VIBRATIONS OF A FRAME STRUCTURE WITH A CRACK

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**Abstract.** The study of the dynamic properties of frames is very important in structural design, as they are the cornerstone for many resistant structures. A crack on a structural member introduces a local flexibility which is a function of the crack depth. This new flexibility condition changes the dynamic behavior of the structure. The knowledge of the influence of the crack on the characteristic dynamic parameters of the frame makes it possible to determine both the position of the crack and its magnitude in damaged frames. In this paper, the crack is modeled by means of an elastically restrained hinge using Chondros’ formulation while the analytical analysis is based on variational calculus and the Euler-Bernoulli beam theory to describe the transversal displacements of the frame members. In the laboratory a device was built to measure experimentally the natural frequencies of steel frames. The first ten natural frequencies of in plane vibration of cracked L-frames are obtained, different locations and depths of the crack are considered. The experimentally measured frequencies are compared with the frequencies obtained from the proposed analytical solution, the finite element method and with values reported in previous studies published on the subject by other authors.
1 INTRODUCTION

The problem of the influence of a crack in a welded joint on the dynamic behavior of a structural member has been studied in a thorough paper by Chondros and Dimarogonas, 1980.

The static and dynamic analysis of beams with single or multiple concentrated damages, produced by cracks, has received an increasing interest in recent years.

A very complete description of the state of the art in the field with the mention and description of the most important work was done by Caddemi and Morassi, 2013.

There it is explained that generally, it is assumed that the amplitude of the deformation is enough to maintain the crack always open, this model offers the great advantage to be linear and, therefore, it leads to efficient formulations for solving both static and dynamic problems.

From the first studies, it became clear that localized damage produces a local reduction in the stiffness of the beam (Thomson, 1943). Many models have been proposed in the literature to describe open cracks on beams, the flexibility modeling of cracks is quite common (Adams et al., 1978). In the case of beams under plane flexural deformation, a crack is modeled by inserting a massless, rotational elastic spring at the damaged cross section (Gudmundson, 1983; Sinha et al., 2002).

The aim of this study is to verify, through an experimental device, the accuracy of the representation of the crack by a rotational spring.

2 STRUCTURAL MODEL AND ANALYTICAL SOLUTION

We deal with the vibration of L-frames assuming an internal crack in different positions of the horizontal part of the frame.

The two parts of the L-shaped geometry are joined at right angle, with the end of one of them clamped and the end of the other elastically restrained. Figure 1 depicts the structure under study.

The position of the crack is defined by the coordinate \( l_z \) and locally affects the flexural stiffness of the L-frame. It is modeled as a massless, rotational elastic spring at the damaged cross section connecting the two adjacent segments of the beam (Gudmundson, 1983; Sinha et al., 2002).

The magnitude adopted for the flexibility constant of the equivalent spring \((\alpha_c)\), is obtained by means of the expression proposed by Chondros et al., 1998 which was found with fracture mechanics methods.

\[
\beta_c = \frac{6\pi (1-v^2)h}{EI} \phi_c(z);
\]

with

\[
\phi_c(z) = 0.6272 z^2 - 1.04533 z^3 + 4.5948 z^4 - 9.973 z^5 + 20.2948 z^6 - 33.0351 z^7 + 47.1063 z^8 - 40.7556 z^9 + 19.6 z^{10},
\]

where \( z = h_c / h \), \( h \) is the height of the cross section and \( h_c \) is the depth of the crack.
The frame-structure has three beam members: the beam $FO$ of length $l_1$, the beam $OP$ of length $l_2$ and the beam $PH$ of length $l_3$; each of them having uniform properties. The beams are modeled using the Euler-Bernoulli beam theory.

The external end $H$ is a classical clamped support and the external end $F$ is supported by two translational springs of stiffness $t_w$ and $t_u$ and a rotational spring of stiffness $r_z$. At point $P$, and modeling the crack, there is an internal hinge elastically restrained against rotation between beams 2, $OP$, and 3, $PH$, this semi-rigid connection is materialized by a rotational spring of stiffness:

$$r_m = \frac{1}{\beta_C}$$ (2)

The flexural rigidity, the mass density, the length and the area of the cross section of each beam are $EI_i$, $\rho_i$, $l_i$ and $A_i$, with $i=1, 2, 3$.

Three co-ordinate systems are located as they are shown in Fig. 1, and its origins are taken to be at points $F$, $O$ and $P$ of each beam. At abscissa $x_i$ ($0 \leq x_i \leq l_i$), $w_i$ is the transverse displacement of the beam $i$, $\theta_i = \partial w_i/\partial x$ is the section rotation and $u_i$ is the axial displacement at any time $t$. The deformation of a beam in $x$ direction is not taken into account, since the beams are considered infinitely rigid in the axial direction.
The sign convention used for the positive shear force spins an element clockwise (up on the left, and down on the right). Likewise the normal convention for a positive bending moment elongates the reference fiber of the beam indicated by the dotted line. Fig. 2 shows the sign convention to be employed.

![Sign convention for positive shear force (Q) and bending moment (M)](image)

Figure 2: Sign convention for positive shear force (Q) and bending moment (M)

For free vibration, the bending moment and the shear force expressions are:

\[ Q(x_i,t) = E_i I_i \frac{\partial^3 W_i(x_i,t)}{\partial x_i^3}; \quad M_i(x_i,t) = E_i I_i \frac{\partial^3 W_i(x_i,t)}{\partial x_i^2} \]  \hspace{1cm} (3)

To express equations in dimensionless form, the non-dimensional parameter is introduced:

\[ X_i = \frac{x_i}{l_i}; \quad \text{with} \quad X_i \in [0,1] \quad \forall i = 1,2,3. \] \hspace{1cm} (4)

The displacements \( u_i \) and \( w_i \), and \( \theta_i \) may be expressed in terms of the dimensionless coordinates as follows

\[ W_i(X_i,t) = \frac{w_i(x_i,t)}{l_i}; \quad \theta_i(X_i,t) = \frac{\partial W_i(X_i,t)}{\partial X_i} = \frac{\partial w_i(x_i,t)}{\partial x_i}; \quad U_i(X_i,t) = \frac{u_i(x_i,t)}{l_i}. \] \hspace{1cm} (5)

The characteristics of beam 1 are used as ‘reference’:

\[ EI = E_1 I_1, \quad \rho A = \rho_1 A_1, \quad l = l_1, \] \hspace{1cm} (6)

to define the ratios:

\[ \nu_{EI} = \frac{E_1 I_1}{EI}, \quad \nu_{\rho A} = \frac{\rho_1 A_1}{\rho A}; \] \hspace{1cm} (7)

and the dimensionless spring stiffness:

\[ T_w = t_w \frac{l^3}{EI}, \quad T_u = t_u \frac{l^3}{EI}, \quad R_i = R_i \frac{l}{EI}, \quad R_m = R_m \frac{l}{EI}. \] \hspace{1cm} (8)

Finally the dimensionless frequency coefficient is expressed as:

\[ \lambda^2 = \frac{l^4 \omega^2 \rho A}{EI}, \] \hspace{1cm} (9)

where \( \omega \) is the circular natural frequency of the vibrating system in radians per second.

Under the described conditions and applying the technique of variational calculus (Grossi,
2010), (Ratazzi et al., 2012) the governing differential equations of the problem and the boundary and continuity conditions are:

\[
\frac{\partial^4 W_1}{\partial X_1^4}(X_1, t) + k_1^4 \frac{\partial^4 W_1}{\partial t^2}(X_1, t) = 0; \tag{9}
\]

\[
\frac{\partial^4 W_2}{\partial X_2^4}(X_2, t) + k_2^4 \frac{\partial^4 W_2}{\partial t^2}(X_2, t) = 0; \tag{10}
\]

\[
\frac{\partial^4 W_3}{\partial X_3^4}(X_3, t) + k_3^4 \frac{\partial^4 W_3}{\partial t^2}(X_3, t) = 0; \tag{11}
\]

with \( k_j^4 = \frac{\rho A_j}{E_j I_j} \), \( i = 1, 2, 3 \).

\[
v_j W_j(1, t) = 0; \tag{12}
\]

\[
v_j W_j(1, t) = v_j W_j(0, t); \tag{13}
\]

\[
\frac{\partial W_1}{\partial X_1}(1, t) = \frac{\partial W_1}{\partial X_1}(0, t); \tag{14}
\]

\[
\frac{v_{El_j}}{v_j} \frac{\partial^2 W_1}{\partial X_1^2}(1, t) = \frac{v_{El_j}}{v_j} \frac{\partial^2 W_1}{\partial X_1^2}(0, t); \tag{15}
\]

\[
\frac{v_{El_j}}{v_j} \frac{\partial^2 W_2}{\partial X_2^2}(1, t) - R_m \left( \frac{\partial W_1(0, t)}{\partial X_1} - \frac{\partial W_2(1, t)}{\partial X_1} \right) = 0; \tag{16}
\]

\[
\frac{v_{El_j}}{v_j} \frac{\partial^2 W_2}{\partial X_2^2}(0, t) - R_m \left( \frac{\partial W_1(0, t)}{\partial X_1} - \frac{\partial W_2(1, t)}{\partial X_1} \right) = 0; \tag{17}
\]

\[
\frac{v_{El_j}}{v_j} \frac{\partial^2 W_3}{\partial X_3^2}(1, t) + T_u v_j W_j(0, t) = 0; \tag{18}
\]

\[
\frac{v_{El_j}}{v_j} \frac{\partial^2 W_3}{\partial X_3^2}(0, t) - \frac{l_1^4}{E_l} \rho A_l v_j \frac{\partial^2 W_3}{\partial t^2}(0, t) = 0; \tag{19}
\]

\[
\frac{v_{El_j}}{v_j^2} \frac{\partial^2 W_3}{\partial X_3^2}(0, t) + T_u v_j W_j(0, t) - \frac{l_1^4}{E_l} \rho A_l v_j \frac{\partial^2 W_3}{\partial t^2}(0, t) = 0; \tag{20}
\]

\[
\frac{v_{El_j}}{v_j^2} \frac{\partial^2 W_3}{\partial X_3^2}(0, t) - R_z \left( \frac{\partial W_j(0, t)}{\partial X_1} \right) = 0; \tag{21}
\]

\[
v_j W_j(1, t) = 0; \tag{22}
\]

\[
\frac{\partial W_j}{\partial X_j}(1, t) = 0. \tag{23}
\]

The last term in expression (20) is due to the rigid body vertical translation of beam 1 of length \( l_1 \), which is a consequence of assuming infinity axial rigidity.
Using the well-known separation of variables method (Grossi, 2010), solution of Eqs. (9) to (11), free vibrations of the system can be expressed in the form.

\[ W_i(X_i,t) = \sum_{n=1}^{\infty} W_{in}(X_i) e^{i\omega t}; \]  
\[ W_2(X_2,t) = \sum_{n=1}^{\infty} W_{2n}(X_2) e^{i\omega t}; \]  
\[ W_3(X_3,t) = \sum_{n=1}^{\infty} W_{3n}(X_3) e^{i\omega t}. \] 

The functions \( W_{in}, W_{2n} \) and \( W_{3n} \) represent the corresponding transverse modes of natural vibration of each beam member and are given by

\[ W_{in}(X_1) = C_i \cosh(\lambda_n \alpha_i X_1) + C_5 \sinh(\lambda_n \alpha_i X_1) + C_3 \cos(\lambda_n \alpha_i X_1) + C_4 \sin(\lambda_n \alpha_i X_1); \]  
\[ W_{2n}(X_2) = C_5 \cosh(\lambda_n \alpha_2 X_2) + C_6 \sinh(\lambda_n \alpha_2 X_2) + C_7 \cos(\lambda_n \alpha_2 X_2) + C_8 \sin(\lambda_n \alpha_2 X_2); \]  
\[ W_{3n}(X_3) = C_9 \cosh(\lambda_n \alpha_3 X_3) + C_{10} \sinh(\lambda_n \alpha_3 X_3) + C_{11} \cos(\lambda_n \alpha_3 X_3) + C_{12} \sin(\lambda_n \alpha_3 X_3), \]

where \( \alpha_i = v_i \sqrt{\rho A/(EI)} \) is a mechanical and geometrical parameter, with \( i = 1, 2, 3; \lambda_n = \sqrt{\rho^2 \alpha^2_n/(EI)} \) is the dimensionless frequency coefficient of mode of vibration \( n \) and \( C_1, C_2, \ldots, C_{12} \) are arbitrary constants to be determined.

Replacing Eq. (27), (28) and (29) in Eq. (24), (25) and (26); and these ones in Eq. (12) to (23) a linear system of equations in the unknown constants \( C_1, C_2, \ldots, C_{12} \) is obtained.

For a non-trivial solution to exist the determinant of the coefficient matrix in the linear system of equations should be equal to zero and the roots of the transcendental frequency equation are the dimensionless frequency coefficients \( \lambda_n \) of the mechanical system in Figure 1.

The results were determined by using the Mathematica software (Mathematica, 2012) with at least five significant figures. (They are indicated as “Analytical” in Table 1 and “Analytical (Eigenvalue)” in Tables 2 to 5).

3 EXPERIMENTAL DEVICE

The frame was modeled with a rod of steel 5/8 "× 1/8" \((b=15.875\text{mm}, h=3.175\text{mm})\). Both members of the frame have the same length: 446 mm, cross sectional area and material properties:

\( v_{\rho A} = 1; \nu_{EL} = 1; \forall i = 1, 2, 3; v_i = 2v_{l_1} = 2v_{l_2} = 1. \)

The constructed model was tested on a steel base composed of UPN 100 and linked to it by two welded presses (Fig. 3). According to the mass and inertia differences between the structure and the base, the outer connections can be assumed as two clamped edges (Fig. 4).
The crack was modeled with a thickness of 1mm using a saw (Fig. 5). All precautions were taken so that the cut is made smoothly and its depth is uniform: A piece of hard steel was employed, with a gap where the beam is embedded up to the desired depth. (Fig. 6 a, b)
In order to determine the mechanical parameters of the material of the strip was taken as reference a value widely verified in solid mechanics: the fundamental frequency of a cantilever beam of length 417 mm. built with the strap. It is known that the corresponding eigenvalue is 1.875

The measured frequency: 14.85 Hz, then:

\[ f = \frac{1}{2\pi} \frac{\lambda^2}{l^2} \sqrt{\frac{EI}{\rho A}}; \quad 14.85 Hz = \frac{1}{2\pi} \left( \frac{1.8751}{0.417 m} \right)^2 \sqrt{\frac{E h^2}{12 \rho}} \Rightarrow \sqrt{\frac{E}{\rho}} = 5034.7 \frac{m \text{seg}}{seg}. \]  

(30)

This value is employed in the numerical determinations.

4 FINITE ELEMENT METHOD

Numerical examples are solved by means of the finite element method, using the software ALGOR 23.1 (ALGOR software, 2009). The column and the beam are divided into 100 beams elements respectively, each beam element with three degrees of freedom.

The crack was modeled by a very small beam element, its length is equal to the width of the cut (1 mm) and height equal to the portion of the material that is not affected by cutting.

5 NUMERICAL RESULTS

Table 1 presents the first ten coefficients of natural frequency of vibration of a clamped-clamped frame, without internal hinge. The values obtained by means of the analytical approach are in very good concordance from an engineering viewpoint with those obtained using the finite element method and with particular cases available in the literature. Experimental data show a striking agreement with aforementioned values.
Table 1: First ten natural frequencies for C-C frames.

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_i$</td>
<td>3.9331</td>
<td>4.7235</td>
<td>7.0540</td>
<td>7.8255</td>
<td>10.149</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>3.9222</td>
<td>4.7145</td>
<td>7.0376</td>
<td>7.7588</td>
<td>10.007</td>
</tr>
<tr>
<td>$(f_i)$</td>
<td>56.62</td>
<td>82.08</td>
<td>183.07</td>
<td>225.29</td>
<td>378.99</td>
</tr>
<tr>
<td>$(f_i)$</td>
<td>56.76</td>
<td>83.01</td>
<td>182.50</td>
<td>228.88</td>
<td>382.08</td>
</tr>
<tr>
<td>$(f_i)$</td>
<td>56.70</td>
<td>82.71</td>
<td>183.69</td>
<td>227.86</td>
<td>383.01</td>
</tr>
</tbody>
</table>

Table 2 to 5 exhibit the results for the same frame with a crack artificially produced which varies in depth and location as indicated in each case.

The crack is located in the first third of the horizontal beam (Tables 2 and 3), in the middle (Table 4) and in the second third (Table 5).

Applying Eqs. (1), (2) and (8) one obtains $R_m=8.40$ if the crack depth $h_c$ involves 75% of the height of the section (Tables 3, 4 and 5), and $R_m=44$ for $h_c=0.5$ (Table 2).

Table 2: First ten natural frequencies for C-C frames with a crack ($h_c/h=0.5$) in the third of the horizontal beam.
<table>
<thead>
<tr>
<th>$l_2/l_1$</th>
<th>$h_2/h$</th>
<th>$R_m$</th>
<th>$i=1$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\lambda_i$</td>
<td>3.9117</td>
<td>4.6886</td>
<td>6.9269</td>
<td>7.7234</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$f_i$</td>
<td>56.22</td>
<td>80.78</td>
<td>174.25</td>
<td>217.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$f_i$</td>
<td>56.15</td>
<td>80.57</td>
<td>167.24</td>
<td>217.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$f_i$</td>
<td>56.36</td>
<td>81.35</td>
<td>174.06</td>
<td>219.89</td>
</tr>
<tr>
<td>0.33</td>
<td>0.75</td>
<td>8.40</td>
<td>$\lambda_i$</td>
<td>10.831</td>
<td>13.238</td>
<td>13.939</td>
<td>16.056</td>
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<td></td>
<td></td>
<td></td>
<td>$f_i$</td>
<td>430.26</td>
<td>643.97</td>
<td>714.02</td>
<td>947.31</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$f_i$</td>
<td>446.78</td>
<td>626.22</td>
<td>705.57</td>
<td>927.12</td>
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<td>$f_i$</td>
<td>445.00</td>
<td>643.86</td>
<td>718.84</td>
<td>947.03</td>
</tr>
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</table>

Table 3: First ten natural frequencies for C-C frames with a crack ($h_2/h = 0.75$) in the third of the horizontal beam

<table>
<thead>
<tr>
<th>$l_2/l_1$</th>
<th>$h_2/h$</th>
<th>$R_m$</th>
<th>$i=1$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\lambda_i$</td>
<td>3.8482</td>
<td>4.6617</td>
<td>7.03645</td>
<td>7.8254</td>
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<td>$f_i$</td>
<td>54.41</td>
<td>79.18</td>
<td>181.44</td>
<td>224.73</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$f_i$</td>
<td>52.49</td>
<td>78.13</td>
<td>183.72</td>
<td>227.05</td>
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<td></td>
<td></td>
<td></td>
<td>$f_i$</td>
<td>54.27</td>
<td>79.61</td>
<td>182.32</td>
<td>227.74</td>
</tr>
<tr>
<td>0.50</td>
<td>0.75</td>
<td>8.40</td>
<td>$\lambda_i$</td>
<td>10.698</td>
<td>13.270</td>
<td>14.1316</td>
<td>16.084</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$f_i$</td>
<td>420.66</td>
<td>647.582</td>
<td>733.76</td>
<td>950.08</td>
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<td></td>
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<td>$f_i$</td>
<td>429.69</td>
<td>656.74</td>
<td>741.58</td>
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<td>$f_i$</td>
<td>430.01</td>
<td>649.88</td>
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<td>950.24</td>
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Table 4: First ten natural frequencies for C-C frames with a crack ($h_2/h = 0.75$) in the middle of the horizontal beam
<table>
<thead>
<tr>
<th>$l_2/l_1$</th>
<th>$h_2/h$</th>
<th>$R_{mm}$</th>
<th>$i=1$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>$\lambda_i$</th>
<th>$(f_i)$</th>
<th>$\lambda_i$</th>
<th>$(f_i)$</th>
<th>$\lambda_i$</th>
<th>$(f_i)$</th>
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<th>$(f_i)$</th>
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<tr>
<td>0.66</td>
<td>0.75</td>
<td>8.40</td>
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<td>3.842</td>
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<td>Analytical (Eigenvalue)</td>
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<td>54.02</td>
<td>80.86</td>
<td>177.82</td>
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<td>376.77</td>
<td>Analytical (Hz)</td>
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<td></td>
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<td>52.49</td>
<td>81.18</td>
<td>173.95</td>
<td>217.29</td>
<td>380.86</td>
<td>Experimental (Hz)</td>
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<td></td>
<td></td>
<td></td>
<td>53.84</td>
<td>81.55</td>
<td>177.94</td>
<td>219.26</td>
<td>381.56</td>
<td>FEM (Hz)</td>
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</table>

Table 5: First ten natural frequencies for C-C frames with a crack ($h_2/h = 0.75$) in the second third of the horizontal beam

Figure 7: Dynamic response of the frame with a crack ($h_2/h = 0.75$) in the second third of the horizontal beam
As it can be seen, the experimental values are again in surprisingly agreement with those obtained by means of the analytical procedure and the finite element method. There’s no case where the difference exceeds than 5%, and generally it is less than 2%.

6 CONCLUSIONS

Through the study of the presented simplified model, it has been analyzed the effect of a crack on the natural frequencies of vibration of a clamped L-frame.

It has been demonstrated that the expressions proposed by Chondros et al., 1998 constitute a simple and reliable tool for modeling a crack in a plane flexural problem.

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