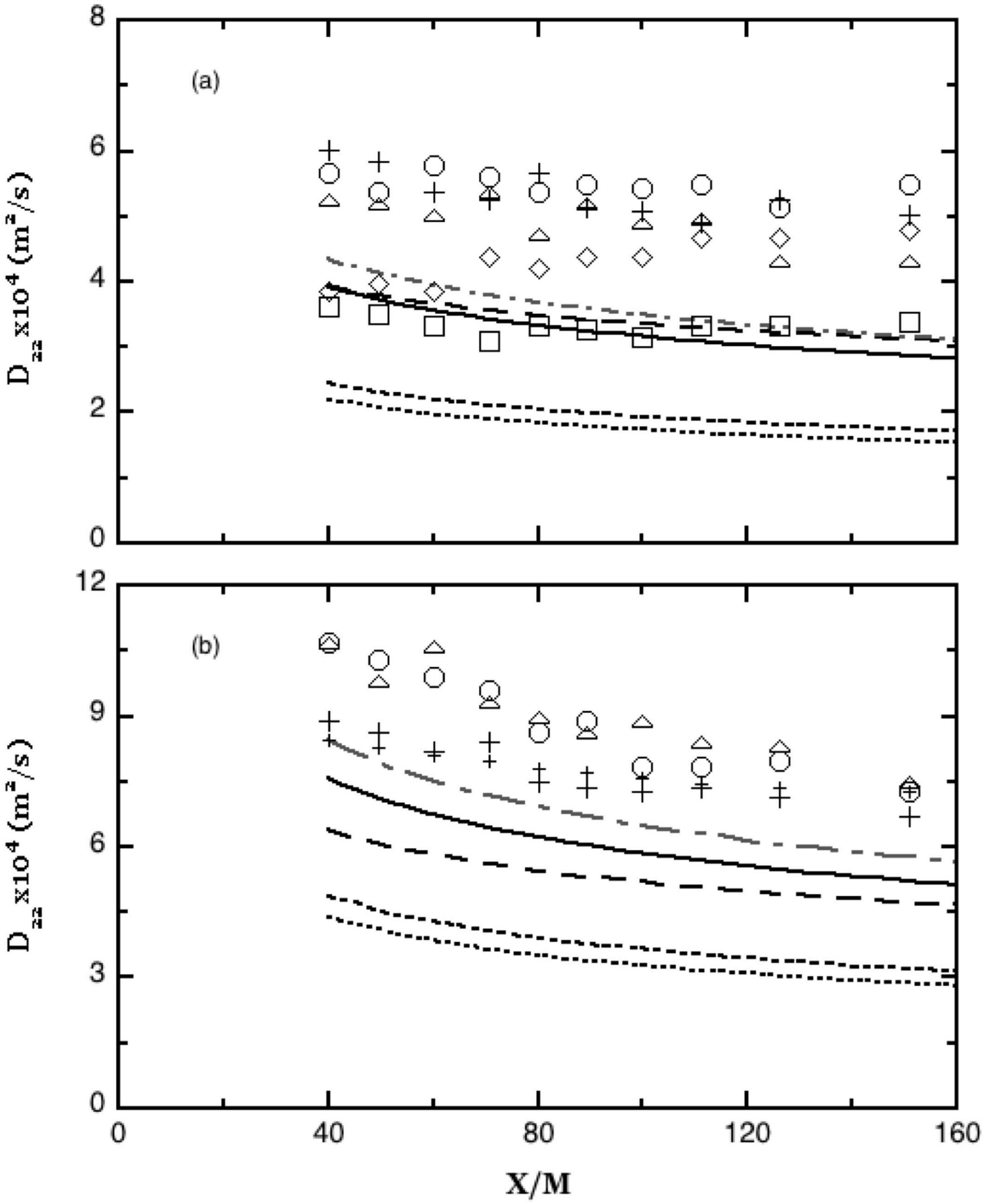


Figure 1: Predicted and measured evolution of thermal eddy diffusivity in decaying grid turbulence. Symbols are data of Sirivat & Warhaft (1983) for (a) U = 3.4 m/s; (b) U = 6.3 m/s. Predictions: ——-, Younis et al. (2004); – – –, Rogers et al. (1989); – . – . –, Gibson (1978); - - - Rubinstein & Barton (1991); . . . . Gradient transport.

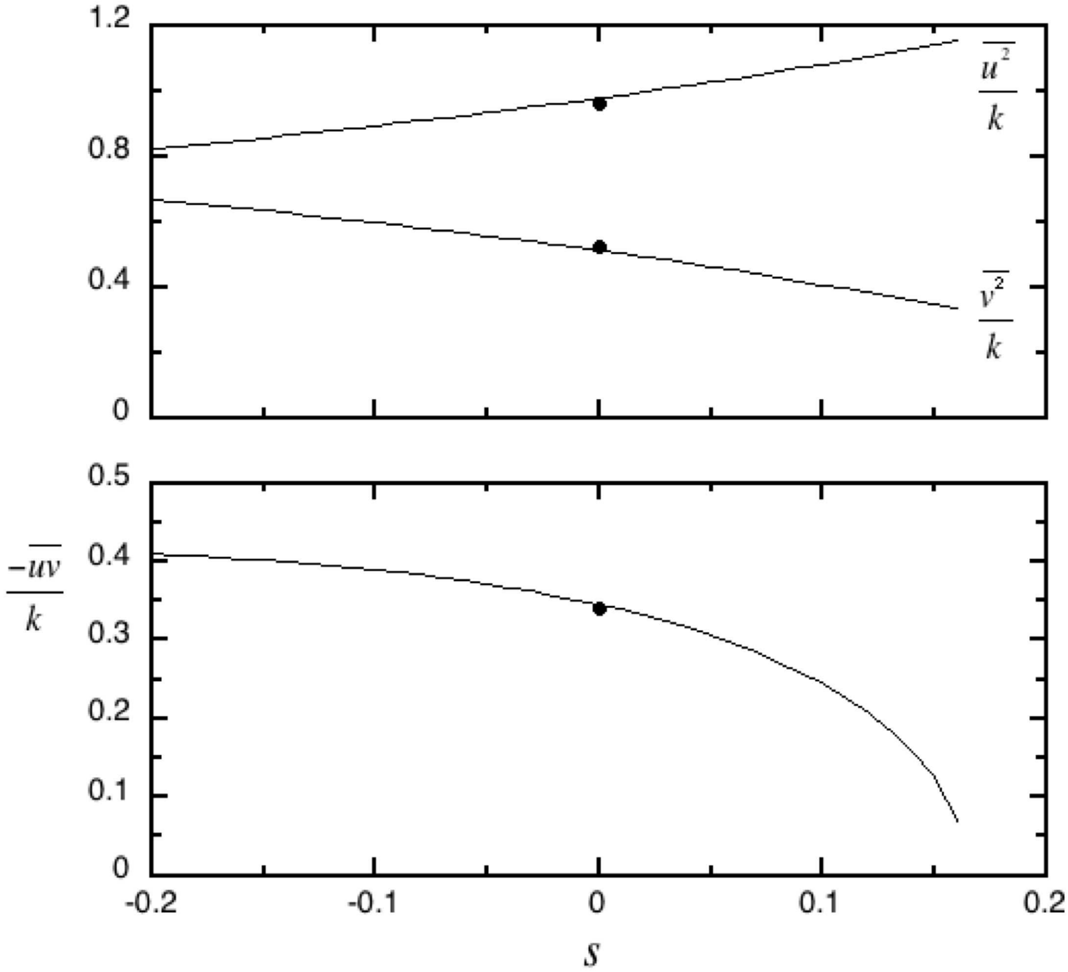


Figure 2: Predicted variation of relative stress levels with curvature parameter. ∘ Data of Champagne et al. (1970)

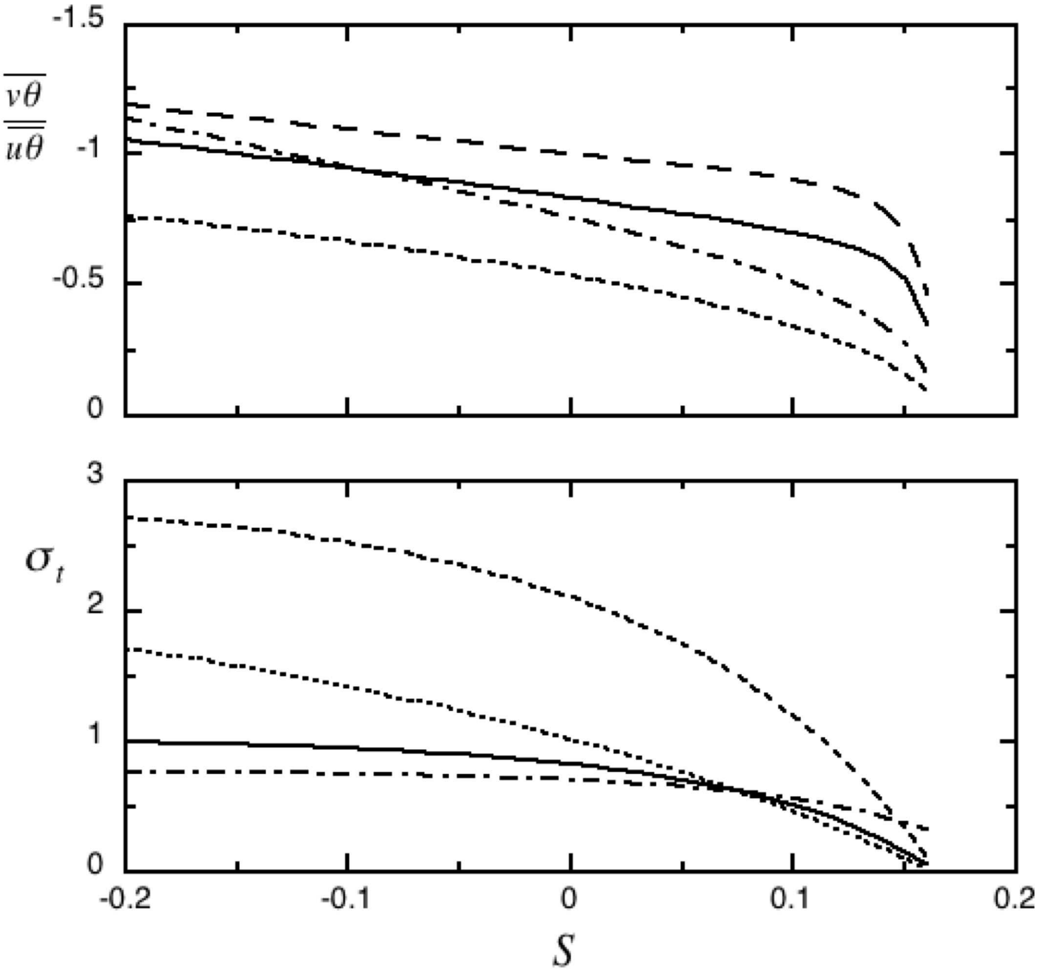


Figure 3: Variation of relative stress levels (top) and the turbulent Prandtl number with curvature parameter. Lines as in Fig. 1.