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TERRESTRIAL REFERENCE FRAME IN THE DATUM DEFINITION OF A FREE TRILATERATION NETWORK

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Abstract. Terrestrial Reference Frames (TRFs) are very useful tools in geodesy, surveying, navigation by land, sea or air, cartography, cadastral works, engineering applications and geophysical investigations. Nowadays, the TRFs are obtained using a wide range of techniques including the Global Positioning System (GPS) among others. The datum problem is one of the central issues in the solution of the inverse problems related with the adjustment of geodetic networks. Many contributions to the study of the geodetic datum problem or zero order design problem (Grafarend, 1974) has been realized since the fundamental works of Meissl (1965,1969) and Blaha (1971), followed by Grafarend and Schaffrin (1974,1976).The relation between minimal constrained solutions have been well established by means of the S-transformation (Baarda, 1973). The concepts and terminology of the Terrestrial Reference System of coordinates and frames were redefined in the 1980s by the astronomical and geodetic communities (Kovalevsky et al., 1989) , and incorporated in the geodetic datum analysis in works such as : a) a review of the algebraic constraints in TRF datum definition by Sillard, P, and Boucher, C. (2001), b) the datum definition in the combination of several particular TRFs from Geodetic Space Techniques for the release of the International Terrestrial Reference Frame by Altamimi, Z ; Sillard, P and Boucher, C. (2002) and, c) the datum definition of geodetic networks within an adjustment model known as Singular Gauss-Markov Model (SGMM) by Vacaflor, J.L. (2008,2010,2011) where the structure of the weights and the selection matrix as well as the role of the Helmert transformation parameters which may have in the datum minimum constraints equations were established. It is (c), the specific context, where a new advance: two representations of the Terrestrial Reference Frame chosen in the datum definition "TRFd" were found for a planar geodetic network and are presented here. Indeed, the main goal of this work is to obtain an explicit and implicit representations of the Terrestrial Reference Frame TRFd(x,y) chosen in the datum definition of a free two-dimensional trilateration network, when the datum is defined by means of three linear conditions equations (minimum constraints), in order to define and realize the origin and the orientation of the Terrestrial Reference System of coordinates TRS(x,y) . The research is realized within the specific adjustment model known as Singular Gauss-Markov Model (SGMM) and in relation with the Restricted LEast Square Solution (RLESS) of the inverse problem: the estimation of the planar coordinates (x,y) of the network points based in observed distances. The explicit representation obtained of the TRFd(x,y) is given by the coordinates (xd,yd) of the chosen datum points and used in the minimum constraints, while the implicit representation is given by four parameters of a plane coordinate Helmert transformation : two translation ,one differential rotation and one scale factor, with respect to a known "a priori" Terrestrial Reference Frame TRFd(xo,yo) chosen in the datum definition.

1 INTRODUCTION

The datum problem is one of the central issues in the solution of the inverse problems related with the adjustment of geodetic networks.

Many contributions to the study of the geodetic datum problem or zero order design problem (Grafarend,1974) has been realized since the fundamental works of Meissl (1965,1969) and Blaha (1971), followed by Grafarend and Schaffrin (1974,1976).

The relation between minimal constrained solutions have been well established by means of the S-transformation (Baarda, 1973). The concepts and terminology of the Terrestrial Reference System of coordinates and frames were redefined in the 1980s by the astronomical and geodetic communities (Kovalevsky et al., 1989) , and incorporated in the geodetic datum analysis in works such as : a) a review of the algebraic constraints in TRF datum definition by Sillard, P, and Boucher, C. (2001), b) the datum definition in the combination of several particular TRFs from Geodetic Space Techniques for the release of the International Terrestrial Reference Frame by Altamimi, Z ; Sillard, P and Boucher, C. (2002) and, c) the datum definition of geodetic networks within an adjustment model known as Singular Gauss-Markov Model (SGMM) by Vacaflor, J.L. (2008,2010,2011) where the structure of the weights and the selection matrix as well as the role of the Helmert transformation parameters which may have in the datum minimum constraints equations were established.

It is (c), the specific context, where a new advance: two representations of a Terrestrial Reference Frame chosen in the datum definition “TRFd” were found for a planar geodetic network and are presented below.

2 TERRESTRIAL REFERENCE FRAME IN THE DATUM DEFINITION OF A FREE TRILATERATION NETWORK

The main goal of this work is to obtain an explicit and implicit representations of the Terrestrial Reference Frame chosen in the datum definition of a free two-dimensional trilateration network, when the datum is defined by means of three linear conditions equations (minimum constraints), in order to define and realize the origin and the orientation of the Terrestrial Reference System of coordinates TRS(x,y).

The TRS(x,y) is a trirectangular trihedron right-handed oriented, its vertex is a point **P** not specified of the Earth’s surface and it is the origin **o** of the Cartesian coordinate system (x,y), the first and second rays are the **ox** and **oy** positive axis respectively with not specified orientations. The third ray is oriented “upward” aligned with the vertical in **P** and it is orthogonal to the others two rays. The scale or length defined of the unit vectors along **ox** and **oy** of the TRS(x,y) is the meter (SI), and it is realized by the observed distances of the trilateration network.

Therefore, we are dealing with: a) a two-dimensional trilateration network constituted by “k” physical points P_i with coordinates (x_i, y_i), $i = 1 \ldots k$ in the $TRS(x, y)$, where “n” distances between these points have been observed, b) the position and orientation of the $TRS(x, y)$ are not *defined* for any epoch causing a datum defect and therefore, a *free* geodetic network, and c) the coordinates (x_i^0, y_i^0), $i = 1 \ldots k$ “a priori” or “approximated” are available from the known reference frame $TRF(x_0, y_0)$.

The lack of definition in the origin and orientation of the TRS(x,y) cause a datum defect and a rank-deficient Singular Gauss-Markov Model (SGMM) for the adjustment of the network (Schaffrin,1985) :

$$y - e = A\xi \quad , r(A) =: q < m < n \ , \ t =: 3 = m - q \ , \ e \sim (0, \sigma_0^2 P^{-1} =: D\{y\}) \quad (1)$$

with,

$n =$ Number of observations; $m =$ Number of unknown parameters ; $r =$ Rank
$t =$ Number of datum defect = 3; $D =$ Dispersion; $P_{nxn} =$ Symmetric positive-definite weight matrix; $o =$ Order; $\sigma_0^2 =$ Unknown (observational) variance component; E=Expectation.

$y_{nx1} =$ Vector of observations (increments)

$y_{nx1} = [y_{ij}] = [(s_{12}^{obs} - s_{12}^0), (s_{13}^{obs} - s_{13}^0), ..., (s_{ij}^{obs} - s_{ij}^0), ..., (s_{k-1,k}^{obs} - s_{k-1,k}^o)]^T$

$e_{nx1} = [e_{ij}] =$ Vector of random errors (unknown)

$E\{e_{nx1}\} = 0$

$s_{ij}^0 = \sqrt{(\Delta x_{ij}^0)^2 + (\Delta y_{ij}^0)^2}$, $i = 1..k$, $j = 1..k$, $i < j$

$\Delta x_{ij}^0 = x_j^0 - x_i^0$; $\Delta y_{ij}^0 = y_j^0 - y_i^0$

$A_{nxm} =$ Design or coefficient matrix ("Jacobian")

$$A_{nxm} = \begin{bmatrix} \alpha_{12} \\ \cdots \\ \alpha_{ij} \\ \cdots \\ \alpha_{k-1,k} \end{bmatrix} ; \ \alpha_{ij_{1xm}} = [0, ..., -\Delta x_{ij}^0, -\Delta y_{ij}^0, ..., \Delta x_{ij}^0, \Delta y_{ij}^0, ..., 0].(1/s_{ij}^0)$$

$\xi_{mx1} =$ Vector of unknown parameters (coordinate increments).

$\xi_{mx1} = X_{mx1} - X_{mx1}^0$

$X_{mx1} =$ Vector of unknown coordinates of the points P_i of the TRF(x,y) expressed in the $TRS(x, y)$.

$X_{mx1} = [x_1, y_1 ... x_k, y_k]^T$

$X_{mx1}^0 =$ Vector of known coordinates of P_i of the "a priori" or "approximated" $TRF(x_0, y_0)$.

$X_{mx1}^0 = [x_1^0, y_1^0 ... x_k^0, y_k^0]^T$

$\xi_{mx1} = [dx_1 \quad dy_1 \quad ... \quad dx_k \quad dy_k]^T$; $dx_i = x_i - x_i^0$; $dy_i = y_i - y_i^0$, $i = 1..k$, $m = 2k$

To complete the datum definition of the network in (1), it is necessary to introduce as minimum three independent condition equations to *define* and *realize* in a given epoch the origin and orientation of the $TRS(x, y)$.

In Vacaflor, J.L. (2010, p.2651), it was shown that this goal can be reached, if in (1), it is introduced a minimum datum constraint:

$$E_{3xm} S_{mxm} \xi_{mx1} = E_{3xm} S_{mxm} E_{mx3}^T P T_{3x1}^* \qquad (2)$$

$$R(A^T) \oplus R(K^T) = \Re^m$$

$$AE^T = 0\ ,\ o(E) = txm\ ,\ r(E) = t$$

$$\Rightarrow R(A^T)\overset{\perp}{\oplus} R(E^T) = \Re^m$$

where,

$$\xi = E^T PT^*$$
$$PT^* = [t_x^*, t_y^*, d\delta^*]^T$$

$$K := ES\ ,\ o(S) = mxm\ ,\ r = r(S) \geq m - q\ ,\ m = 2k$$
$$S := Diag(s_{dx_1}, s_{dy_1}, \ldots, s_{dx_i}, s_{dy_i}, \ldots s_{dx_k}, s_{dy_k})\ ,\ i = 1 \ldots k$$

S: Selection matrix of coordinate differences.

$PT^* =$ Conventionally adopted numerical values of the transformation parameters of the TRS(x,y) with respect to the TRS(xo,yo) where the coordinates of the TRF(xo,yo) are expressed : two translations t_x^*, t_y^* and one differential rotation $d\delta^*$ (of the $TRS(x,y)$) to *define* the position and orientation of the $TRS(x,y)$.

$$E_{3xm} = \begin{bmatrix} 1 & 0 & \ldots & 1 & 0 \\ 0 & 1 & \ldots & 0 & 1 \\ -y_1^0 & x_1^0 & \ldots & -y_k^0 & x_k^0 \end{bmatrix}$$

By assigning the values “0” or “1” respectively to the diagonal elements of S, it is selected or excluded in the first member of (2) the coordinate differences $dx_i = x_i - x_i^0$, $dy_i = y_i - y_i^0$, $i = 1 \ldots k$ of the *selected datum points* P_i and their corresponding numerical values are provide in the second member respectively.

Hereinafter, for the coordinate differences dx_i, dy_i of the datum point P_i, the subscript “d” (datum) will be incorporated and designated as *datum coordinate differences*: $dx_{id} = x_{id} - x_{id}^0$, $dy_{id} = y_{id} - y_{id}^0$, $i = 1 \ldots k$ of the *datum point* P_{id}.

Therefore, for example, if $s_{dx_1} = 1 \Rightarrow dx_1 = x_1 - x_1^0$ is selected for the datum definition in (2), and it will be designate as $dx_{1d} = x_{1d} - x_{1d}^0$ of the datum point P_{1d}, and if $s_{dx_1} = 0 \Rightarrow dx_{1d} = x_{1d} - x_{1d}^0$ is exclude from the datum definition given by (2).

Hence, once the structure of S is established, the minimum constraints (2) define the datum of the network, since:

* Define for a given epoch, the position and orientation of the $TRS(x,y)$ by adopting conventionally the numerical values of the transformation parameters PT_{3x1}^*.

* Realize the position and orientation of the $TRS(x,y)$ by providing the numerical values of the selected datum coordinate differences, $dx_{id} = x_{id} - x_{id}^0$, $dy_{id} = y_{id} - y_{id}^0$, $i = 1 \ldots k$., *or equivalently*, by providing the numerical values of a *selected set of datum coordinates* $\{\ldots x_{id}, y_{id}, \ldots\}$ $i = 1 \ldots k$ (since x_{id}^0, y_{id}^0 are known) of a Terrestrial Reference Frame constituted by the chosen datum points P_{id}, $i = 1 \ldots k$ that it is designated as : *The Terrestrial Reference Frame chosen in the datum definition TRFd(x,y)* of the network.

Hence, if it is designated the *"a priori"* Terrestrial Reference Frame chosen in the datum definition TRFd(xo,yo) of the network, as the set of chosen datum points P_{id} with known coordinates (x_{id}^0, y_{id}^0) taken from the available Terrestrial Reference Frame $TRF(x_0, y_0)$, and taking into account the ways to represent a TRF given by Bähr, H. et. al., (2007, p.16), the TRFd(x,y) can be represented as:

- *Explicit representation*: the TRFd(x,y) is given by the coordinates (x_{id}, y_{id}) of all chosen datum points P_{id} and used in the minimum constraints (2).

-*Implicit representation*: The TRFd(x,y) is given by four transformation parameters $PT_d := [t_{xd}, t_{yd}, d\delta_d, ds_d]^T$ with respect to the TRFd(xo,yo), where t_{xd} and $,t_{yd}$ are two translations, $d\delta_d$ is a differential rotation and ds_d is the scale factor.

3 CONCLUSIONS

In this work, it is obtained explicit and implicit representations of the Terrestrial Reference Frame TRFd(x,y) chosen in the datum definition of a free two-dimensional trilateration network , when the datum is defined by means of three linear conditions equations (minimum constraints), in order to define and realize the origin and the orientation of the Terrestrial Reference System of coordinates TRS(x,y) .

The *explicit* representation of the TRFd(x,y) is given by the coordinates (x_{id}, y_{id}) of all chosen datum points P_{id} and used in the minimum constraints, and the *implicit* representation is given by four transformation parameters $PT_d := [t_{xd}, t_{yd}, d\delta_d, ds_d]^T$ with respect to the *"a priori"* known Terrestrial Reference Frame chosen in the datum definition TRFd(xo,yo) , where t_{xd} and t_{yd} are two translations, $d\delta_d$ is a differential rotation and ds_d is the scale factor.

The research is realized within the specific adjustment model known as Singular Gauss-Markov Model (SGMM) and in relation with the Restricted LEast Square Solution (RLESS) of the inverse problem: the estimation of the planar coordinates (x,y) of the network points based in observed distances.

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