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LARGE STRAIN ELASTO/VISCOPLASTIC CONSTITUTIVE MODEL. THEORY AND NUMERICAL SCHEME.

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Abstract. *In this paper an extension of a large strain elastoplastic constitutive model based on hyperelasticity and multiplicative decomposition of deformation gradient tensor due to García Garino is extended to viscous case following a previous work of Ponthot based on Perzyna type model. The integration of constitutive model is based on numerical scheme originally designed for the elastoplastic problem that naturally includes the rate dependent case. Consequently the algorithm proposed by Ponthot for viscoplasticity is easily taken into account in the framework of hyperelasticity and irreversible thermodynamics of solids. For the case of metals, a unified stress update algorithms for elastoplastic and elasto-viscoplastic constitutive equations submitted to large deformations is obtained. The plastic corrector step is, in case of J2 flow theory material behavior, an extension to the viscoplastic range of the classical radial return algorithm for plasticity. The resulting unified implicit algorithm is both efficient and very inexpensive. Moreover, if there is no viscosity effect (rate-independent material) the presented algorithm degenerates exactly into the classical radial return algorithm for plasticity.*

1 INTRODUCTION

This paper presents preliminary results of a large strain viscoplastic model based on hyperelasticity. The large strain model structure is taken from previous work of García Garino,[1–3] derived in the context of the ideas of Simo and Ortiz[4–6] for the rate independent case. Viscoplastic case, based on Perzyna type model, comes from a work of Ponthot[7] where a unified algorithm for elasto/viscoplastic problems has been proposed.

The kinematics of the resultant constitutive model is based on the multiplicative decomposition of deformation gradient tensor.[8] Stresses can be derived from a hyperelastic potential and the model is written in the framework of internal variables theory and thermodynamics of irreversible solids.[9] The stress update algorithm proposed by Ponthot treats the elasto/viscoplastic problem in a unified way. For a J2-flow material model, it is a simple generalization to rate-dependent problems of the radial return algorithm for rate-independent plasticity, including a generalized consistency condition.

The classical *elastic* predictor - *plastic* corrector split problem is used in order to derive numerical scheme of the model. In this way a fully implicit algorithm is designed. The resultant update algorithm is written in terms of kinematics quantities instead of the usual one defined for the stress tensor. In the work it is shown that the unified elasto/viscoplastic stress update proposed by Ponthot[7] is naturally included in the (previous) numerical structure of rate independent case, as regards the update be rewritten in terms of kinematic variables.

A complete review of the state of the art is not included into the goals of this prelimary work. A comprehensive account of the problem can be found in the textbooks of Lubliner,[9] for the fundamentals, and Ottosen and Ristinmaa[10] both for theory and numerical discussion.

In order to integrate in time the ODE resulting from this kind of problems many algorithms can be developed. The attributes that one strives for are accuracy, reliability, efficiency and ease of computer implementation. The radial return algorithm presented by Wilkins[11] and Maenchen and Sack[12] satisfies all of these attributes. Subsequently developed algorithms have been shown to fall short of Wilkins' method with respect to both simplicity and accuracy, see e.g. Krieg and Krieg,[13] Yoder and Whirley[14] and Ortiz and Popov[15] in a small strain framework.

Consequently, radial return is now extensively used in finit element codes for large-scale computations of elastoplastic behavior, see e.g. Key,[16–18] Hallquist,[19–21] Hughes,[22] Simo, Ortiz and Taylor[23–27] among many others.

This integration scheme is both inexpensive and accurate. In addition, it allows to write down a closed-form expression for the so-called consistent elastoplastic tangent modulus. Use of this consistent modulus (and not the continuum modulus) for the establishment of the global tangent stiffness matrix is essential in preserving the quadratic rate of convergence in Newton's procedure required by implicit algorithms.

However, regarding elastic-viscoplastic modeling of material behaviour, the situation is completely different. At the present time, many different algorithms have been developed in order to integrate elastic-viscoplastic equations, see e.g. Hughes and Taylor[28] , Suliciu,[29] Pan,[30] Rubin,[31] Bruhns and Rott,[32] or Golinval[33] for a valuable discussion. However, none of them actually exhibits the same level of performance as the radial return algorithm for plasticity. Moreover, none of the schemes described in Golinval[33] is amenable to consistent linearization. This fact is highly penalizing and precludes an efficient treatment of viscoplastic problems in large-scale finite element or finite difference codes.

In recent works Carosio and coauthors[34,35] have discussed the problem in the context of contiuum and consistent viscoplasticity and Alfano et al.[36] presented general solución procdedures in elasto/viscoplasticity.

In some works rate dependent Perzyna models are discussed in the framework of large strain models: Wang and Sluys[37] have proposed an incremental model for the elastic problem and the integration of the problem is carried out using a midpoint rule. Ponthot[7] has proposed model based on hypoelasticty for the elastic problem and viscoplastic effects are integrated with a unified (plastic/viscoplastic) stress update procedure. Simo[38] has discussed the problem for a Duvaut-Lyons model type.

The discussed integration scheme proposed in this work is both inexpensive and accurate. In addition, it allows to write down a closed-form expression for the so-called consistent elasto-plastic tangent modulus. Use of this consistent modulus (and not the continuum modulus) for the establishment of the global tangent stiffness matrix is essential in preserving the quadratic rate of convergence in Newton's procedure required by implicit algorithms.[39]

Therefore, it exhibits all its (good) properties, including accuracy, stability and existence of a consistent tangent operator. The fact that the integration procedure is based on a fully implicit backward Euler algorithms also avoids the need to define an instantaneous relaxation time, as is the case in the procedure proposed by Simo and Ortiz.[6,40] Moreover the algorithm is unified in the sense that the same routines are able to integrate both elasto-plastic and elasto-viscoplastic models. The former case is simply obtained by setting the viscosity parameter to zero.

2 LARGE STRAIN ELASTOPLASTIC MODEL

In this point the proposed constitutive model is briefly presented. The elastoplastic constitutive model[1–3] can be written in the three different configurations. However for the purpose of this work it is enough to present the results for the current configuration. First main results of kinematics problem are given followed by a summary of constitutive equations. Finally a particular case of metals is discussed.

2.1 Kinematics

The kinematics of the problem is based on the very well known multiplicative decomposition of deformation gradient tensor $\boldsymbol{F}$ in its elastic and plastic components,[8] as is it shown in equation 1. In figure 1 original and deformed configuration ${}^{o}\Omega$ and ${}^{t}\Omega$, respectively as well as the intermediate configuration ${}^{t}\Omega^{e}$ can be seen.

$$\boldsymbol{F} \;=\; \boldsymbol{F}^{e}\,\boldsymbol{F}^{p} \tag{1}$$

Almansi strain tensor $\boldsymbol{e}$ and its elastic and plastic components, $\boldsymbol{e}^{e}$ and $\boldsymbol{e}^{p}$, respectively, are the

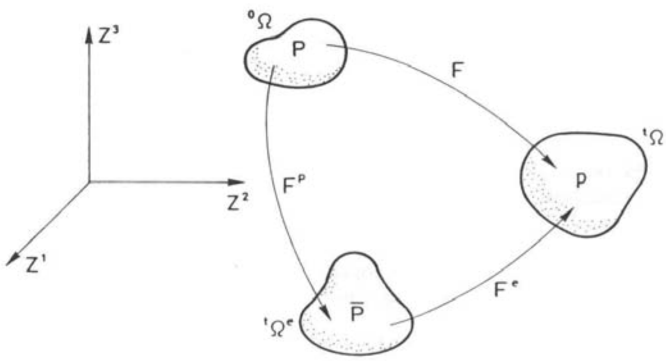

Figure 1: Kinematics of large strain elastoplastic solid: configurations

variables used in the constitutive model. The Almansi strain is defined in equation 2 terms of spatial metric tensor $\boldsymbol{g}$ and Finger tensor $\boldsymbol{b}^{-1} = \boldsymbol{F}^{-T}\,\boldsymbol{F}^{-1}$

$$\boldsymbol{e} = \frac{1}{2}(\boldsymbol{g}\ -\ \boldsymbol{b}^{-1}) \tag{2}$$

The elastic component of Almansi strain tensor is defined in equation 3 in terms of of spatial metric tensor $\boldsymbol{g}$ and elastic component of Finger tensor $\boldsymbol{b}^{e-1} = \boldsymbol{F}^{e-T}\,\boldsymbol{F}^{e-1}$

$$\boldsymbol{e}^e = \frac{1}{2}(\boldsymbol{g}\ -\ \boldsymbol{b}^{e-1}) \tag{3}$$

The rate of deformation tensor $\boldsymbol{d}$ is obtained computing the Lie derivative ${L_v}^{41}$ of Almansi strain tensor, and admits an additive decomposition it its elastic and plastic compenentes $\boldsymbol{d} = \boldsymbol{d}^e + \boldsymbol{d}^p$.

2.2 Constitutive Model

In this section the equations that define the model in the current configuration are presented.

$$\boldsymbol{e} = \boldsymbol{e}^e + \boldsymbol{e}^p \tag{4}$$

$$\boldsymbol{d} = \boldsymbol{d}^e + \boldsymbol{d}^p \tag{5}$$

$$\boldsymbol{\sigma} = \frac{\partial \psi^e(\boldsymbol{e}^e, \boldsymbol{b}^{e-1})}{\partial \boldsymbol{e}^e} \tag{6}$$

$$\dot{\gamma} \geq 0 \quad f \leq 0 \quad \dot{\gamma}\, f = 0 \tag{7}$$

$$\boldsymbol{d}^p = \dot{\gamma}\, \frac{\partial g}{\partial \boldsymbol{\sigma}} \tag{8}$$

$$\mathcal{D}^p = \boldsymbol{\tau} : \boldsymbol{d}^p + \boldsymbol{p} :\ \dot{\boldsymbol{\alpha}} \geq 0 \tag{9}$$

where $\boldsymbol{\sigma}$ denotes Cauchy stress tensor, $\psi^e(\boldsymbol{e}^e, \boldsymbol{b}^{e-1})$ is the elastic free energy, f and g accounts for yield criteria and plastic potential, respectively, and γ is the plastic multiplier. Plastic dissipation is denoted by $\mathcal{D}^p$ and $\boldsymbol{\alpha}$ and $\boldsymbol{p}$ are a proper set of internal variables and their conjugate thermodynamical forces.

2.3 Constitutive Model for Metals

For the case of metals under large strains, the elastic strains are negligible. In this case the tensor $\boldsymbol{F}^e$ approches to the Identity. Consequently tensor $\boldsymbol{b}^{e-1}$ tends to the spatial metric tensor $\boldsymbol{g}$. In this case the distinction between intermediate and current configurations have no meaning and elastic strains are small. Then it is possible to write the elastic componenent of free energy function as a quadratic function of elastic component of Almansi strain tensor $\boldsymbol{e}^e$ and material constants λ and μ as it is shown in equation 10.

$$\psi^e = \left[\frac{1}{2}\lambda \, tr(\boldsymbol{e}^e)^2 \; + \; \mu \, (\boldsymbol{e}^e : \boldsymbol{e}^e)\right] \tag{10}$$

From equation (6) the Cauchy stress tensor results:

$$\sigma = \lambda \, tr(\boldsymbol{e}^e) \, \mathbf{1} \; + \; 2 \, \mu \, \boldsymbol{e}^e \tag{11}$$

This model has been used previously by the authors[1–3] as an alternative to the neohookean models proposed by another authors.[4–6]

Plasticity is taken into account by means of an associative flow rule $f = g$. The yield function is the very well known Von Mises or J2 model given in equation 12.

$$f(\boldsymbol{\sigma}, \sigma_y) = \bar{\sigma} - \sigma_y = 0 \tag{12}$$

where $\bar{\sigma} = \sqrt{\frac{3}{2}\boldsymbol{s} : \boldsymbol{s}}$ denotes equivalent stress, $\boldsymbol{s}$ is the deviatoric stress tensor and σ_y is the current yield stress.

Flow rule can be written now in terms of yield criteria f:

$$\boldsymbol{d}^p = \dot{\gamma} \quad \boldsymbol{n} \qquad \text{where} \qquad \boldsymbol{n}_{ij} = \frac{\boldsymbol{s}_{ij}}{\sqrt{\boldsymbol{s}_{kl}\boldsymbol{s}_{kl}}} \tag{13}$$

where $(\boldsymbol{n} : \boldsymbol{n} = 1)$ is the unit outward normal to the yield surface and plastic multiplier γ can be computed from the Kuhn Tucker conditions given in equation (7).

The hardening law relates yield stress σ_y and the rate of the effective plastic strain $\dot{\bar{\epsilon}}^p$ defined as $\dot{\bar{\epsilon}}^p = \sqrt{\frac{2}{3}\, \boldsymbol{d}^p : \boldsymbol{d}^p}$ as shown in equation (14).

$$\dot{\sigma}_y = \; h \, \dot{\bar{\epsilon}}^p \;\; = \;\; \sqrt{\frac{2}{3}} \, h \;\; \dot{\gamma} \tag{14}$$

and h is a material parameter that corresponds to the slope of the effective stress vs. effective plastic strain curve under uniaxial loading conditions, also known as hardening module in the case linear hardening.

3 LARGE STRAIN VISCOPLASTIC MODEL

In this section viscoplastic problem is presented and theoretical details are derived with emphasis in flow rule and consistency condition. Main results of elastoplastic kinematics are still valid taking into account that inelastic variables in this case describe the rate dependent case.

$$\boldsymbol{e} = \boldsymbol{e}^{e} + \boldsymbol{e}^{vp} \tag{15}$$
$$\boldsymbol{d} = \boldsymbol{d}^{e} + \boldsymbol{d}^{vp} \tag{16}$$
$$\tag{17}$$

where $\boldsymbol{e}^{vp}$ and $\boldsymbol{d}^{vp}$ are viscous counterpart of plastic components of Almansi strain tensor $\boldsymbol{e}^{p}$ and rate of deformation tensor $\boldsymbol{d}^{p}$ respectively.

Given the uncoupled format chosen for free energy function its elastic component remains unchanged in this case. Then Cauchy stress are computed from equations 6 and 10.

Contrary to the case of rate independent plasticity, the effective stress $\bar{\sigma}$ is no longer constrained to remain less or equal to the yield stress but one can have $\bar{\sigma} \geq \sigma_y$. Therefore we define the *overstress* d as

$$d = \langle \bar{\sigma} - \sigma_y \rangle \tag{18}$$

where $\langle x \rangle$ denotes the Mac Auley brackets defined by $\langle x \rangle = 1/2(x + |x|)$. Clearly, an inelastic process can only take place if, and only if, the overstress d is positive, consequently $f \geq 0$.

For example, classical viscoplastic models of the Perzyna type[42,43] may be considered as penalty regularization of rate-independent plasticity where the consistency parameter has been $\dot{\gamma}$ replaced by an increasing function of the overstress e.g.

$$\dot{\gamma}^{vp} = \sqrt{\frac{3}{2}} \left\langle \frac{\bar{\sigma} - \sigma_y}{\eta(\bar{\epsilon}^{vp})^{1/n}} \right\rangle^{m} \tag{19}$$

where γ^{vp} accounts for *viscoplastic multiplier*, n is a hardening exponent, m is a rate sensitivity parameter, $\bar{\epsilon}^{vp}$ is the equivalent viscoplastic flow and η is a viscosity parameter.

In that case, the evolution equations are still of the form

$$\boldsymbol{d}^{vp} = \dot{\gamma} \quad \boldsymbol{n} \tag{20}$$
$$\dot{\bar{\epsilon}}^{vp} = \sqrt{\frac{2}{3}}\gamma \quad \text{or} \quad \dot{\sigma}_y = h \, \dot{\bar{\epsilon}}^{vp} \tag{21}$$

which are quite similar to the rate-independent case given in equations 13 and 14.

Combining equations (19), (20) and (21) gives:

$$\dot{\bar{\epsilon}}^{vp} = \sqrt{\frac{2}{3}\boldsymbol{d}^{vp} : \boldsymbol{d}^{vp}} = \sqrt{\frac{2}{3}}\,\dot{\gamma}^{vp} = \left\langle \frac{\bar{\sigma} - \sigma_y}{\eta(\bar{\epsilon}^{vp})^{1/n}} \right\rangle^{m} \tag{22}$$

so that, in the viscoplastic range, a new constraint is defined.[7]

$$\bar{f} = \bar{\sigma} - \sigma_y - \eta(\bar{\epsilon}^{vp})^{1/n}(\dot{\bar{\epsilon}}^{vp})^{1/m} = 0 \tag{23}$$

This criterion is a *generalization of the classical von-Mises criterion $f = 0$ for rate-dependent materials*. The latter can simply be recovered by imposing $\eta = 0$ (no viscosity effect), result that has been pointed out in the literature by another authors.[10,36,37]

REMARKS

1. The key point feature of the stress update algorithm due to Ponthot[7] is the viscoplastic generalization of the classical Von Mises criterion given in eq. (23), that follows from the introduction of the viscoplastic parameter given in eq. (19).
2. This *viscoplastic* constraint is sometimes called *dynamic yield surface*[10,42]
3. Simo[38] pointed out that viscosity η plays the role of penalty multiplier in eq. (23), then viscoplasticy can be considered as a regularization of elastoplasticity. Consequently rate independent case ($\eta = 0$) and (nonlinear) elastic case ($\eta = \infty$) are recovered as limite cases.

In the elastic regime, both f and $\bar{f}$ are equivalent since, in that case

$$\dot{\bar{\epsilon}}^{vp} = 0 \qquad \text{and} \qquad \bar{\sigma} \leq \sigma_y \tag{24}$$

so that one has, similarly to plasticity

$$\bar{f} \leq 0 \tag{25}$$

Moreover, from relation (19), it can be noted, that as viscosity η goes to zero (rate-independent case), the consistency parameter γ^{vp} remains finite and positive (though indeterminate) since $\bar{\sigma} - \sigma_y$ also goes to zero. The extended criterion (23) will play a crucial role in the integration algorithm described hereafter. It also allows a generalization of the Kuhn-Tucker which, in the visco-plastic case, can be extended to the following form:

$$\dot{\gamma}^{vp}\,\bar{f} = 0, \qquad \dot{\gamma}^{vp} \geq 0, \qquad \bar{f} \leq 0 \tag{26}$$

4 NUMERICAL SCHEME

In this section the numerical scheme necessary to implement the discussed theoretical model in a finite element code is derived. This scheme is based on a predictor, *elastic*, corrector *easto/viscoplastic* approach. First the elastic problem and plastic (rate independent) problems, derived in previous works of García Garino,[1,3] are presented in sections 4.1 and 4.2 respectively. Then numerical algorithm due to Pothot for viscoplasticity is discussed and recasted in terms of kinematics variables in section 4.3 in order to discuss the resultant integration scheme.

4.1 Elastic Problem

In this problem the plastic quantities remain frozen: $({}^{t+\Delta t}\boldsymbol{F}^{pTR} = {}^{t}\boldsymbol{F}^{p})$. The trial (*elastic*) component of the deformation gradient tensor results:

$$ {}^{t+\Delta t}\boldsymbol{F}^{eTR} = {}^{t+\Delta t}\boldsymbol{F} \; ({}^{t+\Delta t}\boldsymbol{F}^{pTR})^{-1} = \boldsymbol{f} \; {}^{t}\boldsymbol{F} \; ({}^{t}\boldsymbol{F}^{p})^{-1} = \boldsymbol{f} \; {}^{t}\boldsymbol{F}^{e} \tag{27} $$

where $\boldsymbol{f}$ is the incremental deformation gradient tensor. The predictor value of the elastic Finger tensor ${}^{t+\Delta t}\boldsymbol{b}^{e-1TR}$ is:

$$ {}^{t+\Delta t}\boldsymbol{b}^{e-1TR} = \left({}^{t+\Delta t}\boldsymbol{F}^{e-T} \; {}^{t+\Delta t}\boldsymbol{F}^{e-1}\right)^{TR} = \boldsymbol{f}^{-T} \; {}^{t}\boldsymbol{b}^{e-1} \; \boldsymbol{f}^{-1} \tag{28} $$

Finally, the trial stresses $\boldsymbol{\sigma}^{TR}$ are computed from eqn 28 in terms of the predictor value of elastic Almansi strain ${}^{t+\Delta t}\boldsymbol{e}^{eTR} = \frac{1}{2}({}^{t+\Delta t}\boldsymbol{g} \; - \; {}^{t+\Delta t}\boldsymbol{b}^{e-1TR})$.

It is important to note that the elastic problem is reduced to the computation of a closed expression. In this way numerical integration of rate equations, typical of hypoelastic models and usually very expensives, is completely avoided.

4.2 Plastic Problem

In this problem the current configuration remains fixed and the internal variables are updated in order to satisfy the constitutive law. For this problem Simo[5] has proposed to integrate the flow rule in the original configuration:

$$ \dot{\boldsymbol{C}}^{p} = 2 \; \phi^{*}\boldsymbol{d}^{p} = 2 \; \dot{\lambda} \; \phi^{*}\boldsymbol{n} = 2 \; \dot{\lambda} \; \boldsymbol{N} \tag{29} $$

where $\boldsymbol{C}^{p}$ is the plastic component of right Cauchy Green tensor and ϕ^{*} denotes the pull-back operator.[41]

Equation 29 is integrated using a Backward-Euler scheme:

$$ {}^{t+\Delta t}\boldsymbol{C}^{p} - {}^{t}\boldsymbol{C}^{p} = 2 \, \lambda \; {}^{t+\Delta t}\boldsymbol{N} \tag{30} $$

where λ accounts for the numerical counterpart of plastic multiplier $\dot{\gamma}$. Pushing eq (6) forward the spatial configuration, the updated Finger tensor is found:

$$ {}^{t+\Delta t}\boldsymbol{b}^{e-1} = {}^{t+\Delta t}\boldsymbol{b}^{e-1TR} + 2 \, \lambda \; {}^{t+\Delta t}\boldsymbol{n} \tag{31} $$

The factor $2 \, \lambda \; {}^{t+\Delta t}\boldsymbol{n}$ is computed by mean of the radial return algorithm.

4.3 Viscoplastic Problem

Viscoplastic counterpart of rate independent problem presented in previous section can be written in terms of viscoplastic component of right Cauchy Green tensor $\dot{\boldsymbol{C}}^{vp}$. Numerical plastic multiplier has been denoted λ^{vp} for this problem.

$$\overset{\bullet}{\boldsymbol{C}}^{vp} = 2\,\phi^{*}\boldsymbol{d}^{p} = 2\;\overset{\bullet}{\lambda}^{vp}\;\phi^{*}\boldsymbol{n} = 2\,\overset{\bullet}{\lambda}\,\boldsymbol{N} \tag{32}$$

Following the same steps of plastic corrector the updated figer tensor is computed:

$$^{t+\Delta t}\boldsymbol{b}^{e-1} = {}^{t+\Delta t}\boldsymbol{b}^{e-1^{TR}} \; + 2\,\lambda^{vp}\;{}^{t+\Delta t}\boldsymbol{n} \tag{33}$$

From equations (31) and (33) follows that both updates are identical with exception of plastic multipliers λ and viscoplastic multipliers λ^{vp}. Consequently the structure of numerical problem is preserved and rate dependent case is naturally encompassed as a particular case of corrector step.

From equation (3 the elastic component of Almansi strain tensor results in terms of the viscoplastic update of elastic Finger tensor given in equation fingervp:

$$^{t+\Delta t}\boldsymbol{e}^{e} = \frac{1}{2}\,(g\; - \;{}^{t+\Delta t}\boldsymbol{b}^{e-1}) = \frac{1}{2}\,(g\; - \;{}^{t+\Delta t}\boldsymbol{b}^{e-1^{TR}}\; - \;2\,\lambda^{vp}\;\;{}^{t+\Delta t}\boldsymbol{n}) = {}^{t+\Delta t}\boldsymbol{e}^{eTR}\; - \;\lambda^{vp}\;\;{}^{t+\Delta t}\boldsymbol{n} \tag{34}$$

Taking into account equation (11), the viscoplastic correction of elastic component of Almansi strain tensor given in equation (34) is written in terms of Cauchy stress tensor as:

$$^{t+\Delta t}\boldsymbol{\sigma} = {}^{t+\Delta t}\boldsymbol{\sigma}^{TR}\; - \;2\,\lambda^{vp}\,\mu\,{}^{t+\Delta t}\boldsymbol{n} \tag{35}$$

that is the result shown in equation (51), section 6.3 in the work of Ponthot,[7] after integration over the time interval $[t, t+\Delta t]$, with initial conditions given by ${}^{t}\boldsymbol{\sigma}$, ${}^{t}\bar{\epsilon}^{vp}$ and ${}^{t}\sigma_{y}$.

In order to compute the viscoplastic multiplier λ^{vp} an integration procedure very similar to the radial return method of plasticity, proposed by Ponthot[7] is used. The tensor ${}^{t+\Delta t}\boldsymbol{n}$ is approximated by:

$$^{t+\Delta t}\boldsymbol{n} = \frac{^{t+\Delta t}\boldsymbol{s}^{TR}}{\sqrt{^{t+\Delta t}\boldsymbol{s}^{TR} : {}^{t+\Delta t}\boldsymbol{s}^{TR}}} \tag{36}$$

so that the final values are given by

$$^{t+\Delta t}\bar{\epsilon}^{vp} \;=\; {}^{t}\bar{\epsilon}^{vp} + \sqrt{\frac{2}{3}}\lambda^{vp} \tag{37}$$

$$\overset{\bullet}{\bar{\epsilon}}^{vp} \;=\; \frac{^{t+\Delta t}\bar{\epsilon}^{vp} - {}^{t}\bar{\epsilon}^{vp}}{\Delta t} \tag{38}$$

where the (unknown) scalar parameter λ^{vp} stands for

$$\lambda^{vp} = \int_{t}^{t+\Delta t} \overset{\bullet}{\lambda}\;dt \tag{39}$$

REMARK: It is important to point out that the first order approximation introduced in eq. (38) is fully consistent with the approximation introduced in eq. (30).

The λ^{vp} parameter is simply determined by the enforcement of the *generalized consistency condition*, $\bar{f} = 0$, at time $t = t + \Delta t$, i.e.

$$\begin{aligned} \bar{f}(\lambda^{vp}) &= \sqrt{\frac{3}{2}\,[\boldsymbol{s}^{TR} - 2\mu\,\lambda^{vp}\,\boldsymbol{n}] : [\boldsymbol{s}^{TR} - 2\mu\,\lambda^{vp}\,\boldsymbol{n}]} - {}^{t+\Delta t}\sigma_y(\lambda^{vp}) \\ &- \eta\,(\bar{\epsilon}_0^{vp} + \sqrt{\frac{2}{3}}\lambda^{vp})^{\frac{1}{n}}\,(\sqrt{\frac{2}{3}}\frac{\lambda^{vp}}{\Delta t})^{\frac{1}{m}} = 0 \end{aligned} \tag{40}$$

where ${}^{t+\Delta t}\sigma_y$ is a given function of $\bar{\epsilon}^{vp}$ and consequently a given function of λ^{vp}.

The scalar equation (40) is a nonlinear expression where the only unknown parameter is λ^{vp}. It can be easily solved by a local Newton-Raphson iteration. In the particular case where $n = \infty$ (no multiplicative hardening), $m = 1$ (linear dependence between overstress and viscoplastic rate of deformation), and $h =$ constant (linear hardening) a closed form solution of this equation is given by

$$\lambda^{vp} = \frac{1}{2\mu}\frac{\sqrt{\boldsymbol{s}^{TR} : \boldsymbol{s}^{TR}} - \sqrt{\frac{2}{3}}{}^{t}\sigma_y}{1 + \frac{1}{3\mu}(h + \frac{\eta}{\Delta t})} \tag{41}$$

so that it is now obvious that the present algorithm is a generalization to the rate-dependent case of the classical radial return algorithm. This one is exactly recovered (with no numerical difficulty) by setting $\eta = 0$ (no viscosity effect). In the viscous case, one can see that the rate-dependent solution (41) is equivalent to rate-independent solution with a fictitious hardening given by $h^* = h + \eta/\Delta t$.

5 CONCLUSIONS

Preliminary results of a large strain viscoplastic model have been presented. Both theoretical constitutive model and numerical implementation have been discussed.

The structure of elastoplastic model based on hyperelasticity and internal variables theory is mantained and viscoplastic problem is easily taken into account due the uncoupled structure of free energy function.

Consequently the structure of the numerical scheme is preserved, the elastic problem remains with no changes and viscoplastic corrector step ecompass in the structure of plastic corrector when stress update algorithm is recasted in terms of kinematics variables. In this way the numerical format of the problem naturally includes viscoplasticity.

The stress update procedure is easily solved after a local non linear iterations at integration point level for the general case and various closed forms expresions are derived for different particular cases. The discussed procedure recover the results of radial return algorithm for the inviscid case.

Consequently all the advantages that can be obtained from radial return method like simplicity, robustness and computational efficiency are mantained.

The numerical scheme admits a consistent tangent operator that will be presented in next works together with numerical applications.

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