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WALL-LAYER TURBULENT HEAT TRANSFER MODELING FOR PERTURBED FLOWS

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Abstract. A wall-layer turbulent heat transfer model for the streamwise and wall-normal fluxes based on the Reynolds stresses is proposed. A pre-generated dataset obtained from a direct numerical simulation of a perturbed turbulent channel flow with heat transfer was used. The proposed model is based on a transformation of the Reynolds stresses using the wall-normal gradient of the mean temperature and the streamwise velocity. An *a priori* comparison of the new model, and different models proposed in the literature based on the standard, the generalized, and the non-linear gradient-diffusion hypothesis, with the direct numerical simulation dataset shows that the model performs well in comparison to these closures. It returns a quite reasonable *a priori* approximation for the streamwise scalar flux which is frequently under predicted.

1 INTRODUCTION

The prediction of turbulent heat transfer has lately received a significant amount of attention because turbulent heat modeling had been outdated in relation to the actual computational capacity (Nagano and Kim, 1988; Kim and Moin, 1989; Kasagi and Nishimura, 1997; Bataille et al., 2003; Qiu et al, 2008; Rossi and Iaccarino, 2009; Philips et al., 2011). In the past the prediction of turbulent heat transfer in applied problems frequently used the standard gradient-diffusion hypothesis (SGDH), in which the turbulent scalar fluxes are assumed to be proportional to the mean scalar gradient. However, this assumption has significant limitations, mainly in the prediction of the streamwise flux. An early improved alternative to this hypothesis was the generalized gradient-diffusion hypothesis (GGDH) proposed, among others at that time, by Daly and Harlow (1970) (and references in this study), in which the eddy diffusivity was calculated as a function of the Reynolds stress.

Based on these pioneering works many ideas and models have been proposed in the literature for the turbulent heat fluxes as a function of the Reynolds stresses (Kim and Moin, 1989; Launder, 2001; Younis et al., 1996; Abe and Suga, 2001). Kim and Moin in a numerical study on scalar transport based on the analysis of data from direct numerical simulation (DNS) of heat transfer in a turbulent channel flow, suggested using the Reynolds stress to predict the turbulent scalar fluxes, owing to the strong correlation, or similarity, between the streamwise velocity and the temperature, and therefore between the second moments of streamwise momentum and heat.

In fact the similarity or dissimilarity between the velocity and the temperature has been investigated in studies on turbulence with heat transfer, as researchers sought a new finding that could improve turbulent heat transfer prediction. Recently, new studies have been applied to turbulent heat transfer, addressing the velocity and temperature dissimilarity in perturbed turbulent flows (Suzuki et al., 1988; Kong et al., 2001). However, to date no work has addressed the dissimilarity between the Reynolds stresses and the turbulent heat fluxes aiming at turbulent heat flux modeling. In a recent study by the author (Pasinato, 2012), the dissimilarity in perturbed turbulent channel and plane Couette flows with heat transfer was numerically studied using DNS. In this study it is shown that the dissimilarity between the Reynolds stresses and the turbulent heat fluxes in perturbed flow is strongly dependent on the wall-normal gradient of the mean field dissimilarity (or differences between the mean streamwise velocity, U, and temperature, Θ), and the velocity-pressure gradient interaction, which is also ultimately defined by the wall-normal gradient of U.

Based on the results of this previous study, in the present work a RANS model for the wall-layer of a bounded turbulent flow is proposed. This model expresses the streamwise and wall-normal fluxes based on the Reynolds stresses and the gradient of the mean fields. A DNS database generated from this previous work for perturbed turbulent channel flow with heat transfer is used. An *a priori* comparison of the prediction of the new model with the DNS database is presented, in parallel with the prediction of other four explicit algebraic turbulent heat models selected from the literature.

In the following section, the four turbulent heat flux models used in the *a priori* comparison are presented together with the RANS wall-layer model presented in this study. Then in the next section the results of the *a priori* comparison of these models with DNS data and with the new model is discussed. Finally, in the last section, some conclusions are drawn.

2 HEAT FLUX MODELING

2.1 Algebraic models

In this section the focus is on algebraic models for heat transfer. Although the reference is made to 'heat transfer', temperature is considered a passive scalar therefore these models are also valid for mass transfer.

In this work a turbulent variable is considered the sum of a Reynolds-averaged mean and a fluctuation, as in $\hat{u} = U + u$, for the instantaneous streamwise velocity. Thus $U;\ V$ and $u;\ v$ are the means and fluctuations of the streamwise and wall normal velocities, respectively; and Θ and θ, the mean and fluctuation of temperature, respectively. All variables in this study are written in dimensionless form using the kinematic viscosity, ν, the velocity friction, u_τ, and the friction temperature $T_\tau = q_w/\rho\, c_p\, u_\tau$, where q_w is the heat flux at the wall, and c_p and ρ are the constant-pressure specific-heat coefficient and density, respectively; e.g. as in $y^+ = y u_\tau/\nu$ for the wall distance or $\epsilon^+ = \epsilon \times u_\tau^4/\nu$ for the dissipation of the kinetic energy of the turbulence or $\hat{\theta}^+ = (\hat{\theta} - \hat{\theta}_W)/T_\tau$ for the dimensionless instantaneous temperature. However for the sake of simplicity the symbols '$+$' is used only for dimensionless distances.

The simplest and easy-to-use turbulent heat flux model is that based on the *standard-gradient-diffusion-hypothesis* (SGDH)

$$\langle u_i \theta \rangle = -\alpha_t \frac{\partial \Theta}{\partial x_i}; \quad i = 1, 2, 3. \tag{1}$$

where α_t is the turbulent eddy-diffusivity which in most cases is evaluated through a turbulent Prandtl number, $Pr_t = \nu_t/\alpha_t$, where ν_t and α_t are the eddy-viscosity and eddy-diffusivity, respectively.

In this work the Pr_t is set to 0.85 as suggested in the literature (Launder, 2001), with the eddy-viscocity computed from the DNS dataset.

The use of a Pr_t to evaluate the heat fluxes in a turbulent flow using an eddy-diffusivity hypothesis is an extension to turbulence of the Reynolds analogy, which implies an analogy between the turbulent fluxes of streamwise momentum and heat. This approach is computationally efficient because turbulent heat transfer predictions are essentially obtained from the turbulent velocity field at relatively little additional computational cost; however it has received significant criticism mainly related with the non-constancy of the Pr_t and the non-alignment between the turbulent heat flux vector and the mean temperature gradient (Launder, 2001).

Another improved type of algebraic models for the Reynolds-averaged turbulent heat fluxes has been proposed in the literature by a number of workers, for example Daly and Harlow (1970) (and previous works cited in this reference), to express the correlation between the velocity and the scalar fluctuations through the following simple formula called *generalized-gradient-diffusion-hypothesis* (GGDH) model

$$\langle u_i \theta \rangle = -\text{constant} \times \langle u_i u_j \rangle \times \frac{\partial \Theta}{\partial x_k}; \quad i, j, k = 1, 2, 3. \tag{2}$$

The Eqn. (2) implies homogeneous turbulence and negligible contribution from convective transport, which is equivalent to invoking the hypothesis of *local-equilibrium* (Daly and Harlow, 1970; Launder, 2001; Qiu et al, 2008; Rossi et al, 2009), which means that all mean strain generation of the heat fluxes is removed by the pressure fluctuations.

In two-dimensions, the GGDH model is

$$\langle u\theta\rangle = -\alpha_{u\theta}\tau_\theta\left(\langle uu\rangle\frac{\partial\Theta}{\partial x} + \langle uv\rangle\frac{\partial\Theta}{\partial y}\right) \tag{3}$$

$$\langle v\theta\rangle = -\alpha_{v\theta}\tau_\theta\left(\langle uv\rangle\frac{\partial\Theta}{\partial x} + \langle vv\rangle\frac{\partial\Theta}{\partial y}\right) \tag{4}$$

where $\alpha_{u\theta}$ and $\alpha_{v\theta}$ are constants of the model that are set to 0.9 and 0.3 for $\langle u\theta\rangle$ and $\langle v\theta\rangle$, respectively, following the suggestion of Abe and Suga (2001) who did not give a justification of this difference, and τ_θ is assumed the turbulence time-scale κ/ϵ, where κ is the kinetic energy of the turbulence equal to $1/2\langle u_i u_i\rangle$ and ϵ the viscous dissipation of κ.

Some of the criticism the GGDH model has received is related with the impossibility to predict both fluxes with only one constant or that the ratio of the scalar fluxes is $\langle v\theta\rangle/\langle u\theta\rangle = \langle vv\rangle/\langle uv\rangle$ and not $\langle v\theta\rangle/\langle u\theta\rangle = \langle uv\rangle/\langle uu\rangle$ as it is expected in a bounded flow (Kim and Moin, 1989; Abe and Suga, 2001).

Many intended improvements of the GGDH model have been proposed in the literature (Rogers et al., 1989; Abe and Suga, 2001; Younis et al., 1996). Here the non-linear model proposed by Abe and Suga (2001) and the model proposed by Younis et al. (1996) are used. The Abe and Suga model, the so-called *high-order generalized-gradient diffusion-hypothesis* (Ho-GGDH) model is

$$\langle u_i\theta\rangle = -\alpha\tau_\theta\left(\frac{\langle u_i u_k\rangle\langle u_k u_j\rangle}{k}\right)\frac{\partial\Theta}{\partial x_j} \tag{5}$$

which for a two-dimensional flow is

$$\langle u\theta\rangle = -\frac{\alpha_{u\theta}\tau_\theta}{k}\left(\langle uu\rangle\langle uu\rangle\frac{\partial\Theta}{\partial x} + \langle uv\rangle\langle uv\rangle\frac{\partial\Theta}{\partial x} + \langle uu\rangle\langle uv\rangle\frac{\partial\Theta}{\partial y} + \langle vv\rangle\langle uv\rangle\frac{\partial\Theta}{\partial y}\right) \tag{6}$$

$$\langle v\theta\rangle = -\frac{\alpha_{v\theta}\tau_\theta}{k}\left(\langle uu\rangle\langle uv\rangle\frac{\partial\Theta}{\partial x} + \langle vv\rangle\langle uv\rangle\frac{\partial\Theta}{\partial x} + \langle uv\rangle\langle uv\rangle\frac{\partial\Theta}{\partial y} + \langle uu\rangle\langle uu\rangle\frac{\partial\Theta}{\partial y}\right) \tag{7}$$

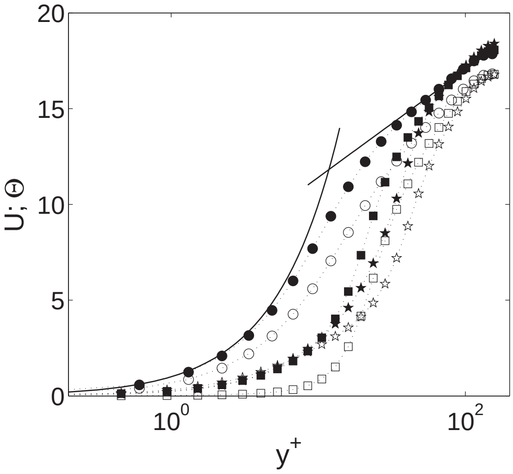
where $\alpha_{u\theta}$ and $\alpha_{v\theta}$ are constants of the model.

In this paper the constant $\alpha_{u\theta}$ is set to 0.57 following the suggestion of Abe and Suga (2001); and $\alpha_{v\theta}$ is set to 0.1, something lower than the value 0.3 used by Abe and Suga (2001) and Rossi et al (2009), owing to some over prediction problems as it is shown below.

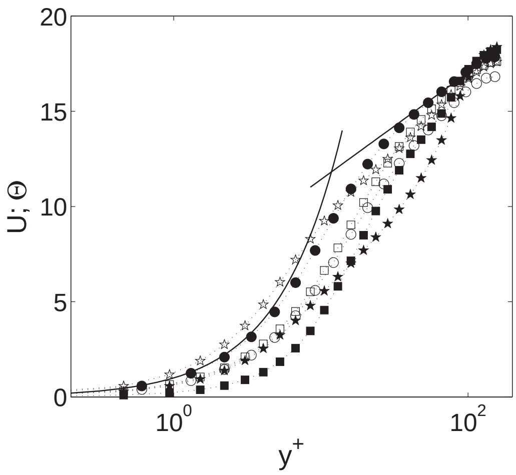
The Abe and Suga model has recently been used by Suga (2004) to predict turbulent heat transfer in a channel with square rib and by Rossi et al (2009) to predict the turbulent dispersion of a scalar from a line source at the wall downstream of a square obstacle, showing in both cases an improvement over the model based on the GGDH.

The second alternative to the GGDH model used here is the model derived by Younis et al. (1996) using 'Tensor Representation Theory', which here is called YSC in reference to its authors. For brevity this model is written only in tensorial form

$$-\langle u_i\theta\rangle = C_1\frac{\kappa^2}{\epsilon}\frac{\partial\Theta}{\partial x_i} + C_2\frac{\kappa}{\epsilon}\langle u_i u_j\rangle\frac{\partial\Theta}{\partial x_j} + C_3\frac{\kappa^3}{\epsilon^2}\frac{\partial U_i}{\partial x_j}\frac{\partial\Theta}{\partial x_j} +$$



(a)



(b)

Figure 1: U (filled symbols) and Θ (open symbols) for channel flow perturbed with (a)blowing and an (b)APGS, on the slot (squares) and W^+ downstream from the slot (stars). Open and filled circles U and Θ for non-perturbed flow, respectively. Solid line, law of the wall, $(1/0.41)ln(y^+) + 5.0$.

$$C_4 \frac{\kappa^2}{\epsilon^2} \left(\langle u_i u_k \rangle \frac{\partial U_j}{\partial x_k} + \langle u_j u_k \rangle \frac{\partial U_i}{\partial x_k} \right) \frac{\partial \Theta}{\partial x_j} \tag{8}$$

where $(C1, C2, C3, C4)$ are constants equal to $= (-0.0455; 0.373; -0.00373; -0.0235)$, resepectively; and κ and ϵ are the kinetic energy of the turbulence and the viscous dissipation.

This model is used in this study since it has been applied to heat transfer with surface blowing by Bataille et al. (2003) showing satisfactory results.

2.2 New model for the wall-layer

The RANS model proposed here for $\langle u\theta \rangle$ and $\langle v\theta \rangle$ is a wall-layer phenomenological model based on physical insights deduced from the dissimilarity analysis between the fluxes of the streamwise momentum $\langle uu \rangle$, $\langle vu \rangle$ and heat $\langle u\theta \rangle$, $\langle v\theta \rangle$ in the wall-layer (Pasinato, 2012).

In this previous study, the dissimilarity between the Reynolds stresses $\langle uu \rangle$, $\langle uv \rangle$ and the turbulent heat fluxes $\langle u\theta \rangle$, $\langle v\theta \rangle$ was studied using DNS, of perturbed turbulent channel and plane Couette flows with heat transfer for $Pr = 1$. It was found that blowing and the adverse pressure gradient step (APGS) were the perturbations that generated the greatest dissimilarity between $\langle uu \rangle$, $\langle uv \rangle$ and the heat fluxes $\langle u\theta \rangle$, $\langle v\theta \rangle$. Figs. 1(a) and 1(b) show the profiles of U and Θ on the slot (used for injection at the wall or the region where the APGS was applied) and at W^+ downstream from the slot for blowing and the APGS; where $W^+ = (W u_\tau/\nu)$ is the dimensionless width of the slot. And Figs. 2(a) and 2(b) show the streamwise and wall-normal fluxes for the same perturbations, for three locations: on the slot, and at $1.5W^+$ and $5W^+$ downstream from the slot.

In this study it was found that the leading terms of the budget of $(\langle uu \rangle - \langle u\theta \rangle)$, or budget of streamwise fluxes dissimilarity, were

$$U \frac{\partial \langle u\phi \rangle}{\partial x} \simeq -\langle uu \rangle \partial \Phi / \partial x - \langle vu \rangle \partial \Phi / \partial y \tag{9}$$

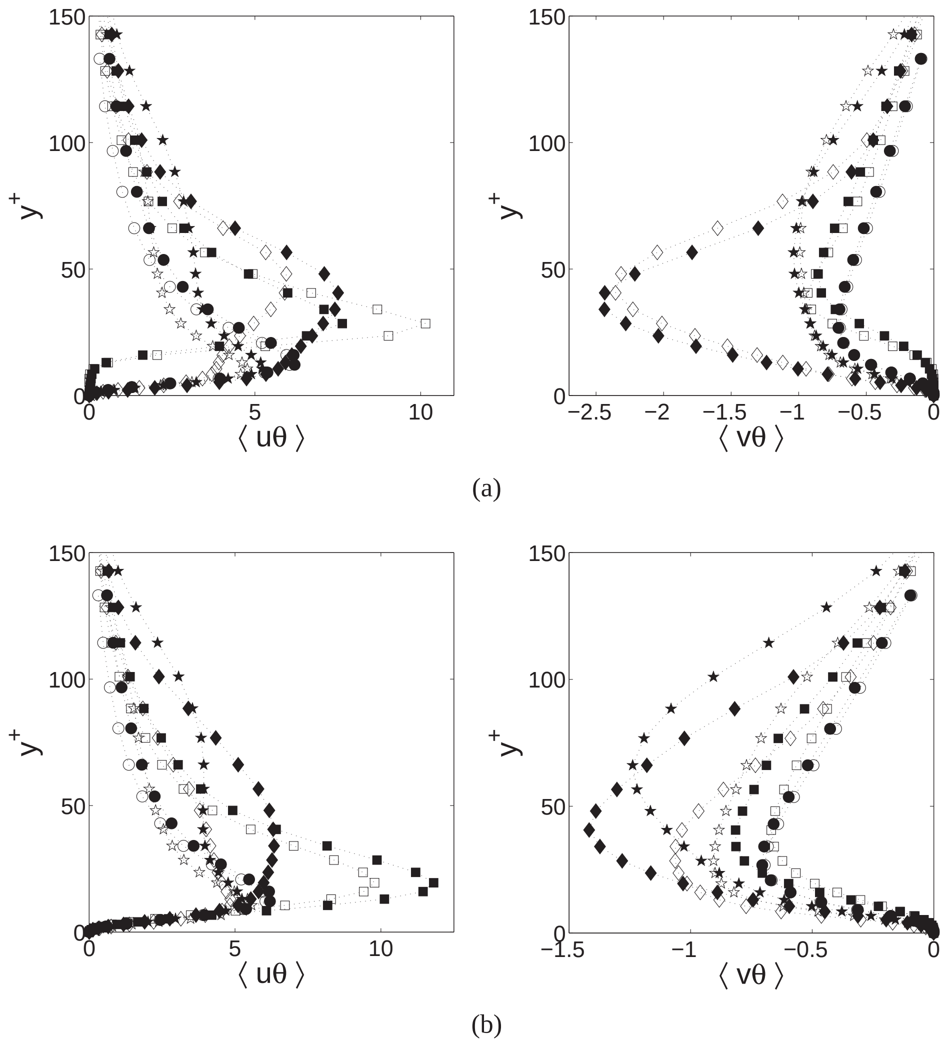


Figure 2: DNS values of $\langle uu\rangle$ and $\langle uv\rangle$(filled symbols); and $\langle u\theta\rangle$ and $\langle v\theta\rangle$(open symbols), for channel flow perturbed with (a)blowing and (b)APGS for $Pr = 1$. Circles, non-perturbed flow; squares, on the slot; diamonds, at $1.5W^+$ and stars $5W^+$ downstream from the slot. Note the different scales.

and for the budget of $(\langle uv\rangle - \langle v\theta\rangle)$, or budget of the wall-normal fluxes dissimilarity, were

$$0 \simeq -\langle vv\rangle\partial\Phi/\partial y - \langle v\partial p/\partial x\rangle \tag{10}$$

where $\Phi = (U - \Theta)$ is the mean dissimilarity, $\phi = (u - \theta)$ the fluctuation of the dissimilarity, and $\langle u\phi\rangle = (\langle uu\rangle - \langle u\theta\rangle)$ and $\langle v\phi\rangle = (\langle uv\rangle - \langle v\theta\rangle)$ are the differences or dissimilarity of the turbulent fluxes.

Since it was the dissimilarity of the wall-normal fluxes which generated the major dissimilarity in the mean fields, according to Eqn. (10), for perturbed flows there was a *quasi-equilibrium* between the velocity-pressure gradient interaction and the production terms. Note that here the 'production' terms can be a source or a sink of dissimilarity.

In Pasinato (2012) it is also shown that almost 80% of the contribution to $\langle v\theta\rangle$, $\langle u\theta\rangle$, $-\langle v\partial p/\partial x\rangle$, $-\langle v\partial p/\partial y\rangle$ and therefore of $\langle vv\rangle$ and $\langle uv\rangle$ in the perturbed flows, occurred during that events at the wall-layer that can be characterized as a combination of $v > 0;\ u < 0;\ \theta < 0$ $(Q22)$ or $v < 0;\ u > 0;\ \theta > 0$ $(Q41)$. In other words, in perturbed as in non-perturbed flow the transport

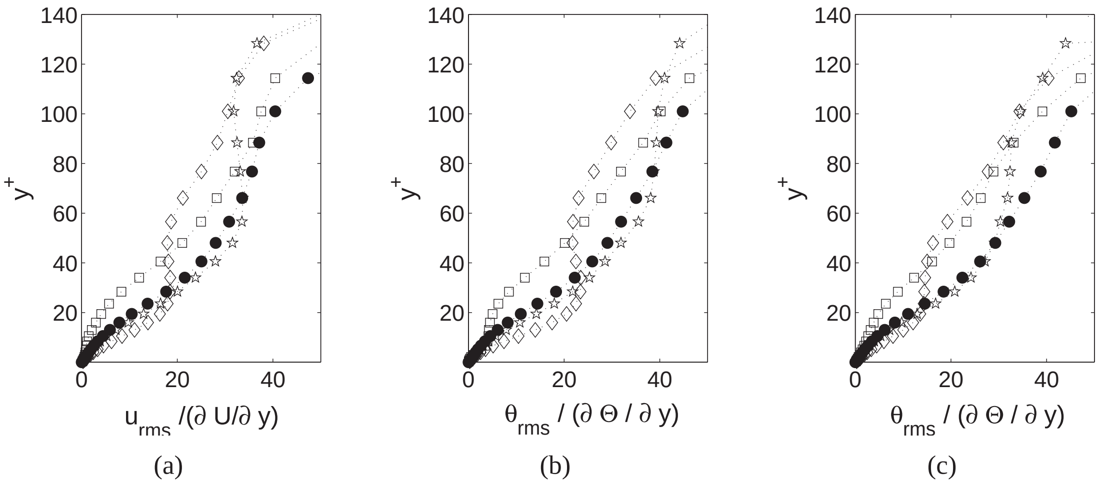


Figure 3: (a)$u_{rms}/(\partial U/\partial y)$; (b)$\theta_{rms}/(\partial\Theta/\partial y)$ for $Pr = 1$; (c)$\theta_{rms}/(\partial\Theta/\partial y)$ for $Pr = 0.5$ for channel flow perturbed with blowing. Filled circles non-perturbed flow; $\square\cdot\square\cdot\square$ on the slot; $\diamond\cdot\diamond\cdot\diamond 1.5W^+$; $\star\cdot\star\cdot\star 5W^+$ downstream of slot.

of momentum and heat is the result of the upwards and downwards motions of fluid in the wall-layer. Since U and Θ changed monotonically in these numerical tests, u and θ can be related with the differences of U and Θ, at the different regions of fluid that exchanged momentum and heat through the turbulent fluxes.

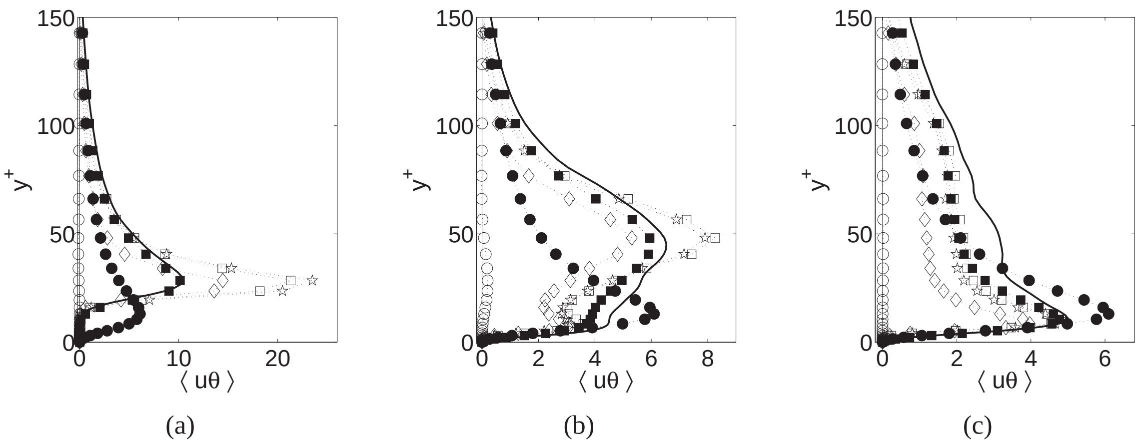
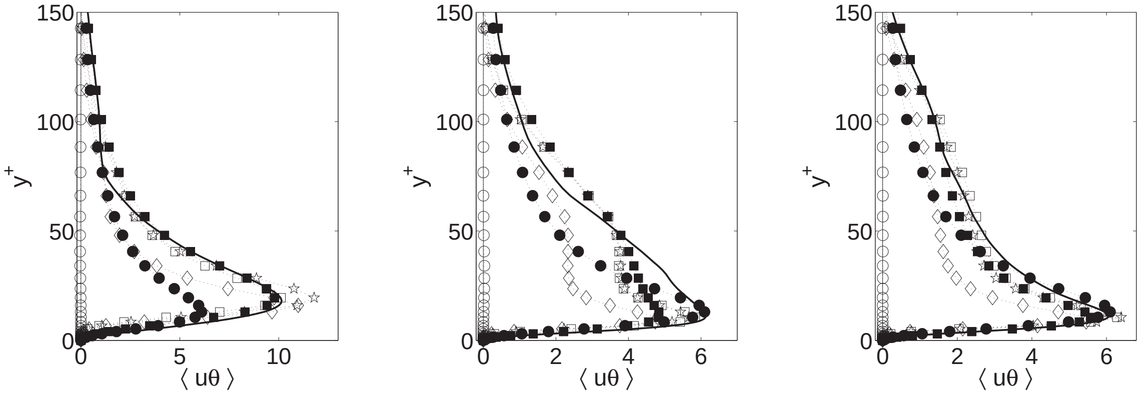


Figure 4: *A priori* comparison of modeled $\langle u\theta\rangle$ with DNS data, for channel flow perturbed with blowing for $Pr = 1$. Filled circles, non-perturbed flow; filled squares, DNS data; solid line, present model; $\circ\cdot\circ\cdot\circ$, SGDF; $\square\cdot\square\cdot\square$, GGDH; $\star\cdot\star\cdot\star$, HoGGDH; $\diamond\cdot\diamond\cdot\diamond$, YSC. (a) on the slot; (b) $1.5W^+$ and (c)$5W^+$ from the slot. Note the different scales.

In conclusion, (1)the differences $(\langle uu\rangle - \langle u\theta\rangle)$ and $(\langle uv\rangle - \langle v\theta\rangle)$ were generated by differences between u and θ, during the upwards and downwards motions of fluid at the wall-layer, responsible of the turbulent transport (events in $Q22$ and $Q41$); (2)u and θ were the result of the wall-normal differences of U and Θ at adjacent regions of fluid that exchanges momentum and heat; (3)the dissimilarity between u and θ at $\langle v\phi\rangle$ and $\langle u\phi\rangle$ was mainly owing to $-\langle v\partial p/\partial x\rangle$, $\langle vv\rangle$, $\langle uv\rangle$ generated by upwards and downwards motions of fluid, and to $\partial\Phi/\partial y$.

Therefore it is expected that the path of one of these motions of fluid exchanging momentum



(a) (b) (c)

Figure 5: Idem Figs. 4(a)-4(c) for APGS and $Pr = 1$. Note the different scales.

and heat, joins points with low and high dissimilarity at its extremes, through which U and Θ would be related in the following way

$$\Theta(l;\tau) \;\simeq\; \Theta(0;0) + \left(\frac{\partial\Theta}{\partial U}\right) dU \tag{11}$$

where l and τ are a lenght and time scale and $u = dU$, $\theta = \Theta(l,\tau) - \Theta(0,0)$.

Assuming that the major changes occur in the wall-normal direction, Eqn. (11) can be written as

$$\frac{\theta}{u} \;\simeq\; \frac{(\partial\Theta/\partial y)}{(\partial U/\partial y)} \tag{12}$$

where the Reynolds-averaging form of Eqn. (12) is

$$\frac{\theta_{rms}}{u_{rms}} \;\simeq\; \frac{(\partial\Theta/\partial y)}{(\partial U/\partial y)} \tag{13}$$

Figures 3(a)-3(c) show the terms of Eqn. (13) along y^+, for channel flow perturbed with blowing.

If the expression (12) is used as an approximation in order to transform uu in $u\theta$ and vu in $v\theta$ in one of these events, the Reynolds-averaged form of the mean fluxes are

$$\langle u\theta\rangle \;=\; \langle uu\rangle \frac{\partial\Theta/\partial y}{\partial U/\partial y} \tag{14}$$

$$\langle v\theta\rangle \;=\; \langle uv\rangle \frac{\partial\Theta/\partial y}{\partial U/\partial y} \tag{15}$$

where $1/(\partial U/\partial y)$ is the large scale time-scale τ_{ls} that here is taken as $1/S$ (since even though for perturbed flow, near the 90% of the magnitude of S is due to $\partial U/\partial y$) is the large-scale or mean flow time-scale, where $S = \sqrt{(2S_{ij}S_{ji})}$ is the module of the mean rate of strain tensor S_{ij}.

Therefore finally the Reynolds-averaged heat model is

$$\langle u\theta\rangle = \tau_{ls}\langle uu\rangle \frac{\partial\Theta}{\partial y} \tag{16}$$

$$\langle v\theta \rangle = \tau_{ls} \langle vu \rangle \frac{\partial \Theta}{\partial y} \tag{17}$$

The novel aspect of this simple model is that both streamwise and wall-normal fluxes are function of the wall-normal gradient of the mean temperature, or mean scalar. On the other hand, the model verifies the expected relations at the wall-layer that $\langle uu \rangle \propto \langle u\theta \rangle$ and $\langle vu \rangle \propto \langle v\theta \rangle$ (Launder, 2001; Rossi and Iaccarino, 2009). Note, however, that modeling $\langle v\theta \rangle$ by Eq. (17) does not agree with expressions like as $\langle v\theta \rangle = -\text{constant} \times \kappa/\epsilon \langle vv \rangle \partial\Theta/\partial y$ suggested by Launder and previous studies cited by him (Launder, 2001), where $\langle vv \rangle$ is used rather than $\langle uv \rangle$.

In other words, the proposed new model does not follow the format of some closures of $\langle v\theta \rangle$ and also of $\langle u\theta \rangle$ proposed in the literature (Daly and Harlow, 1970; Abe and Suga, 2001; Younis et al., 1996). However the *a priri* comparison of the next section shows a good performance of the model for the flows studied here (and flows with other boundary conditions and geometries not shown here for brevity), giving some merit to the new approach.

3 'A PRIORI' COMPARISON

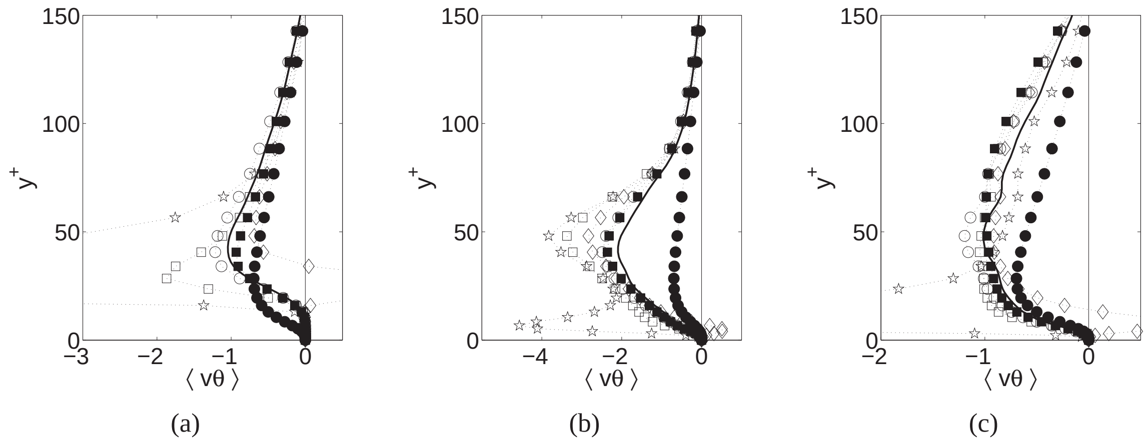
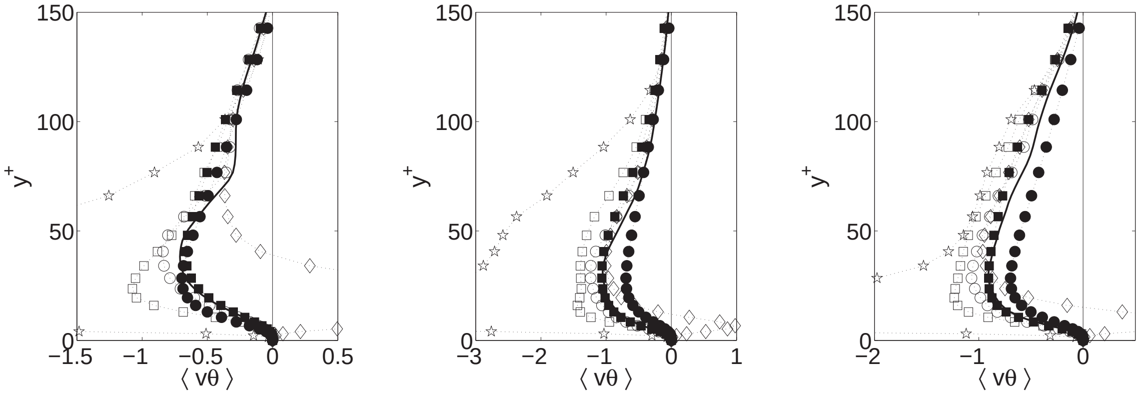


Figure 6: Idem Fig. 4(a)-4(c) for $\langle v\theta \rangle$ for blowing and $Pr = 1$. Note that the scale does not include extreme over predictions.

An *a priori* comparison is presented of the SGDH, GGDH, Ho-GGDH, YSC, and the present model, with DNS data of perturbed turbulent channel flow with heat transfer, for $Re_\tau = 150$ and Pr equal to 1. The perturbation types in this study were blowing and an APGS (Pasinato, 2012).

The algebraic models and the new model presented here are functions of the Reynolds stresses and of the mean field gradient. Although a real validation should be done in a complete test, an *a priori* comparison shows whether there exist, or not, significant errors in the prediction of the turbulent heat using the right values of the mean gradient of Θ, U and of the Reynolds stresses. Even though the DNS dataset can suffer from low-Reynolds effects, these simulations have been validated with experimental data and other DNS studies (Pasinato, 2011). Using the right values of the Reynolds stresses avoids the use of Reynolds stresses modeled data which always would suffer from some level of errors.

On the other hand, blowing and APGS are two convenient cases for testing turbulent heat flux models, because these type of perturbations produce significant modification of the turbu-



(a) (b) (c)

Figure 7: Idem Fig. 4(a)-4(c) for $\langle v\theta\rangle$ for the APGS and $Pr = 1$. Note the different scales, and also that the scale does not include extreme over predictions.

lent fluxes $\langle uu\rangle$, $\langle uv\rangle$, $\langle u\theta\rangle$, $\langle v\theta\rangle$, and of the mean gradients regarding its values in developed conditions (Figs. 1(a)-2(b)). For blowing, Θ undergoes the major changes, while U undergoes the major changes for the APGS. On the other hand, while $\langle uu\rangle$, $\langle uv\rangle$ were clearly higher than $\langle u\theta\rangle$, $\langle v\theta\rangle$ for the APGS. However, for blowing $\langle uu\rangle$ was lower than $\langle u\theta\rangle$ but $\langle uv\rangle$ and $\langle v\theta\rangle$ presented only slight differences.

The comparison uses profiles along y^+ at the middle of the slot and at $1.5W^+$ and $5W^+$ downstream from the slot; and along x^+ for brevity only at y^+ equal to 38. Figures 4(a)-9(b) show the comparison of the predictions for blowing and the APGS for the 5 models with the DNS dataset.

Despite of its simplicity the model presented here has shown a reasonable agreement with the DNS data. It did not present any extremes over or under prediction. The overall behavior of the model is considered good, even though there are some details regarding the prediction in non-perturbed flow afar from the wall and in the recovering region for perturbed flow. The greatest differences between its predictions and the DNS data were for $\langle u\theta\rangle$ at $1.5W^+$ and $5W^+$ from the slot for blowing and at $1.5W^+$ from the slot for the APGS. For $\langle v\theta\rangle$ however the greatest differences were at $1.5W^+$ from the slot for blowing. Also, in general, the prediction of the new model for the APGS case was better than for the blowing case, which was expected since the model presented a slight over prediction, and the APGS case generated, in contrast with the blowing case, lower Reynolds stresses than heat fluxes. In conclusion, in the recovery region the model has shown a slight over prediction of $\langle u\theta\rangle$ (Figs. 8(a) and 9(a)), which produced errors in the angle of the heat vector. These slight over predictions of $\langle u\theta\rangle$ in the recovery region (and for the core region as shown below) can be improved using a different time-scale (other than $1/S$) in that region with less inhomogeneous flow. More is discussed below.

The SGDH, as it is known, presented an extremely under prediction of $\langle u\theta\rangle$ and a reasonable behavior of $\langle v\theta\rangle$ with some slight over predictions. These over predictions show that a greater value of the Pr_t, other than 0.85, should be considered for the flows used here.

The models based on the GGDH and Ho-GGDH presented an improvement of the prediction of $\langle u\theta\rangle$ as regarding the SGDH, but these models presented over predictions, in some cases very high, mainly for $\langle v\theta\rangle$. In general the GGDH model presented, on the slot and near of it, always a better prediction of both heat fluxes than the Ho-GGDH model, which was an unexpected result. However, in the recovery region, which is an almost non-perturbed turbulent flow, both

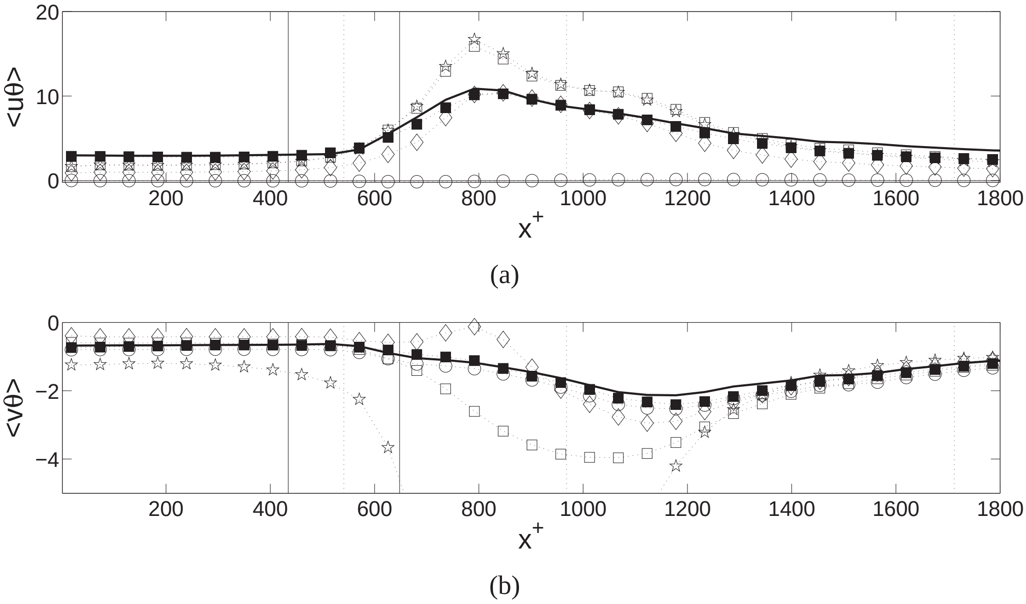


Figure 8: Idem Fig. 4(a)-4(c), but along x^+ at $y^+ = 38$. (a)$\langle u\theta\rangle$; (b) $\langle v\theta\rangle$. Note the different scales. Solid vertical lines denote the slot location, vertical dotted lines denote sections on the middle of the slot, and at $1.5W^+$ and $5W^+$ from the slot. Note also that the scale does not include extreme over predictions.

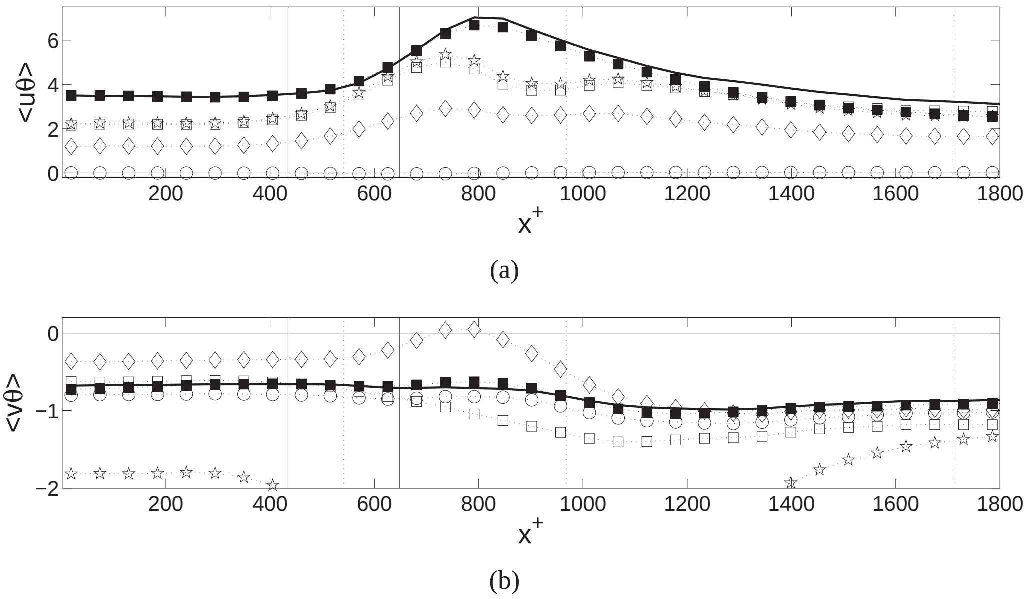


Figure 9: Idem Fig. 5(a)-5(c) for the APGS and $Pr = 1$, but along x^+ at $y^+ = 38$. (a)$\langle u\theta\rangle$; (b) $\langle v\theta\rangle$. Note the different scales, and also that the scale does not include extreme over predictions.

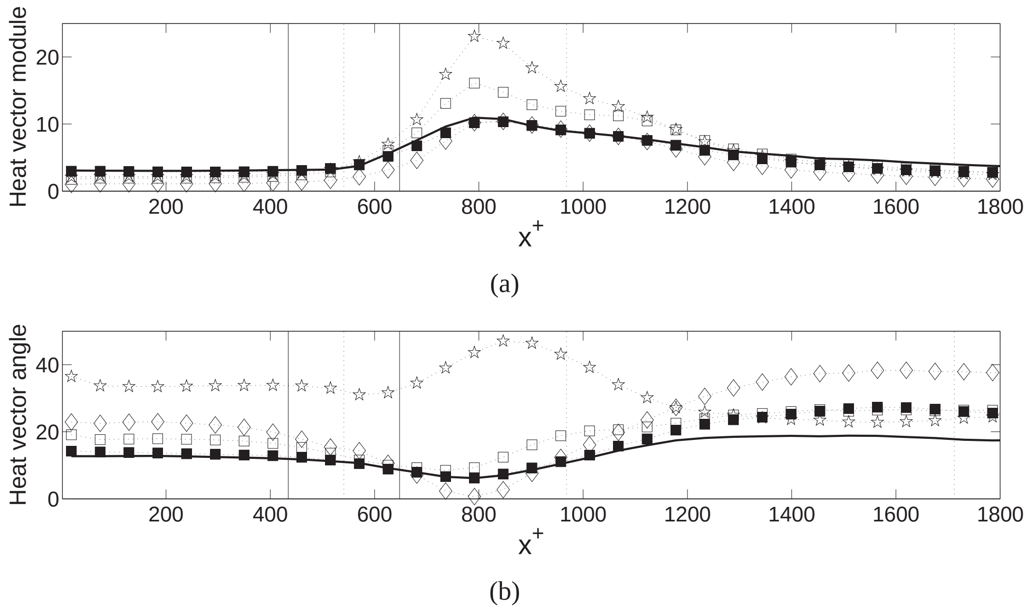


Figure 10: *A priori* comparison of the (a)module and (b)angle (degree) regarding to the horizontal, of the heat flux vector with DNS data, for channel flow perturbed with blowing. Filled squares, DNS data; solid line, present model; $\square \cdot \square \cdot \square$, GGDH; $\star \cdot \star \cdot \star$, HoGGDH; $\diamond \cdot \diamond \cdot \diamond$, YSC.

models performed very well with a slight under prediction of $\langle u\theta\rangle$ and a slight over prediction of $\langle v\theta\rangle$. Nevertheless, the model Ho-GGDH presented extreme over predictions of $\langle v\theta\rangle$ in the buffer region, on the slot and in the recovery region, of the perturbed flows. The term of the Ho-GGDH model which caused these over predictions was $\alpha_{v\theta}(\tau_\theta/\kappa)\langle uu\rangle^2\partial\Theta/\partial y$ in Eq. (7), since the other terms were always less than 1. e.g. for blowing and $Pr = 1$, on the slot and at $y^+ \simeq 25$, $\langle uu\rangle = 7.15$, $\kappa \simeq 4$, $\epsilon \simeq 0.1$, $\tau_\theta/\kappa = 8.45$, $(\partial\Theta/\partial y) \simeq 0.58$ and the total of this terms is $\simeq -25$, where all variable are dimensionless. Although some low-Reynolds effect can be affecting these DNS data, this value of $\langle v\theta\rangle \simeq -25$ seems to be unphysical. For the APGS these extremes over predictions of $\langle v\theta\rangle$ occurred in the buffer region, on the slot and downstream of the slot. $\langle u\theta\rangle$ on the slot and in the buffer region was also over-predicted, but in this case the error was less than a 100% of the DNS value. In other words, in comparison to the SGDH, the model Ho-GGDH presented for turbulent perturbed flows with heat transfer an improvement of the streamwise flux, but a poor prediction of the wall-normal flux in the buffer region, on the slot and downstream of it, in the recovery region.

The YSC model also presented an improvement of the prediction of $\langle u\theta\rangle$ regarding the SGDH model. For blowing the YSC model presented a good prediction on the slot with under predictions afar from the perturbed region, while for the APGS the prediction was better. On the contrary, the wall-normal flux $\langle v\theta\rangle$ was well predicted afar from the slot, even though it presented unphysical oscillations with positive values in the buffer region, on the slot and downstream, generated by the first term in Eqn. (8); e.g for blowing and $Pr = 1$, on the slot and at $y^+ \simeq 25$, $\kappa \simeq 4$, $\epsilon \simeq 0.1$, $(\partial\Theta/\partial y) \simeq 0.58$ and the total of this terms for $\langle v\theta\rangle$ was $-C_1\kappa^2/\epsilon(\partial\Theta/\partial y) \simeq 4.3$, and the sum of the other three terms was approximately -2, resulting in a positive value of 2.3. Although this last term did not generate additional problem in the prediction of $\langle u\theta\rangle$, the model YSC presented for the flows used here better prediction of $\langle v\theta\rangle$ without the first term in Eq. (8).

An important aspect of the turbulent heat prediction is the direction of the turbulent fluxes; e.g. $\beta = atan(\langle v\theta\rangle/\langle u\theta\rangle)$, where β is the angle of the vector regarding to the center line

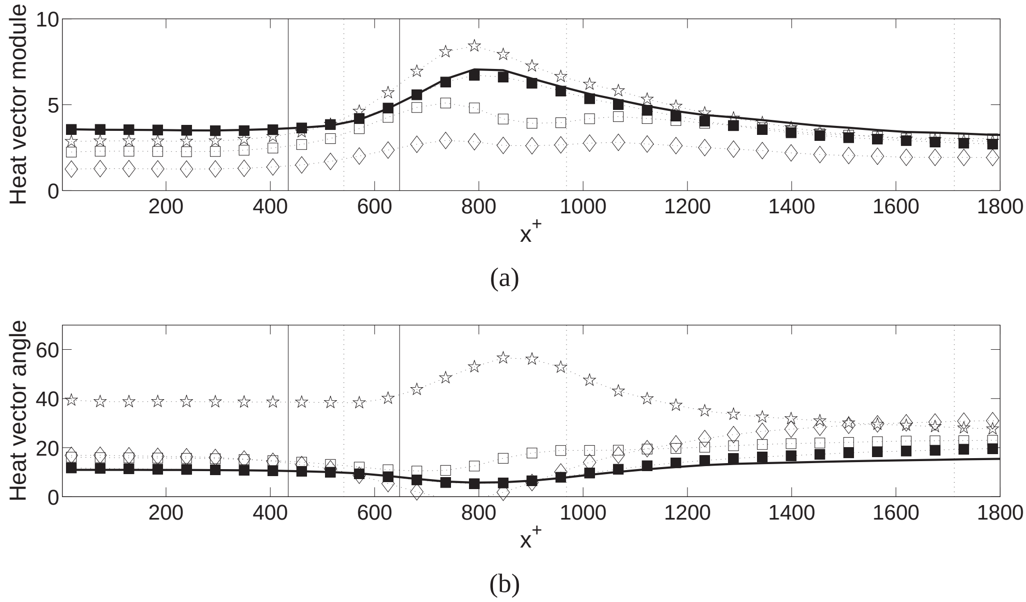
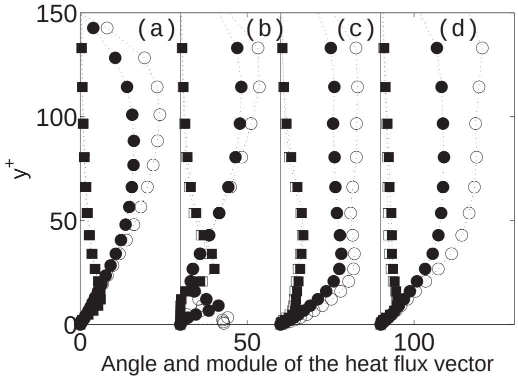


Figure 11: Idem Figs. 10(a) and 10(b) for channel flow perturbed with an APGS. (a)module; (b)angle (degree).

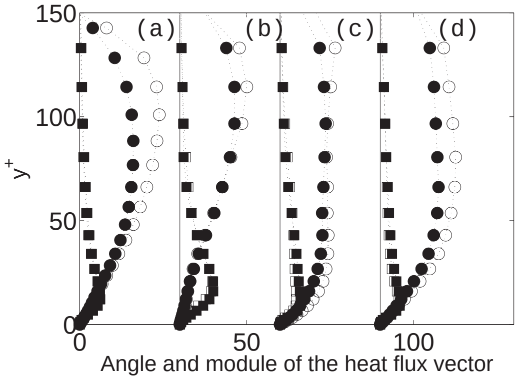
(Kasagi and Nishimura, 1997). A wrong angle with a small module would be not a problem; however a wrong angle in the region with the highest module could be a serious problem. Figs. 10(a)-11(b) show the module and the angle of the heat vector for perturbed flow with blowing and the APGS, along x^+ and at $y^+ \simeq 38$, for the GGDH, HoGGDH, YSC and the present model. Since the SGDH predicted an almost zero or negligible streamwise flux in the whole domain, the prediction of this model is not shown. From the GGDH, Ho-GGDH and YSC models, in general the best prediction of the angle and module was that from the GGDH, followed by that of the YSC model. The Ho-GGDH model prediction of the module and angle was wrong mainly owing to the over prediction $\langle v\theta\rangle$, as it is commented above (Note that here the constant $\alpha_{v\theta}$ was set to a third of the value 0.3 suggested by its authors (Abe and Suga, 2001)). The prediction of the Ho-GGDH model was the worst situation since it presented the highest error in the angle in combination with the highest error in the module of the vector. On the other hand, the YSC model presented an over prediction of the angle in the recovery region owing to a small under prediction of the $\langle u\theta\rangle$ there, but in general its behavior was reasonable. In these figures the model presented here has shown the best prediction of the combination module-angle of the heat vector.

Figs. 12(a) and 12(b) show a complete picture of the distribution along y^+ of the module and angle of the heat vector predicted by the new model presented here, for blowing and the APGS. Note that this model assumes an alignment between both the turbulent fluxes of streamwise momentum and heat. The maximum error in the angle of the heat flux occurred for non-perturbed flow in the core region, and for blowing at $5W^+$ from the slot and also in the core region. This error, however, occurs in combination with very low values of the module of the heat vector. For the APGS the difference between the angles was lower, but again presented the major errors for $5W^+$ from the slot and afar from the wall.

These differences in the angle are revealing that a different time scale should be used afar from the wall (other than $1/S$), in the center and recovery regions, where the flow is less inhomogeneous. However, in order to use, for example, the turbulence time-scale κ/ϵ in an applied case, κ and ϵ should be computed, which indeed represents an important additional computa-



(a)



(b)

Figure 12: *A priori* comparison of the module(squares) and (b)angle(circles) (degree, regarding to the centerline) of the heat flux vector predicted by the present model(filled symbols) with DNS data(open symbols), for channel flow perturbed with (a)blowing and (b)APGS. (a-a), (b-a)non-perturbed flow; (a-b), (b-b)on the slot; (a-c), (b-c) at $1.5W^+$ from slot; (a-d), (b-d) at $5W^+$ from slot. Note the shift (30) of the abscissa of plots (b), (c) and (d) regarding the previous plot.

tional effort, if these variables are not known from the flow calculation.

4 CONCLUSIONS

In this work, Reynolds-averaged expressions for the turbulent heat fluxes $\langle \theta u \rangle$ and $\langle \theta v \rangle$ in the wall layer are proposed, based on the Reynolds stress $\langle uu \rangle$ and $\langle uv \rangle$, the wall-normal gradient of Θ and a large scale time-scale. A pre-generated database obtained from DNS of turbulent channel flow with heat transfer, perturbed with blowing and an APGS was used.

The model is considered incomplete since it is intended only to represent the greatest dissimilarity between the turbulent fluxes of streamwise momentum and heat. Additional studies should be performed in order to find out the behavior of the terms with minor contributions to dissimilarity, to define a more convenient time-scale afar from the wall, besides to test it for other flow characteristics like as streamline curvatures, perturbation where the flow and thermal field present different boundary conditions, among other cases.

The main conclusions of this work are:

1.An *a priori* comparison of the new model has shown that it performs in general well for the perturbed flows used here. It is robust and presented only slight over prediction afar from the wall and afar from the perturbation region.

2.According to the new model, $\langle u\theta \rangle$ seems to be more related to $\partial\Theta/\partial y$ than to $\partial\Theta/\partial x$. Also, $\langle v\theta \rangle$ uses the $\langle uv \rangle$ rather than the $\langle vv \rangle$ second moment, as it is suggested in the literature. Therefore more work and a complete validation should be done of the model in more general situations.

3.The *a priori* comparison of the turbulent heat fluxes predicted by the new model and by the SGDH, GGDH, Ho-GGDDH and YSC models, with DNS data of perturbed flows, has shown a better and more robust performance of the new model.

4.From the SGDH, GGDH, Ho-GGDH and YSC models, the best prediction in the *a priori* comparison with DNS data have been obtained from the GGDH model. The SGDH model presented, as it is known, extreme under predictions of $\langle u\theta \rangle$. The Ho-GGDH model presented important over predictions of $\langle v\theta \rangle$ on the slot and downstream of it, and the YSC model presented oscillations with positive values of $\langle v\theta \rangle$, in the buffer region, on the slot and downstream of it.

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