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OPTIMIZATION OF AIR COOLING CONDITIONS AND TEMPERATURE PROFILES IN A HARD CANDY COOLING TUNNEL

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Abstract. This work presents the optimization of the cooling stage during hard candy production process in order to assure the product quality. A one dimensional (1-D) mathematical model of the transient heat transfer on hard candies has been developed. The resulting PDAEs were converted to a set of non linear algebraic equations using the centered finite difference approximation (CFDM) and were implemented into the optimization environment GAMS (General Algebraic Modeling System). Thus, the temperature profiles in the center and surface of the candy, air cooling temperature and velocity are simultaneously optimized. The model proved to be robust and flexible, achieving convergence for the optimizations made. A study case is presented and discussed in detail. The obtained results show that higher cooling air temperature and lower air velocity increase the product quality considerably.

INTRODUCTION

In many food technological applications the unsteady convective heat transfer ocurring between a fluid flow and a solid food strongly influences on the product quality. The temperature transient in drying, cooling and heating processes on food products has come under strict regulatory control because it causes serious quality problems.

During last years, the application of the mathematical programming models as well as Computational Fluid Dynamic (CFD) and other advanced process modeling tools for simulations and optimizations in food industry have been received much attention (Sun, 2007). In fact, this technique can be efficiently used to understand the complex physical processes involved and to design and optimize different heat transfer operations in order to ensure security and quality in food products. Also nonlinear regression techniques are being applied to determine diffusion coefficients and kinetic parameters in unsteady-state heated and cooled food products which are essential for accurate estimates of food processing and safety (Bower, 2009). Moreover, CDF programs fills a big gap in modeling and optimization problems in food processing involving complex geometry foods, but is not the case presented here.

The goal of this paper is to optimize the operating conditions of the cooling stage in order to improve product quality. The optimization environment GAMS (General Algebraic Modeling System)**,** is used to implement and to solve the mathematical model. The election of using GAMS is due to the fact that this work is the first step of a more ambitious project which will finally lead to the simultaneous optimization of the whole production process of hard candies. According to this, the optimization model presented will be adequately expanded to optimize the design of the entire production process.

Trade-offs among variables describing the cooling process of Hard Candies and quality aspects are investigated. The model will provide important and useful relationships between cooling efficiency and operating conditions, which is crucial from the product quality point of view and operational mode of equipment as well. Potential problems associated with the cooling stage is that the already cooled formed candies present an uneven temperature transient which causes misshapen candies, and being this other cause of rejection.

In order to avoid that the wrapper candies continue cooling, causing misshapen candies, and being this other cause of rejection, the difference of temperature between the centre and the surface on candy should be minimized.

Briefly, the frequent problems connected with the cooling process of hard candies are:

- Deformation due to excess temperature at the exit of the tunnel.
- Fragility in the following step of wrapper because of the sharp cooling.
- Candies sticky between them when the belts velocities are not adequate.
- Stained candies when the belts are badly operated.

The paper is outlined as follows. The second section briefly describes production process for hard candy line as well as the cooling equipment. In the third section, Problem statement, the problem formulation, the assumptions, the mathematical model as well as the resolution procedure are introduced. Optimization results and discussion are presented in the forth section Results and Discussion. Finally, the conclusions are outlined.

1 PRODUCTION PROCESS OF HARD CANDY

During the manufacturing process, granulated sugar, glucose and water are heated in a steam heated vessel for the sugar to be dissolved and then transferred to a batch vacuum cooker and boiled to remove almost all water. Thus the syrup is turned into a candy dough.

The cooker consists of flash and vacuum chambers. Here, cooking temperatures are higher than 140 °C at atmospheric pressure. Then the cooker pressure is modified by applying a vacuum of at least 700 mm Hg during the last few minutes of the evaporation process in order to drive off excessive water and provide the hard candy with the desired humidity. The last cooking stage is the addition and mix of additives (acids, flavoring and coloring agents).

The next step is the dough tempering, where the candy mass is driven to a tempering stainless steel belt which is water refrigerated and then it is mixed to homogenize its temperature. After that, the "forming" process is achieved. Candies at 85 °C are shaped by cutting up a dough roll in a stamp-forming one-process. The formed hard candies are then cooled in a conventional cooling tunnel. Finally, hard candies are individually wrapped.

1.1 Cooling tunnel

Figure 1 illustrates the cooling tunnel. The tunnel has two air ducts (entrance and exit). As shown in Figure 1, the tunnel is composed of three conveyor belts [CB] which are mechanically driven by an engine connected to an adjustable frequency drive [AFD] to vary the residence time of candies.

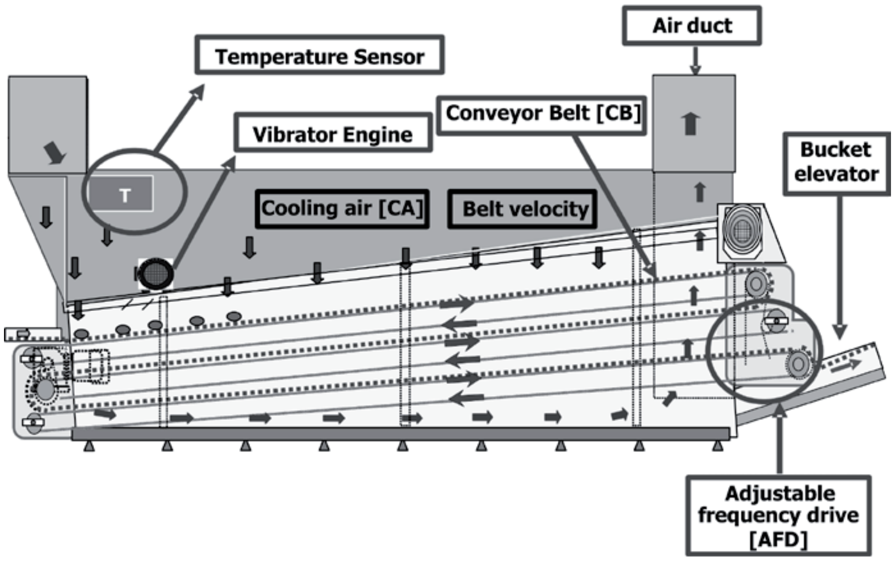

Figure 1: schematic cross-section of the cooling tunnel

2 PROBLEM STATEMENT

The main aim of this paper is to optimize the operating conditions of the cooling stage in the production process of hard candies. More specifically, the goal is to develop a non-linear programming (NLP) model in order to optimize simultaneously the unsteady temperature distribution inside candies and temperature, velocities and flow-rate of air cooling to operate the cooling tunnel.

Candy size and composition, ambient air temperature, heat transfer area and residence time of the candy inside the cooling tunnel are the main input data of the optimization model.

3 ASSUMPTIONS AND MATHEMATICAL MODEL

3.1 Assumptions

The main model assumptions can be summarized as follows:

- Candies are considered as homogeneous and isotropic spheres.
- Model 1-D. Temporal variations of the temperature in the radial direction are contemplated.
- Thermo-physical property variations with the temperature are neglected for the temperature range considered in this work.
- There is no moisture loss. Due to the low water content of hard candy (2.5%), water loss hardly occurs during candies cooling. According to this, moisture intake from air is also negligible, due to the fact that air humidity was monitored and profiles could be considered as constant between the values at tunnel entrance and exit.
- Changes in air temperature and humidity are small enough to produce a negligible effect on the thermal-physical properties of the air. Based on this assumption, the properties of air flow are calculated at the conditions of the dry air entering the system. It is assumed perfect mixture in the air, due to the fact that air has much higher rate than the candy flow and the operation experience.
- The external surface of each sphere is supposed to be surrounded by this cold air with constant properties. This assumption is based on the [0.5–3.0 m/s] range of air velocity (Zou et al., 2006).
- The convective heat-transfer coefficient is computed as the area averaged value of the local heat transfer coefficient (Becker and Fricke, 2004).

3.2 Heat transfer modeling

Based on the above assumptions, the energy balance of the unsteady heat transfer process in hard candies can be formulated as follows:

$$\frac{1}{\alpha_c}\frac{\partial T(r,t)}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} \qquad 0 < r < R \tag{1}$$

where (α) refers to the thermal diffusivity which is defined as follows:

$$\alpha = \frac{k_c}{\rho_c Cp_c} \tag{2}$$

where k_c, Cp_c and ρ_c refer respectively to the candy thermal conductivity, specific heat and density which are the most relevant model parameters for the analysis of food processes and process equipment design. Thermal diffusivity (α) defines how fast heat propagates or diffuses through a material (Singh, 1982), and it is generally affected by composition of the food product (Erdoğdu, 2008).

The boundary condition at the sphere surface (r=R) which is a third class condition is imposed by:

$$k_c \frac{\partial T(r,t)}{\partial r} + h\, T(R,t) = h\, Ta_T, \quad r = R \tag{3}$$

where h and Ta_T refer respectively to the heat transfer coefficient and the cooling air temperature. As indicated in Eq. (3), the heat arriving at the sphere surface by conduction is dissipated into the medium by convection.

As regards the condition at the center r=0, Eq. (4) refers to the adiabatic heat transfer condition by assuming that the product is evenly cooled:

$$\frac{\partial T(r,t)}{\partial r} = 0, \quad r = 0 \tag{4}$$

Heat transfer coefficient (*h*) is computed by the correlation developed by Dincer (1994), which is applicable for air cooling of spherical and cylindrical products:

$$h = \frac{Nu\ k_a}{D_C} = \left(1.56\ \mathrm{Re}^{0.426}\ \mathrm{Pr}^{1/3}\right)\frac{k_a}{D_c} \tag{5}$$

where k_a and D_c refer to the cooling air thermal conductivity and the candy diameter respectively. *Nu* is the Nusselt dimensionless Number.

Re and *Pr* are Reynolds and Prandtl dimensionless number respectively and are computed as follows:

$$\mathrm{Re} = \frac{\rho_a v_a D_c}{\mu_a} \tag{6}$$

$$\mathrm{Pr} = \frac{Cp_a \mu_a}{k_a} \tag{7}$$

where ρ_a, v_a, μ_a, Cp_a and k_a are cooling air density, air velocity, specific heat and thermal conductivity respectively. D_c refers to the candy diameter.

Finally, a uniform initial temperature condition is considered as follows:

$$T(r,0) = T_{inl} \qquad 0 \le r \le R \tag{8}$$

where T_{inl} refers to the inlet candy temperature.

3.3 Discretization of the heat transfer modeling

Equation (1), (3) and (4) were discretized using the central finite difference method (CFDM). The number of discretization points in time and radial direction have been chose equal to 11, defining:

$$\delta = \frac{R}{M} \quad ; \quad \Delta t = \frac{\theta}{N} \tag{9}$$

for the radial and temporal variations, respectively, with M=N=10.

The second-order accurate in time and in space ($0[(\Delta t)^2,(\Delta x)^2]$) is the chosen method and is applied as follows:

First derivatives: the two-point formulae was used for internal nodes, that means i and/or $j \neq 0$, 10. The three-point formulae was applied for boundary nodes, i or j = 0 or 10. Three or four-point formulae were useful to compute the first derivative at a node on the boundary by using more than two grid points on one side of the boundary in order to improve the accuracy of approximation (Ozisik, 1994).

Second derivatives: the central finite difference approximation is used at internal nodes (i and/or $j \neq 0$, 10), and the forward finite difference approximation is applied at the origin node, i = 0, at the boundary condition in the center of the sphere.

Thus, the following constraint computes the approximation of Eq. (1) for internal nodes:

$$\frac{T(i,j+1)-T(i,j-1)}{2\Delta t}=\alpha_c\frac{T(i-1,j)-2T(i,j)+T(i+1,j)}{\delta^2}+ \\ \frac{2}{(i-1)\delta}\left[\frac{T(i+1,j)-T(i-1,j)}{2\delta}\right],\ \mathrm{i:1,2,\ldots,9\ ;\ j:1,2,\ldots,9} \quad (10)$$

The following constraint computes the temperature variation of the internal nodes at final time:

$$\frac{T(i,j-2)-4T(i,j-1)+3T(i,j)}{2\Delta t}=\alpha_c\frac{T(i-1,j)-2T(i,j)+T(i+1,j)}{\delta^2}+ \\ \frac{2}{(i-1)\delta}\left[\frac{T(i+1,j)-T(i-1,j)}{2\delta}\right],\ \mathrm{i:1,2,\ldots,9\ ;\ j:10} \quad (11)$$

The following are the constraints related to the discretizations of Eq. (3) and (4) which are the boundary conditions at the surface and center, respectively:

$$k_c\frac{T(i-2,j)-4T(i-1,j)+3T(i,j)}{2\delta}+h\,T(i,j)=h\,Ta\ ,\quad i:10\ ;\ j:1,2,...,10 \quad (12)$$

$$\frac{T(i,j+1)-T(i,j-1)}{2\Delta t}=\alpha_c\frac{T(i-1,j)-2T(i,j)+T(i+1,j)}{\delta^2},\quad \mathrm{i:0\ ;\ j:1,2,\ldots,10} \quad (13)$$

The initial condition is imposed by:

$$T(i,j)=80\quad ,\quad \mathrm{i:0,1,2\ldots,10\ ;\ j:0} \quad (14)$$

The following constraint is imposed in order to ensure the glassy structure as well as to prevent quality problems (stickiness and deformation):

$$T(i,j)\le 34\ ,\ i:0\ ;\ j:10 \quad (15)$$

3.4 Objective function

In order to analyze the effects of main process parameters, it is important to highlight that not only the required final temperature is important, but also the temperature difference between the surface and the center (ΔT) in each instant of time. In fact, this difference should be minimized in order to avoid a sharp cooling which leads to fragility problems.

According to this, the goal of the paper is to determine the operating conditions which lead to lower and more uniform temperature differences between the center and surface of the candy along the cooling tunnel. Previous results (Reinheimer et al., 2010) showed that temperature difference (ΔT) is higher at the beginning of the cooling process. It reaches a maximum value and then it decreases slowly down. Based on this behavior, an objective function which minimizes the maximum difference is proposed:

$$Min\ OF1=Min\,[\max\Delta T]=Min[\max[T(i=0,j)-T(i=10,j)]] \quad (16)$$

Equations (5) to (16) are basically the model constraints that approximate the transient behavior of the cooling of candies.

The generalized reduced gradient algorithm CONOPT 2.041 was here used as NLP solver

(Drud, 1992). The optimization model involves approximately 140 variables and constraints. It should be noticed that global optimal solutions cannot be guaranteed due to the presence of bilinear terms and logarithms which lead to non-convex constraints.

Intel Core i5 M480 2.67 GHz processor and 8 GB RAM has been used to perform the optimization.

4 RESULTS

The proposed mathematical model was successfully validated (Reinheimer et al., 2010). For this reason the results of validation are not presented here. The model parameter values used for model optimization are listed in Table 1. Lower and upper bounds in some optimization variables are listed in Table 2.Next, the optimized results are discussed.

Parameter	Value
Hard candy composition [w/w %]:	0.276
carbohydrates	97.13
water	2.5
ash	0.18
Hard candy thermal conductivity, k_c [W/ m °C]	0.276
Hard candy thermal diffusivity, α_c [m^2/ s]	1.106 E-7
Hard candy radio, R [m]	0.008
Specific heat capacity of air, Cp_a [J/kg·°C]	1005
Residence time, Θ [s]	500

Table 1: Model parameter values.

Variable	Lower bound	Upper bound
Cooling air velocity, v_a [m/s]	1.2	3
Cooling air temperature, Ta_T [°C]	10	34

Table 2: Lower and upper bounds for the optimization variables.

Figure 2 and 3 show, respectively, the corresponding temperature difference at each instance of time and the temperature profiles at the center and surface of the candy. The optimal value of the variables are: Ta_T = 27.92 °C and v_a = 1.2 m/s (lower bound).

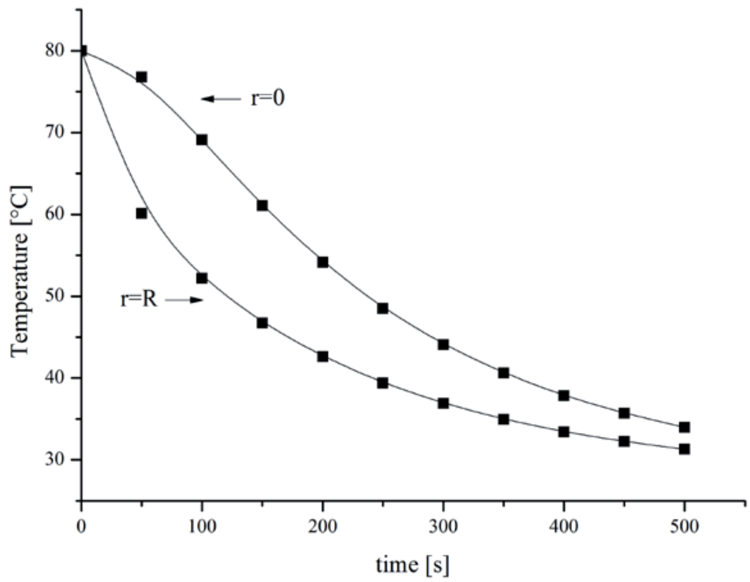

Figure 2: Temperature profiles vs. residence time

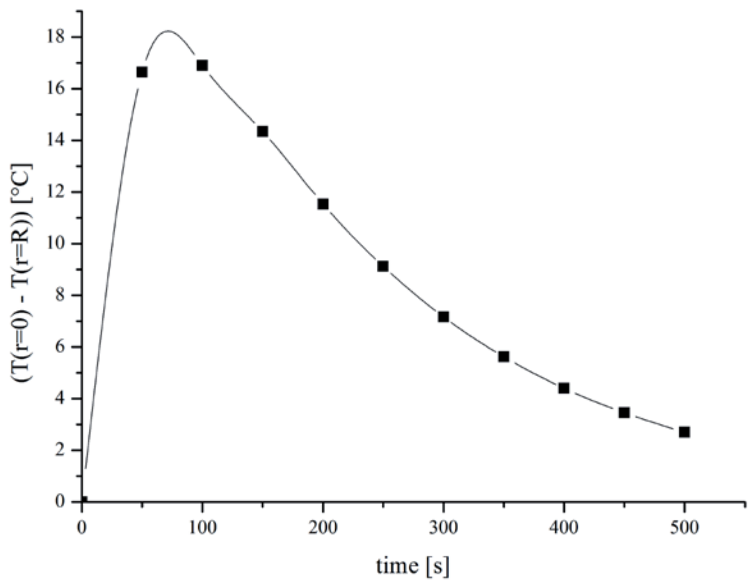

Figure 3: Temperature difference vs. residence time

Figure 2 illustrates optimal temperature profiles indicating the minimum residence time for each objective function in order to reach the maximum admissible temperature at the end of the cooling tunnel (≤ 34 ºC). As it can be clearly seen from Figure 3, the risk of product fragility predicted is high at the beginning of the cooling process because the temperature differences reach the maximum values and then they decrease slowly down. Despite of it, several objective functions have been tested and the presented results showed the best performance (more homogeneous temperature difference and therefore minimum temperature differences). These optimal temperature profiles have also been reproduced by another objective function which involves the minimization of the main operating variables of the cooling tunnel. The minimization of the ratio between v_a and Ta_T also guarantee the more uniform temperature profile.

It is important to remark here, that heterogeneous temperature distributions will always exist due to the high internal heat transfer resistance (low value of thermal conductivity) of hard candies.

Finally, it should be mentioned that scaling on variables and equations has been also implemented in order to facilitate the model convergence. Thus, the proposed problems were solved at a relatively low computational cost. Certainly, all solutions were obtained rapidly in at worst 0.12 CPU seconds after 12 iterations. The model proved to be robust and flexible, achieving convergence in all the several optimization made.

5 CONCLUSIONS

A mathematical model for predicting the temperature variation in hard candies during cooling was presented. The proposed model allows to determine the optimal operating conditions of the cooling tunnel in order to maximize the product quality by minimizing temperature differences in hard candies. A centered finite difference approximation (CFDM) was used to discretize the PDAEs into a set of non linear algebraic equations.

REFERENCES

Becker, B.R. and Fricke, B.A., Heat transfer coefficients for forced air cooling and freezing of selected foods. *International Journal of Refrigeration*, 27: 540-551, 2004.

Bower, J., *Statistical methods for Food Science: Introductory procedures for the food practitioner*, Wiley-Blackwell Publishing, 2009.

Dincer, I, Development of new effective Nusselt – Reynolds correlations for air – cooling of spherical and cylindrical products. *International Journal of Heat and Mass Transfer*, 37: 2781– 2787, 1994.

Drud, A., CONOPT-A Large Scale GRG Code. *ORSA Journal on Computing*, 6: 207-216, 1992.

Erdoğdu, F., A review on simultaneous determination of thermal diffusivity and heat transfer coefficient. *Journal Food Engineering*, 86: 453-459, 2008.

Ozisik, M.N., *Finite difference methods in heat transfer*. CRC Press, Boca Raton, Florida, 1994.

Reinheimer, M.A., Mussati, S., and Scenna, N.J., Influence of product composition and operation conditions on the unsteady behavior of hard candy cooling process. *Journal of Food Engineering*, 101(4): 409-416, 2010.

Singh, R., Thermal diffusivity in food processing. *Food Technology*, 36: 87–91, 1982.

Sun, D.W., *Computational fluid dynamics in food processing*, CRC Press, 2007.

Zou, Q., Opara, L.U. and McKibbin, R., A CFD modeling system for airflow and heat transfer in ventilated packaging for fresh foods: I. Initial analysis and development of mathematical models. *Journal of Food Engineering*, 77:1037-1047, 2006.