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INTEGRITY OF AN OFFSHORE STRUCTURE SUBJECTED TO WAVES: A STOCHASTIC ANALYSIS

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Abstract.

A fatigue analysis procedure was developed to evaluate the structural integrity of a drilling tower welded to an offshore platform. The tower is built from welded steel plates and it has uncertainties on the thickness of the plates and on the welds. The weld between the tower and the offshore platform is critical for fatigue and the knowledge of the probability distribution of the stress cycles on this critical point of the structure is necessary to estimate its fatigue life. The stresses on this point are given by the dynamics of the tower and the excitation of the tower is given by the dynamics of the platform (base excitation) which in turn is given by the wave loads.

INTRODUCTION

An offshore platform and all of its installed equipments should be designed for a long life span. Therefore, it is necessary to evaluate the fatigue resistance of several items during design stage. To evaluate this fatigue resistance the designer needs to obtain the dynamic response of the platform to all the external loads and thereafter to obtain the base excitation over these equipments. Given the base excitation, the dynamic response of the equipments can be calculated and the stress time history on the critical for fatigue points can be obtained and then the fatigue resistance can be determined. During all this process of determining the fatigue resistance of these equipments several assumptions have to be made by the designer and the uncertainty on them have to be considered. Sacramento et al. (2013) proposed a simplified strategy to compute the fatigue damage on a drilling tower welded to an offshore platform using the power spectral density of the wave loads and the probability distribution of the occurrence of sea states. In the present work this fatigue damage will be determined considering the uncertainty on the thickness of the plates and on the thickness of the welds as well.

The evaluation of structural integrity is a required step for the design of any offshore structure. The dynamic response of the structure due to external loads need to be investigated on early stages of the design process in order to avoid significant changes afterwards. The ocean waves are a main source of external loads and as the structure will be subjected to several different sea states during the working life of the equipment, several sea surface elevations and dynamic responses simulations should be accomplished during such investigation phase. Any reduction on the computational effort for these simulations will save working time.

Many different research areas require some source of simulation of ocean waves and structural integrity evaluation. Langley (1987), investigated statistical techniques for estimating the reliability of offshore structures. He studied the reliability of an equipment modeled as a single degree of freedom system based on the available information for the intended location. Kukkanen (1996) presented a procedure for the fatigue analysis of hull structures of ships. A spectral method has been applied to determine stress responses in different short-term condition and the long-term predictions for stress responses have been determined by taking into account the operational conditions of the ship. The estimative of fatigue life of the structure has been determined using Miner's fatigue accumulation hypothesis together with probabilistic models of stress ranges and number of stress cycles. Pérez and Blanke (2002) were interested on testing of applications of ship motion control strategies and needed accurate and simple mathematical models to describe the exerted loads and motions of vehicles. After simulating the sea state for a given wave spectrum the results were related to the ship motion using the Response Amplitude Operators (RAO) for the specific ship. Kukkanen (2003) presented a fatigue analysis procedure for offshore floating structures based on the separation of hydrodynamic load and structural responses, on the effective fatigue load concept and using response interpolation in order to simplify the fatigue analysis calculating just a few directional fatigue effective load cases. Such calculation can be accomplished in early stages of the project and can be easily updated during the development of the project. Forristall (2006) needed to define the height of the deck of oil platforms and obtained statistics for the maximum crest over an area using a combination of analytical theory and numerical simulations. Forristall (2007) investigated the damage caused by hurricanes Ivan, Katrina and Rita to deep water facilities and concluded that crest heights calculated using standard theories hardly could have caused such damage and calculations of the maximum crest height over the area of the deck are able to explain it. Forristall (2011), studied the influence of the diffraction and radiation of the incident waves due to the large columns of a Tension Leg Platform (TLP) and estimated the maximum crests under the structure. Nieslony (2009) presented a method for determination of multiaxial load segments from original service histories and proposed a rainflow procedure for stress cycles counting that will be used in this work. In all these works some kind of ocean wave's simulation and structural integrity evaluation were needed.

This article is organized as follows. Section 1 presents the conceptual model for the sea surface elevation. On Section 2 it is shown the evaluation of the loads over the platform due to the sea surface elevation. These loads will be used on Section 3 to evaluate the dynamics of the platform. The dynamic response of the platform will be used on Section 4 to evaluate the dynamics of the drilling tower. An approximation to the solution of the dynamics of the drilling tower is shown on Section 5 and the fatigue analysis is accomplished on Section 6. On Section 7 the uncertainties on the model and on the method are discussed. Results are shown on Section 8 and conclusions are drawn on Section 9. An overview of the procedure can be seen on Fig. 1.

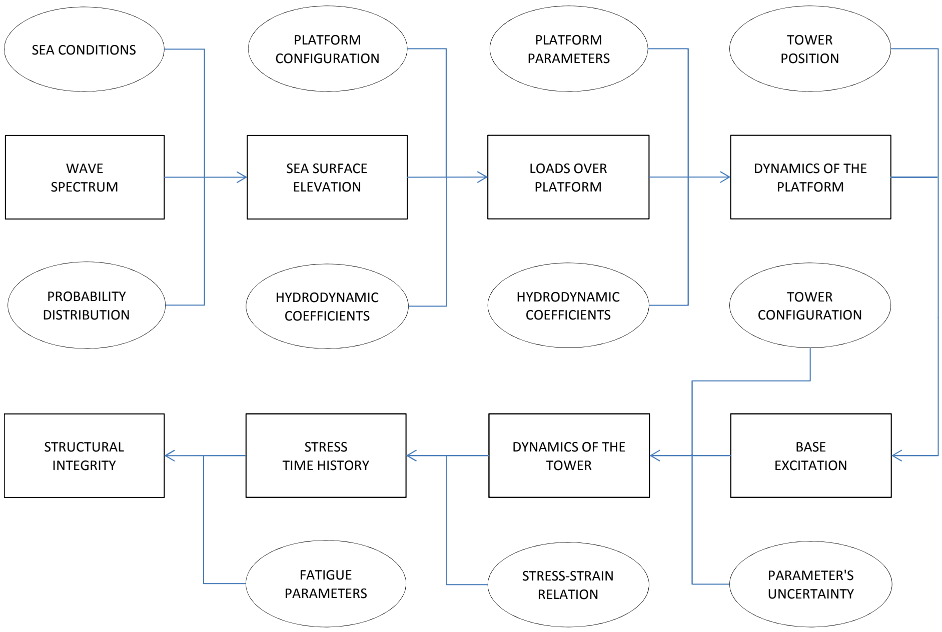

Figure 1: Overview of the procedure

1 SEA SURFACE ELEVATION

A conceptual model to describe the sea surface elevation is given by the sum of a large number of essentially independent regular (sinusoidal) contributions with random phases. In this representation, the sea surface elevation at a location x, y with respect to a X, Y, and Z global coordinate system is given by (Pérez and Blanke, 2002)

$$\zeta(x, y, t) = \sum_{i=1}^{N} \zeta_i(x, y, t) = \sum_{i=1}^{N} \bar{\zeta}_i \cos\left(k_i x \cos\chi + k_i y \sin\chi + \omega_i t + \theta_i\right), \tag{1}$$

where $\zeta_i(x, y, t)$ is the contribution of the regular or harmonic traveling wave components i

progressing at an angle χ with respect to the X direction and with a random phase θ_i. The parameters k_i (wave number), ω_i (wave frequency seen from a fixed position) and $\bar{\zeta}_i$ (constant wave amplitude) characterize each component. For each realization, the phase angle θ_i of each component is chosen to be a random variable with uniform distribution on the interval $[-\pi, \pi]$. This choice ensures the stationarity of $\zeta_i(x, y, t)$ (Pérez and Blanke, 2002).

In any particular sea state, the sea surface elevation presents irregular characteristics. After the wind has blown constantly for a certain period of time the sea elevation surface becomes stationary. In this case the sea is referred to as *fully-developed*. If the irregularity of the observed waves are only in the dominant wind direction so that there are mainly uni-dimensional wave crests with carrying separation and remaining parallel to each other the sea is referred to as a *long-crested* irregular sea (Pérez and Blanke, 2002). For a fully-developed sea the Pierson-Moskowits (PM) spectrum for the wave amplitudes is given by (Benaroya and Han, 2005)

$$S^0_{\zeta\zeta}(\omega) = \frac{8.1 \times 10^{-3} g^2}{\omega^5} \exp\left(-0.0324\left(\frac{\sqrt{g/H_S}}{\omega}\right)^4\right), \tag{2}$$

where g is the gravitational constant and H_S is the significant wave height,

In order to predict the possible sea surface elevations that the offshore platform can be subjected to it is necessary to know the values of significant wave heights and its probability of occurrence for the location where it will be installed. The Weibull distribution can be used and its cumulative distribution function is given by

$$F(H_S) = 1 - \exp\left(-\left(\frac{H_S - \gamma}{\beta}\right)^m\right) \quad \text{for} \quad \gamma < H_S, \tag{3}$$

where γ, β and m are the Weibull parameters that can be determined by least-squares methods, provided that significant wave height data over a long period of time are available. The National Data Buoy Center (NBDC) provides historical data about significant wave height collected from several stations all over the world.

The use of a sea state representation with a large number of uncorrelated sources of uncertainty in nonlinear wave body interactions leads to a computational task which may become prohibitively expensive when the statistics of extreme loads and responses are necessary. The Karhunen-Loève (KL) is an optimal basis to construct a reduced order model of the sea surface elevation in the sense that the projection on to the subspace generated by this basis contains the maximal amount of energy for a given number of trial functions (Loève, 1977) and given the power spectral density of the signal the KL basis fits the signal with the minimum number of uncorrelated sources of uncertainty (Sclavounos, 2012). An application for reduced-order models is given by Ritto et al. (2011).

The use of KL basis to represent a stochastic process is based on two assumptions, the process is stationary in time and ergodic. For long periods of time the sea surface elevation is not a stationary process and as the statistical distribution of H_s is normally determined by measuring the value of H_s at three hourly interval over an extended period (Langley, 1987) the process can be considered stationary only for a three hour period.

2 WAVE LOADS

After the sea surface elevation was obtained it is necessary to evaluate the loads over the equipment to be designed. An example of application of the proposed procedure will be given where the equipment to be designed is similar to the drilling tower mounted on a platform shown on Fig. 2.

Some simplifications on the geometry of the platform have been made and each leg of the platform will be considered to have a cylindrical shape and the connections between the legs were removed. The draft of the platform, the depth of the submerged volume of the body measured from undisturbed sea surface, has been modified in order to compensate the differences on the geometry. A sketch of the simplified platform is shown on Fig. 3.

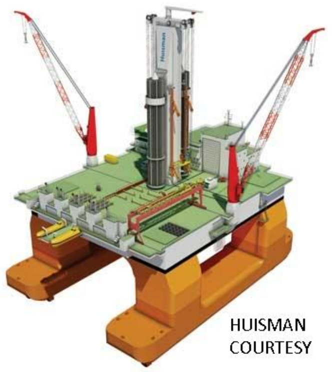

Figure 2: Drilling tower mounted on a platform

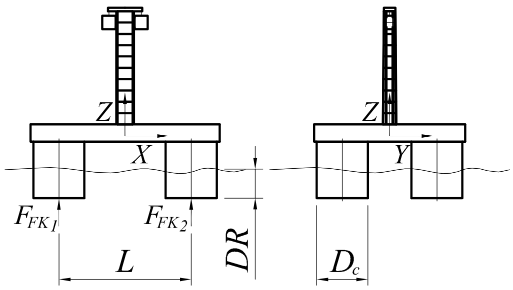

Figure 3: Sketch of the platform

The Froude-Krilov force on the bottom of each cylinder is given by (Journée and Massie, 2001)

$$F_{FK_j} = c \sum_{i=1}^{N} \zeta_i^*(x_j, y_j, t) \quad \text{for} \quad j = 1, \ldots, 4, \tag{4}$$

where ζ_i^* is the effective wave elevation at the bottom of the cylinder, x_j, y_j are the coordinates of the center of the cylinder and c is the restitution coefficient given by

$$c = \rho_w g \frac{\pi}{4} D_c^2, \tag{5}$$

where ρ_w is the water mass density and D_c the diameter of the cylinder. The effective wave elevation at the bottom of the cylinder for deep water is given by

$$\zeta_i^*(x_j, y_j, t) = e^{-k_i DR} \zeta_i(x_j, y_j, t), \text{for} \quad i = 1, 2, \ldots, N, \quad \text{for} \quad j = 1, \ldots, 4, \tag{6}$$

where DR is the draft and $\zeta_i(x_j, y_j, t)$ is the component of the sea surface elevation at the center of the cylinder. For deep water the wave number is given by

$$k_i = \frac{\omega_i^2}{g} \quad \text{for} \quad i = 1, 2, \ldots, N. \tag{7}$$

The Froude-Krilov forces are obtained from an integration of the pressures on the body in the undisturbed wave. As part of the waves will be diffracted there are two additional force components, one proportional to the effective vertical acceleration and one proportional to the effective vertical velocity, therefore the total wave force on the bottom of each cylinder is given by

$$F_{w_j} = \sum_{i=1}^{N} \left\{ a\ddot{\zeta}_i^*(x_j, y_j, t) + b\dot{\zeta}_i^*(x_j, y_j, t) + c\zeta_i^*(x_j, y_j, t) \right\} \quad \text{for} \quad j = 1, \ldots, 4, \tag{8}$$

where a is the hydrodynamic mass coefficient and b is the hydrodynamic damping coefficient.

3 DYNAMICS OF THE PLATFORM

When obtaining the dynamics of the platform the global coordinate system will be used. The equations of motion for the six degrees of freedom of the platform, influenced by external loads are given by

$$\sum_{j=1}^{6} \{(M_{ij} + A_{ij})\ddot{x}_j + B_{ij}\dot{x}_j + C_{ij}x_j\} = F_i, \quad \text{for} \quad i = 1, 2, \ldots, 6, \tag{9}$$

where i = 1 to 6 are surge, sway, heave, roll, pitch and yaw motions, x_j is the displacements of harmonic oscillation in or about direction j, M_{ij} are solid mass or inertia coefficients, A_{ij} are hydrodynamic mass or inertia coefficients, B_{ij} are hydrodynamic damping coefficients and C_{ij} are restitution coefficients and F_i is the harmonic exciting wave force or moment in direction i. In this work only the response of the platform along Z direction and about Y direction will be considered. The total wave loads on the bottom of each cylinder are the only external loads considered. It will be considered that the dynamics of the platform has no influence on the sea surface elevation.

4 DYNAMICS OF THE DRILLING TOWER

The drilling tower mounted on the platform consists of a tower used to support two lifting systems. The base of the tower is welded to the platform and this weld is critical for fatigue. The base excitation on the tower is obtained by means of a coordinate transformation of the response of the platform to the x, y and z local coordinate system located at the base of the tower. The Fig. 5 shows the model of the tower.

The tower will be considered a beam clamped to the platform and free on the other end and the normal stress due to the bending about the z direction will be calculated. As the mass of the tower is much smaller than the mass of the platform it will be considered that the dynamics of the tower does not affect the dynamics of the platform. The differential equation for a beam in bending is given by

$$-\frac{\partial^2}{\partial x^2}\left[EI_z(x)\frac{\partial^2 v(x,t)}{\partial x^2}\right] + f_y(x,t) = m(x)\frac{\partial^2 v(x,t)}{\partial t^2} \quad \text{for} \quad 0 < x < L, \tag{10}$$

where $v(x, t)$ is the displacement on y direction of any point x and instant t, $f_y(x, t)$ is the load per unit length and $I_z(x)$ is the inertia area moment about the z direction, the direction x cross the geometric center of transverse cross sections. The Euler-Bernoulli theory has been used. For more information about the dynamics of the drilling tower see Sacramento et al. (2013).

5 APPROXIMATION OF THE SOLUTION

As the tower has a variable cross section it will be necessary to obtain an approximation of the solution to the dynamics of the structure. One of the possible ways to obtain such approximation is through the discretizing of the equations that describe the dynamics of the structure using the Finite Element Method (FEM). The equations will be discretized using one-dimensional elements with two nodes and three degrees of freedom per node as shown on Fig. 4. The assembly of the elements can be seen on Fig. 6 and the approximation of the dynamics of the structure is given by

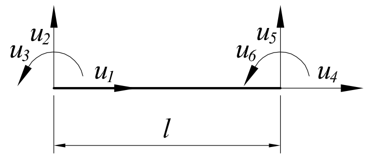

Figure 4: One-dimensional element

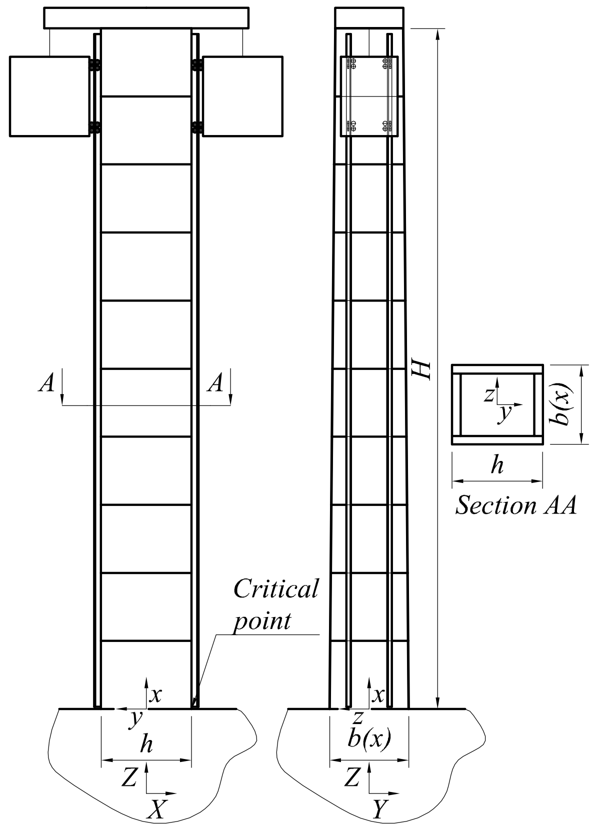

Figure 5: Sketch of the tower

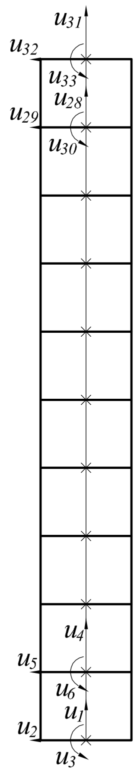

Figure 6: Assembly of the elements

$$[M]\ddot{\mathbf{X}}(t) + [C]\dot{\mathbf{X}}(t) + [K]\mathbf{X}(t) = \mathbf{F}(t), \tag{11}$$

where $[M]$, $[C]$ and $[K]$ are the global matrices of mass, damping and stiffness of the assembly of elements, $\mathbf{X}$ are the degrees of freedom of the approximation of the dynamics and $\mathbf{F}$ are the external loads over the tower. For more information about the approximation of the solution see Sacramento et al. (2013).

Since the dynamic response of the system will have to be computed successively it will be necessary to reduce the required computational effort. A reduced basis composed of the eigenvectors of the system will be used to represent the dynamic response of the system.

Being $[\Phi]$ the $m \times m$ matrix of normalized eigenvectors of the system, where m is the number of degrees of freedom of the system, and $[\Phi_r]$ a $m \times n$ $(n \ll m)$ matrix containing the first n eigenvectors of the system, the following coordinate transformation can be accomplished

$$\mathbf{X}(t) \approx [\Phi_r]a(t) \tag{12}$$

Substituting the Eq. (12) into Eq. (11) it is obtained

$$[M][\Phi_r]\ddot{a}(t) + [C][\Phi_r]\dot{a}(t) + [K][\Phi_r]a(t) \approx \mathbf{F}(t) \tag{13}$$

Projecting the dynamics of the system on the reduced subspace, $[\Phi_r]$, it is obtained

$$[M_r]\ddot{a}(t) + [C_r]\ddot{a}(t) + [K_r]a(t) = \mathbf{F_r}(t) \tag{14}$$

where

$$[M_r] = [\Phi_r]^T[M][\Phi_r], \tag{15}$$

$$[C_r] = [\Phi_r]^T[C][\Phi_r], \tag{16}$$

$$[K_r] = [\Phi_r]^T[K][\Phi_r] \tag{17}$$

and

$$[F_r](t) = [\Phi_r]^T\mathbf{F}(t) \tag{18}$$

it can be noted the dimension of the system is now $n \times n$.

6 FATIGUE ANALYSIS

In order to determine the fatigue strength of any equipment it is necessary to calculate the cumulative damage on its structure caused by cyclic loads. The expected cumulative damage for the total working life of the equipment at every point of the structure considered critical for fatigue should not exceed a critical level (Kukkanen, 1996). In this work the fatigue life will be calculated based on the S-N fatigue approach under the assumption of linear cumulative damage, the Palmgren-Miner rule, given by

$$D = \frac{n}{N}, \tag{19}$$

where n is the number of stress cycles in a constant stress range S and N is the number of cycles to failure at the same constant stress range. The S-N curve for a given material and structural joint is then given by

$$NS^{m_f} = C_f, \tag{20}$$

where m_f is the fatigue strength exponent and C_f is the fatigue strength coefficient. Only the ranges of stress cycles will be considered in determining the fatigue endurance as the mean stresses have no influence on the fatigue resistance of welded connections. The chosen S-N curve for a given joint takes into account the local stress concentrations created by the joint themself and by the weld profile and the design stress can be considered the stress adjacent to the weld. If the weld is situated in a region of stress concentration the nominal stress should be multiplied by an appropriate stress concentration factor (DNV, 2010a).

In this work the structural integrity of the welded connection of the base of the tower to the deck of the platform will be investigated as it is critical for fatigue and a failure in this connection would be catastrophic. Due to this criticality, DNV (2010b) recommends the use of a full penetration weld and a non destructive examination after the welding process in order to check for the existence of cracks or bubbles on the weld. The cross section of the tower is shown on Fig. 5. For more information about the fatigue damage evaluation see Sacramento et al. (2013).

7 MODEL UNCERTAINTIES

The inherent uncertainties on the parameters or operators of any mechanical system must be considered during the evaluation of its fatigue resistance. Since the probability density function for the random variables that represent such parameters or operators are not always available in advance for the designer it is necessary a strategy to obtain the necessary probability density functions.

If there is not enough information available for the random variables the Principle of Maximum Entropy can be used to obtain an approximation of the required probability density function Shannon (1948), Jaynes (1957a) and Jaynes (1957b). This principle states that:

"Among all the probability distributions consistent with the prescribed conditions the one that maximizes the uncertainty (entropy) should be chosen"

Being n the number of the welds of the tower, W a random vector with n components and pW_i the probability density function for the $i = 1, 2, \ldots, n$ thicknesses of the welds, the entropy related to pW_i is given by

$$S\left(pW_i\right) = -\int_{-\infty}^{+\infty} pW_i(w) \ln\left(pW_i(w)\right) dw \qquad \text{for } i = 1, 2, \ldots, n \tag{21}$$

The only available information about the random variables is the fabrication tolerance, $Wmin_i < W_i < Wmax_i$. By using the Principle of Maximum Entropy the obtained probability density function is

$$pW_i(w) = \mathbb{1}_{[Wmin, Wmax]}(w)\Pi_i \frac{1}{Wmax_i - Wmin_i} \qquad \text{for } i = 1, 2, \ldots, n \tag{22}$$

therefore, the random variables are independents with uniform probability distribution. The same distribution applies to the thickness of the plates.

8 RESULTS

The expected damage on the critical point of the tower shown on Fig. 5 will be calculated. The working life of the equipment is 20 years. The main parameters of the platform and of the tower are shown on Tab. 1.

The Pierson-Moskowitz spectrum has been used to identify the frequency composition of the sea surface wave elevation. The spectrum is shown on Fig. 7.

Table 1: Main parameters of the equipment

Diameter of the legs of the platform	D_c	27m
Distance between the legs	L	70m
Draft of the platform	DR	15m
Mass of the platform	M_P	15.000ton
Height of the tower	H	60m
Height of the cross section	h	8m
Width of the cross section	$b(x)$	7m to 6m
Thickness of the plates of the tower	t	24mm to 15mm
Hydrodynamic mass coefficient	a	760ton
Hydrodynamic damping coefficient	b	68ton/s

For each simulation the phase angles for the different frequencies of sea surface elevation were randomly chosen between 0 and 2π rad. The probability density function for the phase angles was considered to be uniform.

On Fig. 8 one can see a snapshot of the sea surface elevation obtained using the PM spectrum. The sea surface elevation was calculated and a realization of the elevation at two of the cylinders of the platform is shown on Fig. 9.

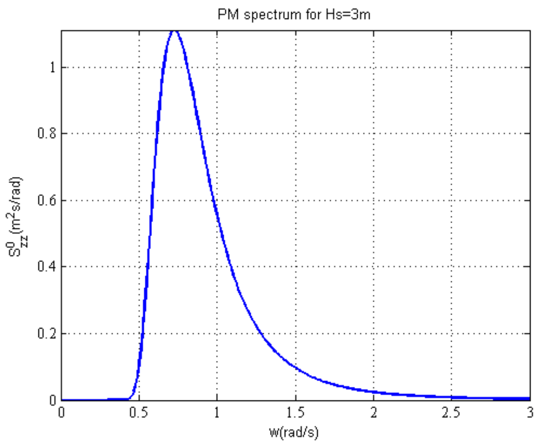

Figure 7: PM spectrum

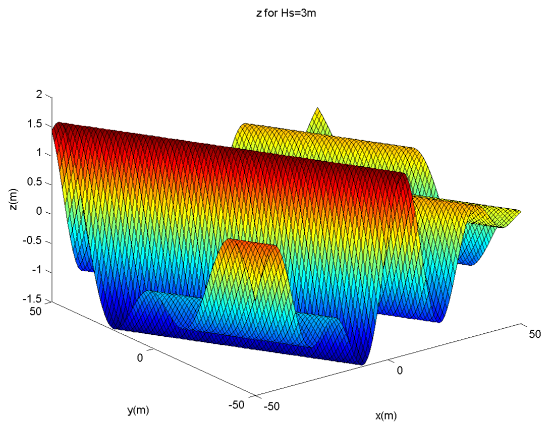

Figure 8: Sea surface elevation

A KL decomposition of the sea surface elevation was accomplished in order to reduce the computational cost of calculating the dynamic response of the platform. The simulation was carried out for 10s. After the simulation the KL basis has been obtained and the results have been approximated using only 6 modes. The construction of KL basis took 5% of the necessary time to construct the original model. On Fig. 10 one can see a comparison between the original result from simulation and the result obtained from reduced-order model for all the points on the field at a given instant. A good agreement between both results can be noted.

The Froude-Krilov forces on the bottom of the cylinders are the only external forces acting on the platform. A realization of theses forces is shown on Fig. 11. The incidence angle χ considered will be zero, therefore the Froude-Krilov force on two of the cylinders will be $F_1(t)$ and at the other two cylinders will be $F_2(t)$. The dynamics of the platform is given by Eq. (23). The restitution coefficient is given by Eq. (5). The remaining parameters are given on Tab. 1. A realizations of platform displacement is shown on Fig. 12.

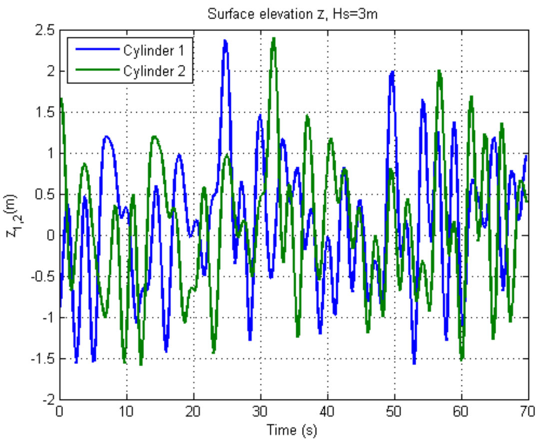

Figure 9: Sea surface elevation at cylinders

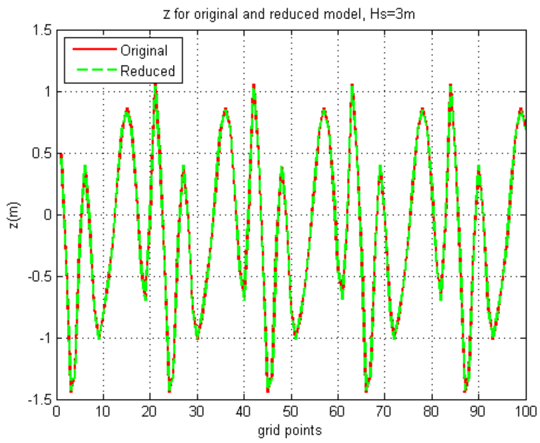

Figure 10: Original x reduced-order model

$$\begin{bmatrix} M_P + 4a & 0 \\ 0 & I_{yy} + 2aL^2 \end{bmatrix} \begin{Bmatrix} \ddot{z} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 4b & 0 \\ 0 & 2bL^2 \end{bmatrix} \begin{Bmatrix} \dot{z} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} 4c & 0 \\ 0 & 2cL^2 \end{bmatrix} \begin{Bmatrix} z \\ \theta \end{Bmatrix} = \begin{Bmatrix} 2\left(F_1 + F_2\right) \\ 2\left(F_2 - F_1\right) L \end{Bmatrix} \tag{23}$$

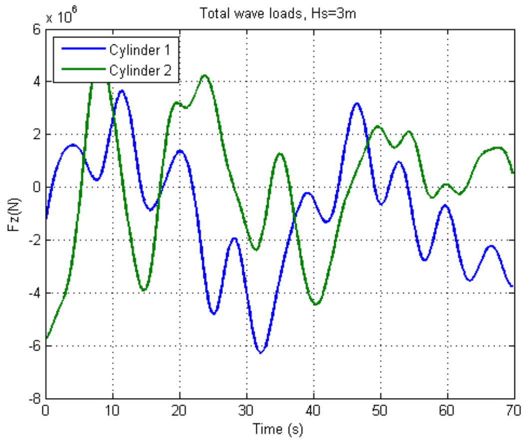

Figure 11: Total wave loads at cylinders

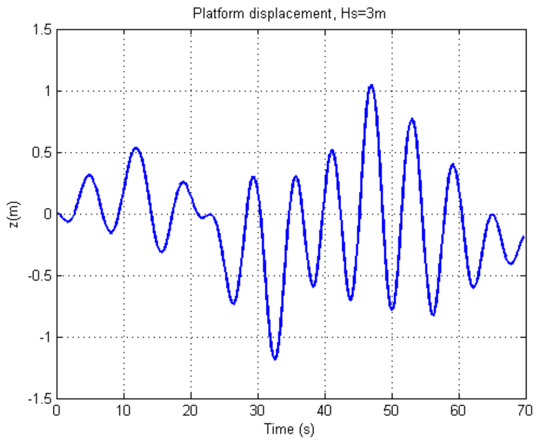

Figure 12: Platform displacement

The lifting load will be considered a concentrated mass at the free end of the beam. As the equipment is not always lifting the maximum load and there are limitations for the maximum load depending on the sea condition it is necessary to estimate during design phase of the equipment the rate of use of the equipment for each expected sea condition. The Weibull parameters for the area where the structure will be installed are $\gamma = 0.84$, $m = 1.6$ and $\beta = 1.6$. The expected working years per each significant wave height are shown on Tab. 2.

The maximum lifting load of the equipment is 800ton and this is the maximum value of the concentrated mass, M_c. The rates of utilization of the equipment under each sea condition are given on Tab. 3.

Table 2: Significant wave height probability

H_S (m)	Prob.	Working years
1	0.0248	$t_1 = 0.496$
2	0.4252	$t_2 = 8.503$
3	0.3514	$t_3 = 7.028$
4	0.1474	$t_4 = 2.948$
5	0.0413	$t_5 = 0.827$
6	0.0084	$t_6 = 0.169$
7	0.0013	$t_7 = 0.026$
8	0.0002	$t_8 = 0.003$

Table 3: Rates of utilization of the equipment

Condition	-	1	2	3	4	5	6	7	8	9	10
h_S	m	1	1	1	1	1	2	2	2	2	2
Time	years	$\frac{t_1}{5}$	$\frac{t_1}{5}$	$\frac{t_1}{5}$	$\frac{t_1}{5}$	$\frac{t_1}{5}$	$\frac{t_2}{5}$	$\frac{t_2}{5}$	$\frac{t_2}{5}$	$\frac{t_2}{5}$	$\frac{t_2}{5}$
Conc. mass	ton	0	$\frac{M_c}{4}$	$\frac{M_c}{2}$	$\frac{3M_c}{4}$	M_c	0	$\frac{M_c}{4}$	$\frac{M_c}{2}$	$\frac{3M_c}{4}$	M_c
Condition	-	11	12	13	14	15	16	17	18	19	20
h_S	m	3	3	3	3	3	4	4	4	4	4
Time	years	$\frac{t_3}{5}$	$\frac{t_3}{5}$	$\frac{t_3}{5}$	$\frac{t_3}{5}$	$\frac{t_3}{5}$	$\frac{t_4}{5}$	$\frac{t_4}{5}$	$\frac{t_4}{5}$	$\frac{t_4}{5}$	$\frac{t_4}{5}$
Conc. mass	ton	0	$\frac{M_c}{4}$	$\frac{M_c}{2}$	$\frac{3M_c}{4}$	M_c	0	$\frac{M_c}{4}$	$\frac{M_c}{2}$	$\frac{3M_c}{4}$	M_c
Condition	-	21	22	23	24	25	26	27	28		
h_S	m	5	5	5	6	6	6	7	8		
Time	years	$\frac{t_5}{3}$	$\frac{t_5}{3}$	$\frac{t_5}{3}$	$\frac{t_6}{3}$	$\frac{t_6}{3}$	$\frac{t_6}{3}$	t_7	t_8		
Conc. mass	ton	0	$\frac{M_c}{4}$	$\frac{M_c}{2}$	0	$\frac{M_c}{4}$	$\frac{M_c}{2}$	0	0		

After the definition of the expected working conditions the dynamic simulations using the reduced order model can be accomplished. A realization of the time history of the stress cycles at the critical point is shown on Fig. 13. It can be noted that there are several small range cycles along the time history. Such small range cycles are accounted for during the fatigue damage evaluation procedure but have no significant contribution for the damage. The steady-state part of the response is a stationary and ergodic process, therefore only one realization is needed, since a convergence check is accomplished. An histogram of the values of stress at critical point during the simulation is shown on Fig. 14. It can be noted that this histogram can be approximated by a gaussian probability density function with zero mean.

The rainflow procedure proposed by Nieslony (2009) was used to determine the quantity of stress cycles per stress block. Only the stress ranges above 40MPa have been considered since values below this threshold cause no fatigue damage to the structure. The obtained histogram is shown on Fig. 15.

The parameters for the considered SN curve are given by $m_{f1} = 3$, $C_{f1} = 10^{12.592}$, $m_{f2} = 5$ and $C_{f2} = 10^{16.320}$ and the stress limit at 10^7 cycles is 73.1MPa. The stress concentration factor for the weld detail is 1.0. The simulation period was 1000s and it was considered to be representative of a 3 hours sea state for the given significant height.

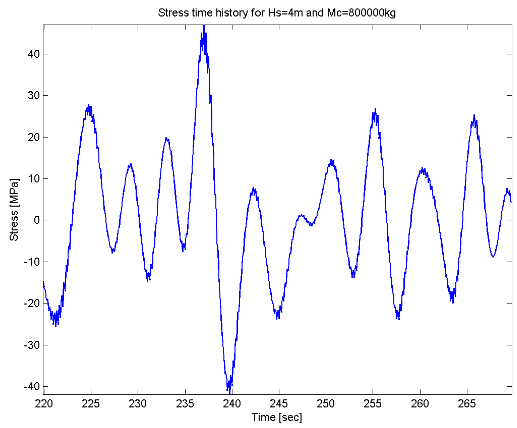

Figure 13: Stress time history at critical point

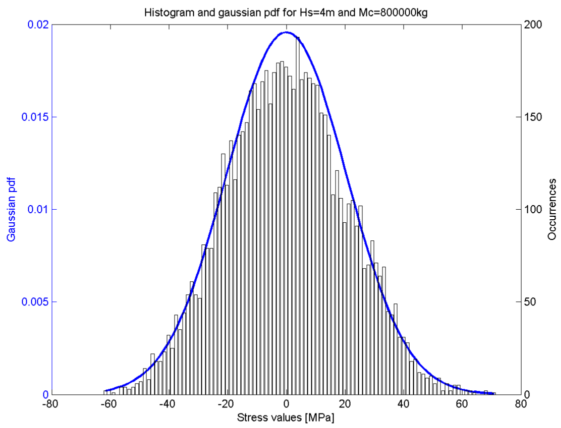

Figure 14: Histogram and Gaussian pdf

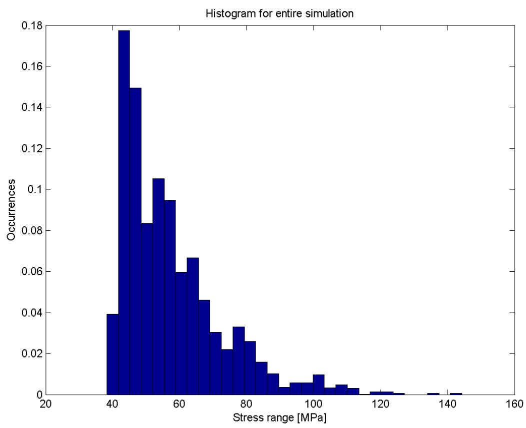

Figure 15: Histogram for entire simulation

The drilling tower is built from ten sections with different thickness welded to each other and welded to the deck of the platform. Each section in turn is built from four steel plates with same thickness, as shown on Fig. 5. Despite of the recommendation for doing a non destructive examination after the welding process, DNV (2010b), there is always some level of misalignment between the plates and the welds can not be considered to have its nominal thickness all over its length. The thickness of the steel plates is not constant all over its area as well.

Such variations on the parameters of the structure must be considered during the evaluation of the fatigue resistance of the equipment otherwise the obtained value may be to conservative. The thicknesses of the welds between the sections of the tower will be considered a random variable ranging from 80% to 100% of the thickness of the plates. Due the uncertainty on the manufacturing process of the steel plates the thickness of the plates within the length of each finite element will be considered a random variable ranging from 100% to 105% of the nominal thickness of the plates. A correlation length between the thickness of the different welds and between the thickness of the plates within each finite element has been considered.

During the Monte Carlo simulation for the evaluation of the mean value and variance of the fatigue resistance it is necessary to simulate all the sea and loading conditions for each trial of the random parameters. Since the quantity of sea and loading conditions is significative it is necessary to reduce the computational effort as much as possible.

Table 4: Influence of the uncertainty

Fatigue Damage				First Natural Frequency [Hz]			
Min.	Max	Mean	Std. Var.	Min.	Max	Mean	Std. Var.
0.57	1.04	0.71	0.15	2.99	3.01	3.00	0.004

The Tab. 4 shows a few statistics for the results obtained after the Monte Carlo simulation. It can be noted that the uncertainty on the on the thickness of the weld and on the thickness of the plates can cause some calculated fatigue damage to be above the acceptable level of 1 but has little influence on the dynamics of the structure.

9 CONCLUSIONS

In this preliminary work a computational model was developed to evaluate the structural integrity of a drilling tower welded to an offshore platform with uncertainties on the thickness of the welds and on the thickness of the plates. The base of the tower is excited by the dynamics of the platform which in turn is excited by the ocean waves. By using reduced order models for the simulation of the sea surface elevation and for the simulation of the dynamics of the tower several working conditions and geometries of the welds and plates can be accomplished with reduced computational effort. The approximation of the histogram of stress cycles by a probability density function can be used in future works to avoid the need of new finite element analysis in case of changes on the design of the equipment. The method can be used for the fatigue evaluation of equipments with more complex geometries and since there are more information about the expected working conditions and environmental conditions a better estimative of the expected fatigue damage can be obtained.

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