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ESTIMATES FOR TWO-PHASE NONLINEAR CONDUCTORS VIA ITERATED HOMOGENIZATION

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Abstract. The purpose of this work is to determine the overall electrical response of two-phase nonlinear conductors in terms of the electrical responses of its constituent phases and their geometrical arrangement. The focus is on material systems consisting of a random dispersion of micrometric particles in a continuous matrix, operating under stationary conditions.

Several strategies have been proposed by now to determine the overall response of material systems exhibiting a linear relation between the current density and the electric field. One particular strategy consists in identifying special classes of microgeometries that reproduce the essential geometrical features of the composite microstructure while at the same time allow the exact computation of the overall response via homogenization theory. These so-called "solvable microgeometries" include, for instance, laminates, composite sphere assemblages, and checkerboards. The works of Maxwell (A Treatise on Electricity and Magnetism, Clarendon Press, 1873) and Bruggeman (Ann. Phys. 416 (1935), pp. 636--664) showed that the set of solvable microgeometries could be enlarged by following iterative procedures whereby the constituent phases in a solvable microgeometry are themselves identified with solvable microgeometries at a lower length scale, thus producing hierarchical microgeometries of increasing complexity whose overall response can be determined via iterated homogenization. For instance, Maxwell constructed particulate composites with an isotropic response by iterating anisotropic laminated microgeometries; the resulting microgeometries are known as sequential laminates, and have been generalized by Tartar (Research Notes in Math. 125, Pitman Publishing Ltd., 1985) and others. Bruggeman, in turn, constructed particulate composites with arbitrary volume fractions of particles by iterating microgeometries with dilute volume fractions of particles; this approach is known as the differential scheme, and has been generalized by Norris et al. (J. Mech. Phys. Solids 33 (1985), pp. 525--543) and others. Sequential laminates and differential schemes have proved very useful in estimating the overall response of two-phase linear conductors and linear composites more generally.

The use of iterative schemes in the context of nonlinear conductors has remained limited, by contrast, mainly because iteration is encumbered by the fact that the overall response of nonlinear composites does not exhibit, in general, the same functional dependence on the electric field as the local responses. The first attempts to estimate the overall reponse of two-phase nonlinear conductors via sequential laminations are due to Ponte Castañeda (Phil. Trans. R. Soc. Lond. A 340 (1992), pp. 531—567), followed by the works of Li & Douglas (Q. App. Math. LIII (1995), pp. 433—464) and Hariton & deBotton (Proc. R. Soc. Lond. A 459 (2003), pp. 157—174). Other types of nonlinear solvable microgeometries have been found by Lukkassen and co-workers (see, for instance, Braides & Lukkassen, Math. Mod. Meth. Appl. Sci. 10 (2000), 47--71). While valuable contributions, these works were restricted to isotropic local responses and fairly limited classes of microstructures. In the present contribution we construct a much more general class of sequential laminates that can represent two-phase composites with general local responses ---isotropic as well as anisotropic ones--- and with fairly general particulate microstructures ---including isotropic and ellipsoidally anisotropic particle dispersions. This is achieved by following an iterative scheme recently pursued by Idiart (J. Mech. Phys. Solids 56 (2008), pp. 2599—2617) in the mathematically related context of viscoplasticity, which hinges upon the simultaneous use of a nonlinear differential scheme and sequential laminations. Since the resulting estimates are realizable, by construction, their predictions are guaranteed to satisfy all pertinent bounds, to exhibit the required convexity properties, and to agree exactly to second order with the small-contrast expansion of Blumenfeld & Bergman (Phys. Rev. B 40 (1989), pp. 1987—1989).

The resulting scheme is used to generate results for power-law materials and to study the effect of constitutive nonlinearity on the overall electrical response of two-phase conductors.