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AN EXTENDED ANALYSIS FOR FLOW SPLITTING IN MULTIPLE, PARALLEL-BOILING CHANNELS SYSTEMS

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Abstract. Previous results are generalized to consider multiple boiling channels systems. The analysis is consistent with the approximations usually adopted in the use of systems codes (like RELAP5 and TRACE5) commonly applied to perform safety analyses of nuclear power plants. The problem is related to multiple, identical, parallel boiling channels, connected through common plena. Flow splitting without reversal was computationally found and to explain this behavior a theoretical model limited in scope was developed. The unified analysis performed and the confirmatory computational results found are summarized in this paper. New maps showing the zones where this behavior is predicted are shown considering again twin pipes. Multiple pipe systems have been found not easily amenable for analytical analysis when dealing with more than three parallel pipes. However, the particular splitting found (flow dividing in 1 standalone pipe flow plus N-1 identical pipe flows) has been verified up to twelve pipes, involving calculations in systems with even and odd number of pipes using the RELAP5 systems thermal-hydraulic codes. Although not shown in this paper, results have been also generalized to consider the flow in systems of identical, parallel, condensing, inverted U-tubes.

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1. INTRODUCTION

In nuclear power plants such as PWR, BWR or PHWR reactors, natural circulation is one of the mechanisms for decay heat removal after a transient or accident. In this type of installations multiple parallel tubes are a common system configuration. The complexity of the flow pattern may be exemplified by simply mentioning that after a very small break loss of coolant accident (SBLOCA), the natural circulation flow map go through different regimes, from single or two-phase natural circulation to reflux condensation regime, see e.g. D'Auria and Frogheri (2002) and Lazarte and Ferreri (2012). The main task is verifying whether those designs can avoid the development of instabilities in transients under nominal and postulated accident conditions or, also important from the point of view of nuclear safety evaluations, that system codes used for such safety evaluations are capable of capturing the instabilities threshold. There is plenty of consolidated literature on this subject and a recent review (Ruspini et al., 2014) confirms this assertion. The behavior of a set of parallel tubes or inverted U-tubes under natural or forced circulation systems in single and two phase flow with common inlet an outlet may become complex. It is well-known that these configurations can develop several types of instabilities, classified as static or dynamic. In the first case only steady-state conservation laws are required for instability prediction.

In a previous study, see Lazarte and Ferreri (2014), the present authors analyzed steady asymmetric flow splitting coming from static instabilities in thermodynamic conditions representative of a SBLOCA in an integral test facility (SEMISCALE) that have steam generators with two different pipes which, for the sake of results symmetry checks were considered identical with average height. This lead to non-symmetric flow splitting. Because of this, twin channels systems have been considered in detail and the basic mechanism leading to the asymmetric flow splitting was elucidated, a map was constructed showing the conditions for such behavior and the approximate, theoretical results verified using the systems thermo-hydraulic code RELAP5.

Other authors have studied flow rate distribution, see Minzer Minzer al. (2006) and Baikin et al. (2011) in very long evaporating parallel pipes and performed a stability analysis and the corresponding experiments related to solar energy collecting pipes.

In this paper previous results from the present authors are extended with a unified analysis to consider multiple but still small (up to twelve) number of N identical boiling pipes systems. In this case, the possible flow splitting becomes even more complicated and above N=4 the behavior is studied through system codes. A peculiarity of the results is that under certain conditions, the splitting consists in a large fraction of mass flow in one pipe and the corresponding remaining fraction distributed in equal mass flow rate in the N-1 pipes. No reverse flow exists. The conditions for such a behavior has been established and represented in maps numerically computed with an in-house developed code and verified using thermal-hydraulic systems codes, namely RELAP5 and TRACE5. However, reference to these results refers only to RELAP5. Not surprisingly, modification of simulation options (such as user options on models or numerical schemes) with the systems codes may lead to different predictions.

2. TWO PARALLEL BOILING CHANNELS

A typical single boiling channel has a characteristic curves as it is shown in Figure 1. Depending on operating conditions, the classical sigmoidal curve has a local minimum and a maximum that typically separates liquid and vapor single phases, respectively. In between, there is a two-phase flow region. The shape of the curve strongly depends on the relative contribution to pressure drop of gravitational, acceleration, form and friction effects. Plenty of literature describes this curve and analyze the channel behavior depending on boundary conditions and may be found at Ruspini et al. (2014), Kakak and Bon (2008) and Kakak and Veziroglu (1983). For a constant inlet pressure, there could be one or three possible mass flow rates.

Different approximations to analyze this problem are considered in what follows.

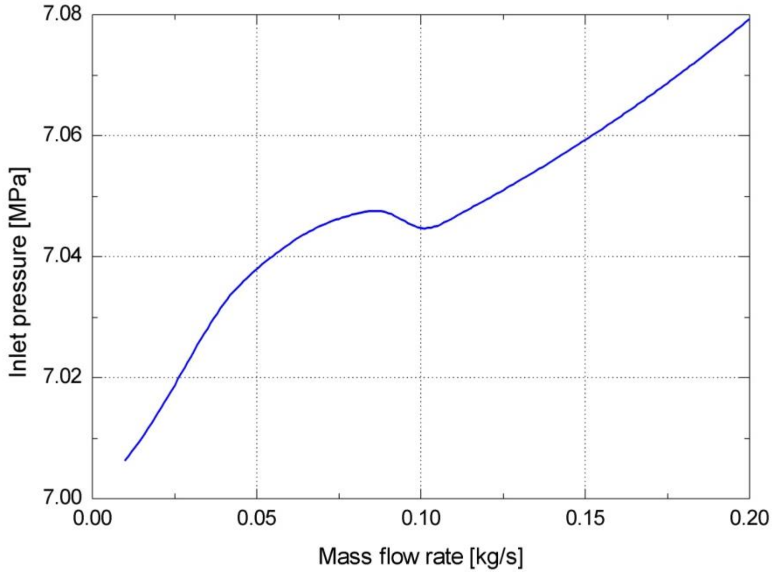

Figure 1: Single boiling channel characteristic curve.

2.1. Lumped parameter model

Let us consider a single boiling pipe and homogeneous, equilibrium fluid model (HEM) as shown in Figure 2. Heat flux is uniform along the tube and three regions may be observed: liquid only, steam only and two-phase regions.

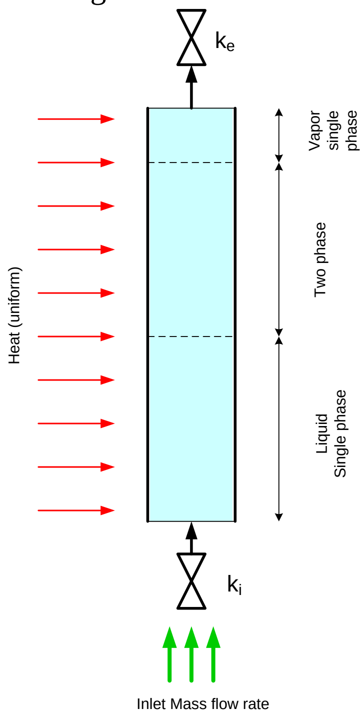

Figure 2: Single boiling channel.

The pressure drop along such a boiling channel (neglecting the vapor phase) can be written as:

$$\Delta P = g\, Z_B\, \rho_f + g\,(L - Z_B)\,\rho_m + \frac{k_i W^2}{2A^2 \rho_f} + \frac{f\left(\frac{Z_B}{\rho_f} + \frac{L - Z_B}{\rho_m}\right) W^2}{2A^2 D} + \frac{k_e W^2 \phi_{2\phi}^2}{2A^2 \rho_f} \qquad (1)$$

where Z_B is the non-boiling length, W is the mass flow rate, A is the channel area, g the gravity acceleration, L the channel length, f the friction factor (assumed constant in this analysis), ρ_f is the liquid density at saturation and ρ_m a mean density in the two phase region calculated at half outlet quality. Parameters k_i and k_e are the concentrated friction values at the inlet and outlet of the individual channels, respectively. $\phi_{2\phi}^2$ is the two-phase friction factor depending on $(v_f + v_{fg} x)^{-1}$ that acts as mixture density and equals 1.0 when the fluid is saturated with $x = 0$. Using the above definition for $\phi_{2\phi}^2$, the mean density (ρ_m) in equation (1) is obtained dividing the saturation density ρ_f by $\phi_{2\phi}^2$ evaluated at half outlet quality.
Some useful (and commonly used) definitions are:

$$N_p = \frac{Q\ v_{fg}}{h_{fg}\, v_f W}, \quad N_S = \frac{\Delta h_{in}\ v_{fg}}{h_{fg}\, v_f}, \quad N_{fr} = \frac{fL}{2D} \qquad (2)$$

where N_p, N_S and N_{fr} are the phase change, subcooling and friction dimensionless numbers; v_{fg}, is the difference between the gas and liquid specific volumes and h_f and h_{fg} are the saturation and latent heat, respectively.
Suppose now two parallel, identical (twin) channels and, hence, the pressure drop through then are equal; i.e. $\Delta P_1 = \Delta P_2$. It is well known that this system may have several stable solutions. This means, for equal pressure drop, different mass flow rate each tube. This is a static analysis.

In several cases, the pressure drop along of the channel is assigned and kept constant as a boundary condition. However, in this case, the inlet mass flow rate and outlet pressure will be fixed. From mass conservation: $W_1 + W_2 = W_T$ (constant).

Defining: $N_{pM} = \frac{Q\ v_{fg}}{h_{fg} v_f W_T}$ and $N_{pi} = \frac{Q\ v_{fg}}{h_{fg} v_f W_1}$ and setting:

$$\phi = \frac{W_1}{W_T} \qquad 1 - \phi = \frac{W_2}{W_T}$$

then

$$N_{p1} = \frac{N_{pM}}{\phi}\ , \quad N_{p2} = \frac{N_{pM}}{1 - \phi}$$

The dimensionless magnitude N_{pM} is the phase change number corresponding to a channel with same imposed heat as the single channel but with the total system mass flow rate. The channel non-boiling length Z_{B1} and Z_{B2} are defined as the positions at which the fluid becomes saturated, that is

$$Z_{B1} = \frac{N_s\, L}{N_{p1}} = \frac{L\, N_s\, \phi}{N_{pM}} \qquad Z_{B2} = \frac{N_s\, L}{N_{p2}} = \frac{L\, N_s(1-\phi)}{N_{pM}} \tag{3}$$

Each pressure drop term will be evaluated in what follows. Recall that the expression of ΔP for channel 2 will be the same as for channel 1, exchanging ϕ by $(1-\phi)$. The friction term in (1), using the definition of the dimensionless numbers, reads

$$\Delta P_{1f} = f\frac{{W_T}^2\phi^2}{2A^2D\,\rho_f}L\left(N_s\frac{\phi}{N_{pM}} + \left(1-\frac{N_s\phi}{N_{pM}}\right)\left(1-N_s+\frac{N_{pM}}{\phi}\right)\right) + \frac{k_i{W_T}^2\phi^2}{2A^2\rho_f} + \frac{k_e{W_T}^2\phi^2(1-N_s+\frac{N_{pM}}{\phi})}{2A^2\rho_f} \tag{4}$$

where $\phi_{2\phi}^2$ was replaced by $1-N_s+N_{pM}\phi^{-1}$.
The gravity contribution, results,

$$\Delta P_{1g} = g\ Z_{B1}\ \rho_f + g\,(L - Z_{B1})\,\rho_{m1} = g\,N_s\,\rho_f\frac{\phi}{N_{pM}} + \frac{g\,L\left(1-N_s\frac{\phi}{N_{pM}}\right)\rho_f}{\left(1-N_s+\frac{N_{pM}}{\phi}\right)} \tag{5}$$

Once again, the value ρ_{m1}may be calculated at the half exit quality. Adding (5) to (7), the total pressure drop along channel 1 become

$$\Delta \mathrm{P}_1 = \frac{k_i{W_T}^2\phi^2}{2\mathrm{A}^2\rho_\mathrm{f}} + \frac{k_e{W_T}^2\phi^2\left(1-N_s+\frac{N_{pM}}{\phi}\right)}{2A^2\rho_f} + f\frac{{W_T}^2\phi^2}{2A^2D\rho_f}L\left(N_s\frac{\phi}{N_{pM}} + \left(1-N_s\frac{\phi}{N_{pM}}\right)\left(1-N_s+\frac{N_{pM}}{\phi}\right)\right) \tag{6}$$

Pressure loss $\Delta\mathrm{P}_2$ has the same expression as (6), (7) and (8) changing ϕ by $1-\phi$. Since $\Delta\mathrm{P}_1 = \Delta\mathrm{P}_2$ then

$$G(1-\phi) = \Delta\mathrm{P}_1\frac{A^2\rho_f}{N_{fr}\,{W_T}^2} = \Delta\mathrm{P}_2\frac{A^2\rho_f}{N_{fr}\,{W_T}^2} = G(\phi) \tag{7}$$

$$G(\phi) = k_{im}\phi^2 + k_{em}\phi^2\left(1-N_s+\frac{N_{pM}}{\phi}\right) + 2\phi^2\left(\frac{N_s\,\phi}{N_{pM}} + \left(1-N_s\frac{\phi}{N_{pM}}\right)\left(1-N_s+\frac{N_{pM}}{\phi}\right)\right) \tag{8}$$

For simplicity, k_{em} and k_{im} denote k_e and k_i divided by N_{fr}. The objective is to find the fixed points of the equation $G(\phi) - G(1-\phi) = 0$. In previous work by these authors (Lazarte and Ferreri, 2014), gravity terms were neglected simplifying the equation. The authors showed that the expression for pressure drop is a polynomial of degree three and the fixed points and a stability region can be obtained. However when gravity contribution is included, the following expression should be added to the function $G\ (\phi)$

$$\frac{Ns\phi}{2LN_{pM}Fo^2N_{fr}}+\frac{1}{N_{fr}Fo^2}\frac{\left(1-\frac{N_s\phi}{N_{pM}}\right)}{\left(1-N_s+\frac{N_{pM}}{\phi}\right)} \tag{9}$$

where Fo is the Froude number defined as

$$Fo^2=\frac{v^2}{gL}=\frac{W_T^2}{4A^2\rho_f^2gL} \tag{10}$$

The velocity v is the inlet velocity and it was expressed as function of the total mass flow rate W_T. The area corresponds to one channel flow area.

When gravity is added to the pressure drop term, the function $G(\phi)$ becomes more complex. Actually, the function may be represented by third degree polynomial plus the ratio between a linear and a quadratic function of ϕ. Therefore, at least five fixed points may be obtained and the obvious solution is $\phi = 0.5$ (equal flow in each channels). On the contrary to case without gravity contribution, fixed points cannot be obtained in a closed analytical form. The above simplified model will be tested with numerical values depicted in Table 1 to find the stability regions and the characteristic curve for two twin parallel channels. Moreover, as it would be shown below, the stability map for multiple equal parallel tubes may be computed using just the same procedure for two tubes but considering a particular splitting or using expression coming from stability analysis.

Nomenclature	Value	Variable / units
D	0.0124	Inner channel diameter (m)
k_i	23	Inlet concentrated pressure losses (-)
k_e	5	Outlet concentrated pressure losses (-)
f	0.01	Friction factor (-)
L	3.66 or 12	Total length of the U-tube (m or ft)
ρ_f	739.86	Liquid saturated density (kg/m^3)
ρ_g	36.6	Vapor saturated density (kg/m^3)
μ	9.46x10^{-5}	Liquid Dynamic viscosity at saturation (Pa s)
W_i	0.1	Mass flow rate per channel i (kg/s)
p	7	Absolute pressure (MPa)
T	20ºC	Inlet temperature (ºC)
Q	125 kW	Power delivered to each pipe.

Table 1 Numerical values used in the calculations,
like in Ambrosini and Ferreri (2006).

2.2. Pressure drop characteristic curve for two parallel tubes

As shown in Figure 1 above, depending on the external characteristic the system (a single boiling tube), may experiment different steady states. For instance, for a constant external pressure the system could have at most three mass flow rate, namely: W_i with i =1 to 3. When two pipes are joined together, then up to nine steady states may arise, *i.e.*,

For channel 1: W_{11}, W_{12}, W_{13}

For channel 2: W_{21}, W_{22}, W_{23}

The nomenclature is: the first index identify channel number and the second the possible solution. For both channels connected to the same inlet and outlet plena:

$$W_T = \sum_{i,j=1}^{3} W_{ij} \tag{11}$$

Then, at most 3^N different mass flow rates with N the number of pipes. When the pipes are identical among them, several solutions are the same and can be disregarded. For two identical pipes, at most 6 possible solutions may be obtained.

Straightforward application of the lumped parameter model considering the values shown in Table 1, allows obtaining the inlet pressure curve (constant inlet mass flow rate and outlet pressure. The procedure was:

a) For each inlet pressure within a range, all possible mass flow rates are determined.
b) Since both channels have the same mass flow rate, then for each inlet pressure the total mass flow rare is determined as the sum of individual channels mass flow (Figure 3).
c) The inlet pressure is plotted for each mass flow as it is shown in Figure 3

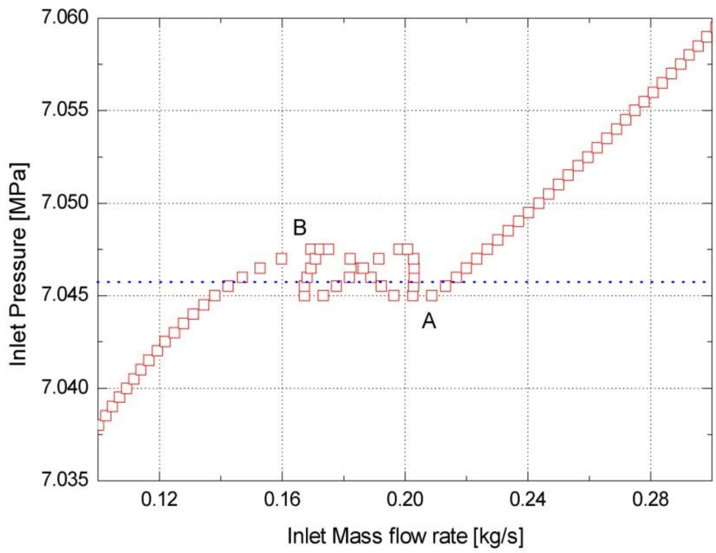

Figure 3: Pressure drop characteristic curve for two parallel channels.

Figure 3 shows a typical "∞" shape. For large and small mass flow rate the system behaves as one single channel working with a single phase fluid (liquid or vapor). In between there is a complex region showing interaction and feedback between channels (between points A and B). Since, both pipes are identical then the numbers of mass flow possible states are less than 9; as it was stated above, for a constant inlet pressure (dotted line in the figure) there are 6 possible solutions. The shape of the curve depends on the characteristic curve of each channel, which in turn is a function of friction losses and heat flux delivered to the fluid.

Recalling that the fraction of flow splitting $\phi_1 = W_1/W_T$, a flow splitting map as function of the total mass flow rate is depicted in Figure 4.

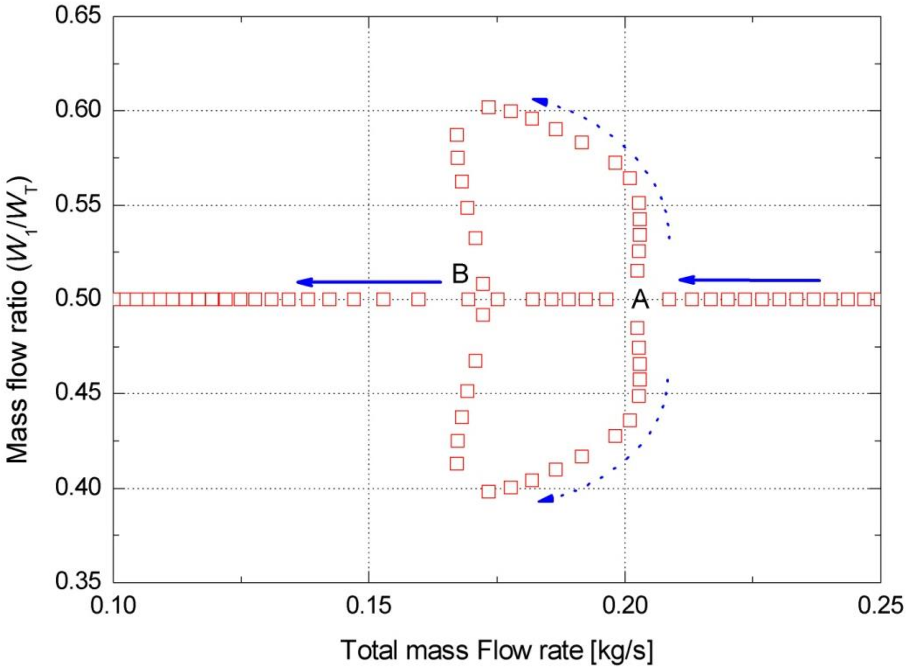

Figure 4 Flow splitting in twin parallel channels as a function of total mass flow rate. Results from lumped parameter model and parameters values from Table 1

In Figure 4 it may be observed that as the mass flow rate decreases from 0.2 to 0.16 kg/s, three possible states appear. Two of them correspond to symmetric flow splitting. The total flow rate at which flow splitting begins coincides with the position at which the derivative of the pressure drop curve changes the sign (at A and B).

2.3. Stability analysis of two parallel boiling channels

The momentum conservation equation for a channel, named k, may be written as

$$m_k \frac{dW_k}{dt} = P_{in}(W_T) - P_{out} - \Delta P_k(W_k) \tag{12}$$

Being $m_k = L_k/A_k$, W_T and W_k is the mass flow rate for the k channel, respectively. Rewriting (12) for two twin channels, reads

$$m_1 \frac{dW_1}{dt} = P_{in}(W_T) - P_{out} - \Delta P_1(W_1) \tag{13}$$

$$m_2 \frac{dW_2}{dt} = P_{in}(W_T) - P_{out} - \Delta P_2(W_2) \tag{14}$$

Recalling that $W_1 + W_2 = W_T$, and $m = m_1 = m_2$ (from hereinafter all channels are assumed identical among them) adding (13) and (14) results

$$m \frac{dW_T}{dt} = 2P_{in}(W_T) - 2P_{out} - \left(\Delta P_2(W_2) + \Delta P_1(W_1)\right) \tag{15}$$

In order to solve the transient behavior of twin boiling channels, for instance equations (13) and (15) should be solved together. The other channel is obtained by the difference. Perturbing the mass flow rate in both channels and the total as $W_k = W_{k0} + \delta W_k$, where W_{k0} corresponds to the steady state solution and δW_k are the infinitesimal perturbations. Replacing in (13),

$$m\frac{dW_{10}}{dt} + m\frac{d\,\delta W_1}{dt} = P_{in}(W_{T0}) + \frac{dP_{in}}{dW_T}\delta W_T - P_{out} - \Delta P_1(W_{10}) - \frac{\mathrm{d}\Delta P_1}{dW_1}\delta W_1 \tag{16}$$

and using steady-state condition for W_{k0}, then the perturbed momentum equation for channels 1 and 2,

$$m\frac{d\,\delta W_1}{dt} = \frac{dP_{in}}{dW_T}\delta W_T - \frac{\mathrm{d}\Delta P_1}{dW_1}\delta W_1 \tag{17}$$

$$m\frac{d\,\delta W_2}{dt} = \frac{dP_{in}}{dW_T}\delta W_T - \frac{\mathrm{d}\Delta P_2}{dW_2}\delta W_2 \tag{18}$$

Now, considering that the perturbation may be written as $\delta w_i = \beta_i\, e^{\lambda t}$, calling $\alpha_s = \frac{dP_s}{dW_s}$, being s = 1,2 or T, the resulting equations are:

$$\begin{gathered} m_1\beta_1\lambda = \alpha_T(\beta_1 + \beta_2) - \alpha_1\beta_1 \\ m_2\beta_2\lambda = \alpha_T(\beta_1 + \beta_2) - \alpha_2\beta_2 \end{gathered} \tag{19}$$

or

$$\begin{gathered} (m_1\lambda + \alpha_1 - \alpha_T)\beta_1 - \alpha_T\beta_2 = 0 \\ -\alpha_T\beta_1 + (m_2\lambda + \alpha_2 - \alpha_T)\beta_2 = 0 \end{gathered} \tag{20}$$

Finally, the above system equations could be written in matrix form. The determinant of the system of equations is:

$$\begin{vmatrix} (m\lambda - \alpha_T + \alpha_1) & -\alpha_T \\ -\alpha_T & (m\lambda - \alpha_T + \alpha_2) \end{vmatrix} = 0 \tag{21}$$

The characteristic equation is: $\lambda^2 + \mathrm{a}_1\lambda + \mathrm{a}_2$ with

$$\begin{gathered} a_1 = \alpha_1 + \alpha_2 - 2\,\alpha_T \\ a_2 = \alpha_1\alpha_2 - \alpha_T(\alpha_1 + \alpha_2) \end{gathered} \tag{22}$$

The system is stable to perturbations when the real part of the characteristic equation roots are negative, *i.e.*

$$(\alpha_2 + \alpha_1)\left(\frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2} - \alpha_T\right) \geq 0, \text{or } a_1 < 0 \tag{23}$$

$$(\alpha_2 - \alpha_T) + (\alpha_1 - \alpha_T) \geq 0, \text{ or } a_2 < 0 \tag{24}$$

For a constant mass flow rate or a vertical external characteristic curve,

$$\alpha_T = \frac{dP_{in}}{dW_T} = -\infty \tag{25}$$

It could easily verified that the system is stable if

$$(\alpha_2 + \alpha_1) \geq 0 \tag{26}$$

or

$$\frac{d\Delta P_1}{dW_1} + \frac{d\Delta P_2}{dW_2} \geq 0 \tag{27}$$

Since ΔP_1y ΔP_2 are functions of ϕ (the mass flow rate fraction) then

$$\frac{d\Delta P_1}{d\phi} - \frac{d\Delta P_2}{d\phi} = \frac{d(\Delta P_1 - \Delta P_2)}{d\phi} \geq 0 \tag{28}$$

For twin parallel boiling channels, using the lumped parameter model disregarding the gravity contribution the stability region could be easily obtained, given by

$$\frac{2}{N_{pM}}[(k_{em} + 2)N_{pM}^2 + N_{pM}(2 + k_{em} + k_{im} - 4N_S - N_S k_{em}) + 3N_S^2(2\phi^2 - 2\phi + 1)] \geq 0 \tag{29}$$

with the constrain the $N_{pm} \geq N_s/\phi$. For $\phi = \frac{1}{2}$, the system is stable for N_{pM} outside the shaded area in Figure 5 that shows the stability map for a two twin boiling parallel channel with vanishing gravity force contribution; the inner area corresponds to the instability area. It has to be remarked that the horizontal axis is N_{pM} variable defined as the phase change number of channel with total system flow rate and the power of individual channel. *i.e.* $N_{p1} = 2\, N_{pM}$ since $\phi = 0.5$. The red lines indicate the thermodynamic quality of one in a single channel.

As stated above, a stability map may be built using analytical expression (29) for twin parallel channels. However, in what follows a larger number of channels will be considered, so an analytical expression may be complex to obtain. Another, but equivalent way for plotting the stability map for twin channels, is evaluating the pressure drop over the N_s-N_{pM} plane for a constant total mass flow rate $\phi = 0.5 \pm \Delta$, for each channel. This corresponds to use equation (27). Here Δ is a small parameter uses to calculate a difference around 0.5. The difference of pressure drop in each channel divided by 2Δ gives the plotted discrete approximation of the derivative.

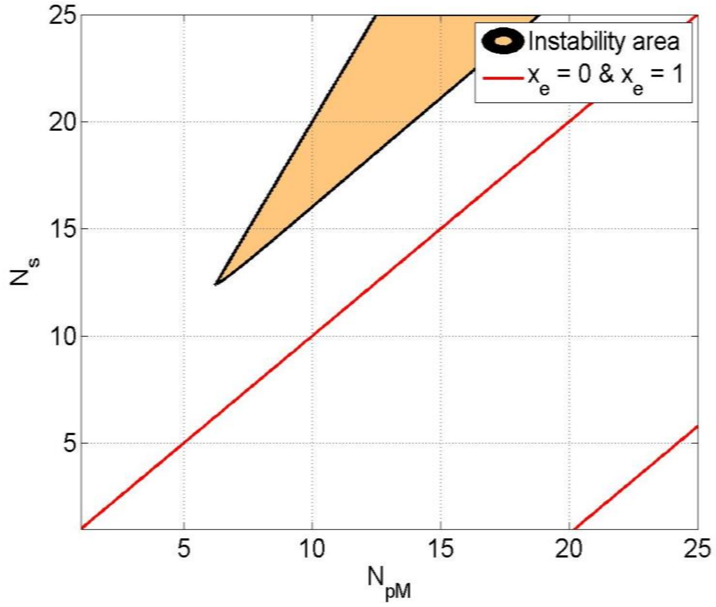

Figure 5: Stability map obtained by lumped parameter model for twin boiling parallel pipes. No gravity contribution. Parameter values from Table 1.

Figure 6 shows the stability region with gravity contribution. It may be observed by comparison with Figure 5 that gravity tends to stabilize the system for a constant subcooling number. This is consistent with the results obtained by Popov et al. (2000), indicating that the critical subcooling number (minimum subcooling number) increases when gravity is included. On the other hand, it should be remarked that absence of gravity is conceptually different than considering a horizontal channel. Basically, pressure drop in the two phase region should also take into account the pipe flow pattern even considering homogeneous and equilibrium model.

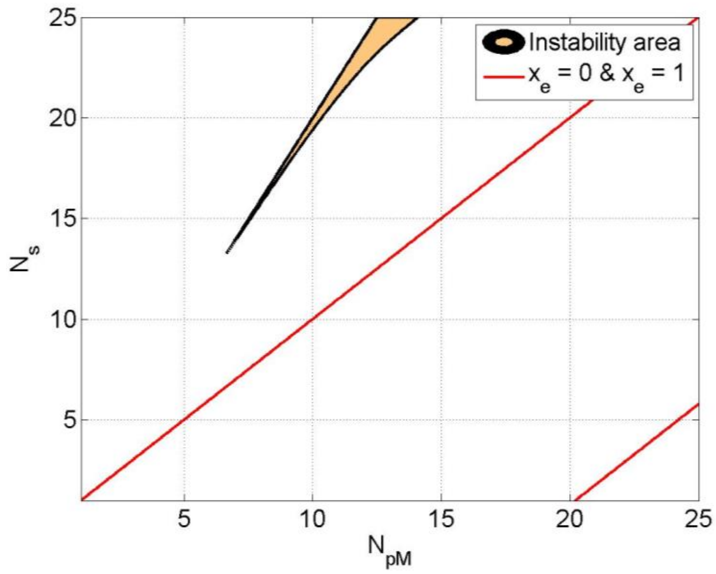

Figure 6 Stability map obtained by lumped parameter model for two twin boiling parallel pipes considering gravity contribution and using values from Table 1.

2.4. Parametric studies

It is usual to perform parametric studies to check the effects of different system parameters on its overall behavior. For instance, in Figures 5 and 6 the effect of gravity in the stability curve was clearly shown. Keeping all parameters constant and switching on gravity contribution the unstable region becomes greater and the minimum value (vertex) moves to lower N_s values. The concentrated pressure losses at the outlet tend to destabilize the system, so increasing the outlet concentrated pressure coefficient k_e to 10 or 50 as is shown respectively in Figures 7 and 8, the instability regions are enlarged and the minimum subcooling number moved to lower values, equivalent to higher inlet temperatures.

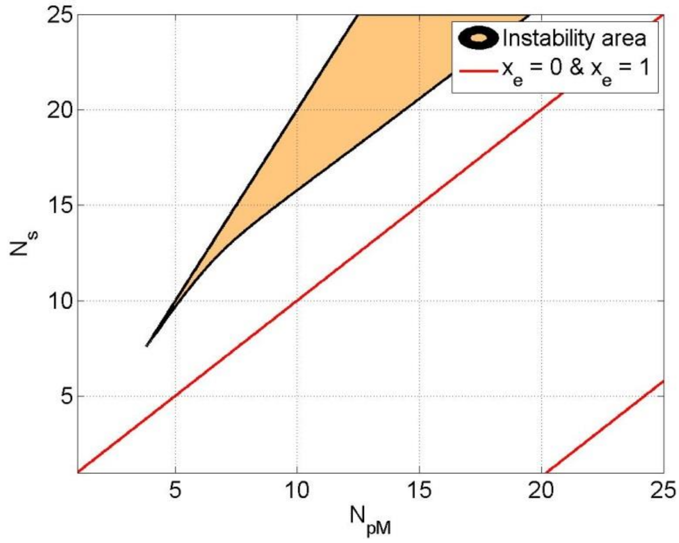

Figure 7 Stability map obtained by lumped parameter model for two twin boiling parallel pipes, considering gravity contribution and using values from Table 1 with one exception: k_e = 10.

In Figure 9, instead of a flow splitting, a RELAP5 simulation shows mass flow rate out-of-phase oscillation. This oscillation shows a sign change indicating intermittent flow reversal. Static instabilities and dynamic instabilities in boiling channel may come up together, and both could be distinguished due to the oscillation characteristic time. For instance, density wave instabilities have larger oscillatory frequency in relation to pressure drop oscillation.

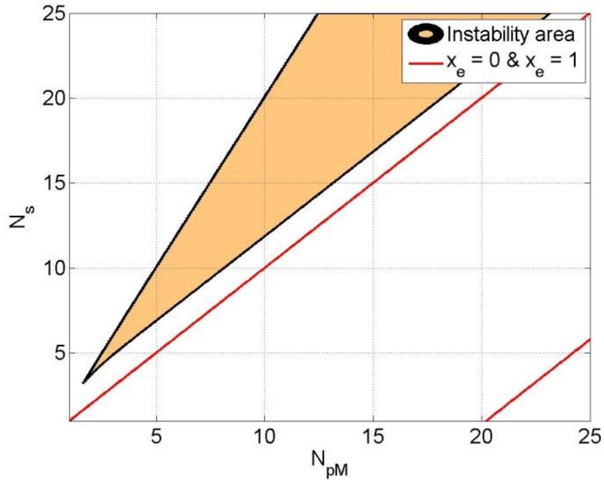

Figure 8 Stability map obtained with lumped parameter model for two twin boiling parallel pipes, considering gravity contribution and using values from Table 1 but k_e = 50.

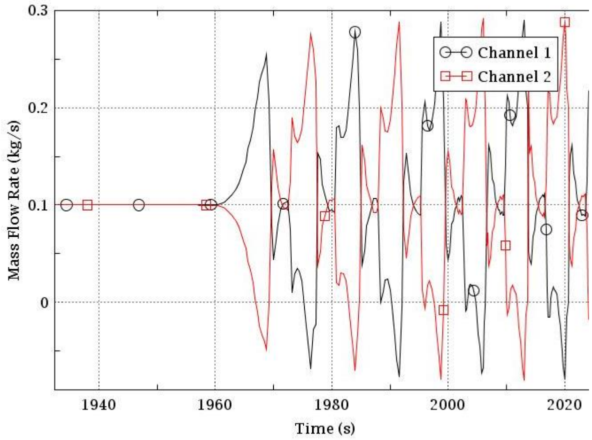

Figure 9 Calculated RELAP5 mass flow rate for two twin boiling parallel pipes with the same conditions of Figure 8.

3. MULTIPLE PARALLEL BOILING CHANNELS

In this section an extended model to get the stability regions for three and multiple identical boiling pipes system is performed. The procedure shown here could be easily extended to a larger number of channels. A stability map is built, as above, evaluating the stability condition related with pressure drop over the N_s-N_{pM} plane for a constant total mass.

3.1. Three parallel boiling channels

For three parallel channels, we follow the same procedure as in Section 2.3, namely: writing the momentum conservation equation for each channel, perturbing their flow rates with respect to the steady state values and rearranging, the system characteristic equation is determined from the vanishing determinant of the following matrix.

$$\begin{vmatrix} (m\lambda - \alpha_T + \alpha_1) & -\alpha_T & -\alpha_T \\ -\alpha_T & (m\lambda - \alpha_T + \alpha_2) & -\alpha_T \\ -\alpha_T & -\alpha_T & (m\lambda - \alpha_T + \alpha_3) \end{vmatrix} = 0 \qquad (30)$$

It was assumed, as above, that the three channels have the same length and flow area and its ratio is denoted by *m*. The characteristic equation has the form:

$$\begin{aligned} \mathrm{P}(\lambda) = \ & \alpha_1\alpha_2\alpha_3 - \alpha_1\alpha_2\alpha_T - \alpha_1\alpha_3\alpha_T - \alpha_2\alpha_3\alpha_T \\ & + m(\alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3 - 2(\alpha_1 + \alpha_2 + \alpha_3)\alpha_T)\lambda + m^2(\alpha_1 + \alpha_2 + \alpha_3 \\ & - 3\alpha_T)\lambda^2 + m^3\lambda^3 \end{aligned}$$

To determine the stability regions, the conditions to get a negative real part of the characteristic equation roots must be found. Applying the Hurwitz theorem said conditions are:

$$(\alpha_1 + \alpha_2 + \alpha_3 - 3\alpha_T) \geq 0 \tag{31}$$

$$\alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3 - 2(\alpha_1 + \alpha_2 + \alpha_3)\alpha_T \geq 0 \tag{32}$$

$$\alpha_1\alpha_2\alpha_3 - \alpha_1\alpha_2\alpha_T - \alpha_1\alpha_3\alpha_T - \alpha_2\alpha_3\alpha_T \geq 0 \tag{33}$$

Recalling that $\alpha_T \to -\infty$, then the first condition may be easily satisfied (the characteristic curve of each channel are smooth functions of the mass flow). The second condition, (32) has two terms. The sign of first depends on individual values of α's, which are upper bounded. The sign of the latest term of (32), $-2(\alpha_1 + \alpha_2 + \alpha_3)\alpha_T$, is positive if so $(\alpha_1 + \alpha_2 + \alpha_3)$. Then, a necessary (but not sufficient) condition is:

$$\alpha_1 + \alpha_2 + \alpha_3 \geq 0 \tag{34}$$

The third condition could be analyzed in the following way. Replacing $\alpha_T = -|\alpha_T|$ reads

$$\alpha_1\alpha_2\alpha_3 \left(1 + |\alpha_T|\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3}\right)\right) \geq 0 \tag{35}$$

Condition (35), neglecting the first term in brackets, may be rewritten

$$\alpha_1\alpha_2\alpha_3 \left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3}\right) \geq 0$$

or

$$\prod_{k=1}^{3} \alpha_k \left(\sum_{k=1}^{3} \frac{1}{\alpha_k}\right) \geq 0 \tag{36}$$

The above expression is a necessary condition for getting stability region for static instabilities for three boiling parallel channels. It should be note that stability depends on a single channel stability condition since just one of the α_k values of any of the channel changes sign, the whole system becomes unstable. This result is in agreement with the results of Akagawa et al., 1971. However, flow instability may trigger different ways of flow splitting that should be taken into account.

3.2. Multiple parallel boiling channels

For any number of parallel boiling channels, the procedure becomes rather cumbersome. Writing the momentum conservation equation for each channel, perturbing their flow rates with respect to the steady state values and rearranging, *N* differential equations are obtained, namely:

$$m_k \frac{d\,\delta W_k}{dt} = \frac{dP_{in}}{dW_T}\delta W_T - \frac{d\Delta P_k}{dW_k}\delta W_k \tag{37}$$

where subindex k represents the channel number. Once again, replacing $\delta w_k = \delta_k\, e^{\lambda t}$, the remaining algebraic equations are:

$$m_k \beta_k \lambda = \alpha_T \sum_{j=1} \beta_j - \alpha_k \beta_k \tag{38}$$

m_k is quotient between length and channel area, $\alpha_k = \frac{dP_k}{dW_k}$ with k = 1,2…N or T (for the total mass flow rate. Writing the set of equation in matrix form:

$$\begin{vmatrix} (m_1\lambda - \alpha_T + \alpha_1) & -\alpha_T & \cdots & \cdots & -\alpha_T \\ -\alpha_T & (m_2\lambda - \alpha_T + \alpha_2) & -\alpha_T & -\alpha_T & -\alpha_T \\ \vdots & -\alpha_T & \ddots & -\alpha_T & \vdots \\ \vdots & \vdots & -\alpha_T & \ddots & -\alpha_T \\ -\alpha_T & \cdots & \cdots & -\alpha_T & (m_N\lambda - \alpha_T + \alpha_N) \end{vmatrix} = 0$$

Stability regions are determined by finding the conditions where the real part of the characteristic equation roots is negative. Analytical conditions could be written based on the coefficient of the characteristics equations but are difficult to be applied. From Hurwitz theorem, it could be derived that a necessary condition for having negative real roots is that all characteristics coefficient should be negative. However, the reverse is not true. The stability conditions for four channels are:

$$\begin{gathered} \sum_{k=1}^{4} \alpha_k - 4\,\alpha_T \geq 0 \\ -3\alpha_T \sum_{k=1}^{4} \alpha_k + \sum_{\substack{i,j=1 \\ i\neq j}}^{4} \alpha_i \alpha_j \geq 0 \\ -2\alpha_T \sum_{\substack{i,j=1 \\ i\neq j}}^{4} \alpha_i \alpha_j + \sum_{\substack{i,j,k=1 \\ i\neq j\neq k}}^{4} \alpha_i \alpha_j \alpha_k \geq 0 \\ \prod_{k=1}^{4} \alpha_k - \alpha_T \sum_{\substack{i,j,k=1 \\ i\neq j\neq k}}^{4} \alpha_i \alpha_j \alpha_k \geq 0 \end{gathered} \tag{39}$$

Using a similar procedure performed previously for three channels, it may be shown that the necessary conditions are:

$$\sum_{k=1}^{4} \alpha_k \geq 0 \tag{40}$$

and

$$\prod_{k=1}^{4} \alpha_k \left(\sum_{k=1}^{4} \frac{1}{\alpha_k} \right) \geq 0 \tag{41}$$

Both equations (41) and (40) are necessary conditions for stability and, as it could be shown, these expressions for different number of identical channels are similar. For instance, equations (34) and (40) could be extended to *N* parallel identical channels, yielding

$$\sum_{k=1}^{N} \alpha_k \geq 0 \quad \text{or} \quad \sum_{k=1}^{N} \frac{d\Delta P_k}{dW_k} \geq 0 \tag{42}$$

Recalling that the fraction of mass flow though a channel is:

$$\phi_k = \frac{W_k}{\sum_{j=1}^{N} W_j} \tag{43}$$

When bifurcation or the conditions for several multiple steady states are met in *N* parallel channels, the splitting will develop in pairs. This hypothesis will be verified shortly with the system code. The latter means that *N-1* channel will have the same flow and only one splits. The flow ratio for *N-1* is equal, and hence $\phi_1 = \phi_2 = \cdots = \phi_{k-1} = \phi$. This could be expressed as:

$$\phi = \phi_1 = \frac{W_1}{\sum_{j=1}^{N-1} W_j + W_N} = \cdots = \frac{W_3}{\sum_{j=1}^{N-1} W_j + W_N} = \cdots \frac{W_{N-1}}{\sum_{j=1}^{N-1} W_j + W_N} \tag{44}$$

and the fraction of the individual channel as function of the other *N-1* channel results:

$$\phi_N = \frac{\sum_{j=1}^{N} W_j - \sum_{j=1}^{N-1} W_j}{\sum_{j=1}^{N} W_j} = 1 - (N-1)\phi \tag{45}$$

Since $d\phi_k = W_T^{-1} dW_k$

$$\frac{1}{W_T} \sum_{k=1}^{N} \frac{d\Delta P_k}{d\phi_k} = \frac{1}{W_T} \left((N-1) \frac{d\Delta P_1}{d\phi} - \frac{1}{(N-1)} \frac{d\Delta P_N}{d\phi} \right) \geq 0 \tag{46}$$

Considering that $\Delta P_1 = \Delta P_N$

$$(N-1)^2 \frac{d\Delta P_1(\phi)}{d\phi} - \frac{d\Delta P_1(1-(N-1)\phi)}{d\phi} \geq 0 \tag{47}$$

The latter expression is the necessary condition for *N* parallel tubes; with the condition that splitting is channel to channel. The quantity $\Delta P_1(\phi)$ and $\Delta P_1(1-(N-1)\phi)$ means that pressure drop ΔP_1 is evaluated at expression between brackets. In addition the stability area is also delimited by $N_s = N_p(1-(N-1)\phi)$ that correspond to single to two-phase separation.

4. RELAP5 CALCULATIONS

RELAP5 is a best estimate systems code extensively used for nuclear safety evaluations related to nuclear power plants. It provides numerical results using closure correlations and steam tables. Results heavily depend on user choices. The present authors discussed in detail the effects of different code options and fluid models; see Lazarte and Ferreri (2014). It is important to state here that the predictions using this thermo-hydraulic system code confirmed the approximate model predictions. The reader is referred to said reference for the detailed analysis.
Figure 10 shows a sketch of the geometry implemented for a four identical pipes system. The pipes were divided into 48 cells. The inlet mass flow rate is imposed with a time dependent junction, the outlet pressure is fixed with a time dependent volume. Simulations have been be performed using HEM and the two fluid model.

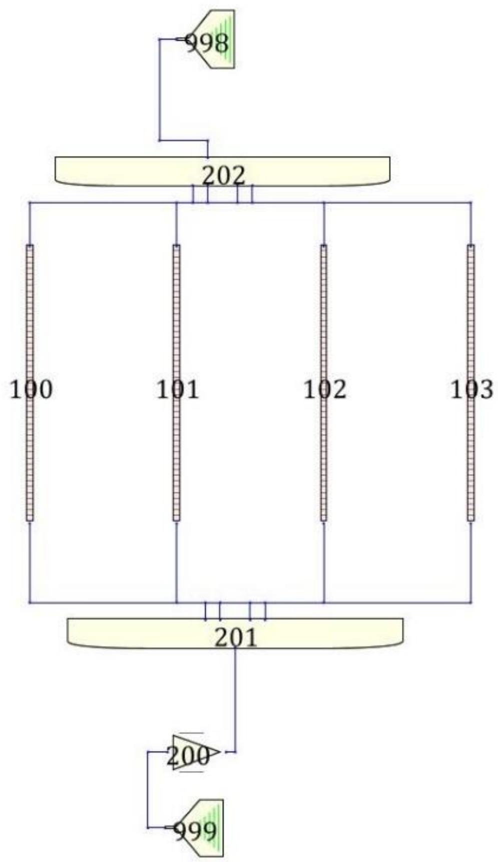

Figure 10 RELAP5 nodalization for a system with four identical pipes

It is interesting to verify the behavior of a system composed of three or four parallel channels. For simulating this case, additional pipes have been added in parallel to the twin pipes case and identical to the others. The inlet mass flow rate and power delivered to the channel were changed accordingly. It must be noted that the same rate of power delivered to the fluid was not used in all cases. The system behavior for three parallel channels using RELAP5 with and without gravity and the HEM or two phase flow model is shown in Figure 11. Considering vanishing gravity there is a flow splitting at 1300 s, corresponding in this case to 125 kW. The flow splitting is asymmetrical; that is, one pipe of them increases its flow rate while the other channels reduce proportionally their flow rate and is equal among them. This behavior supports the hypothesis used for deriving the instability area in three pipes systems.

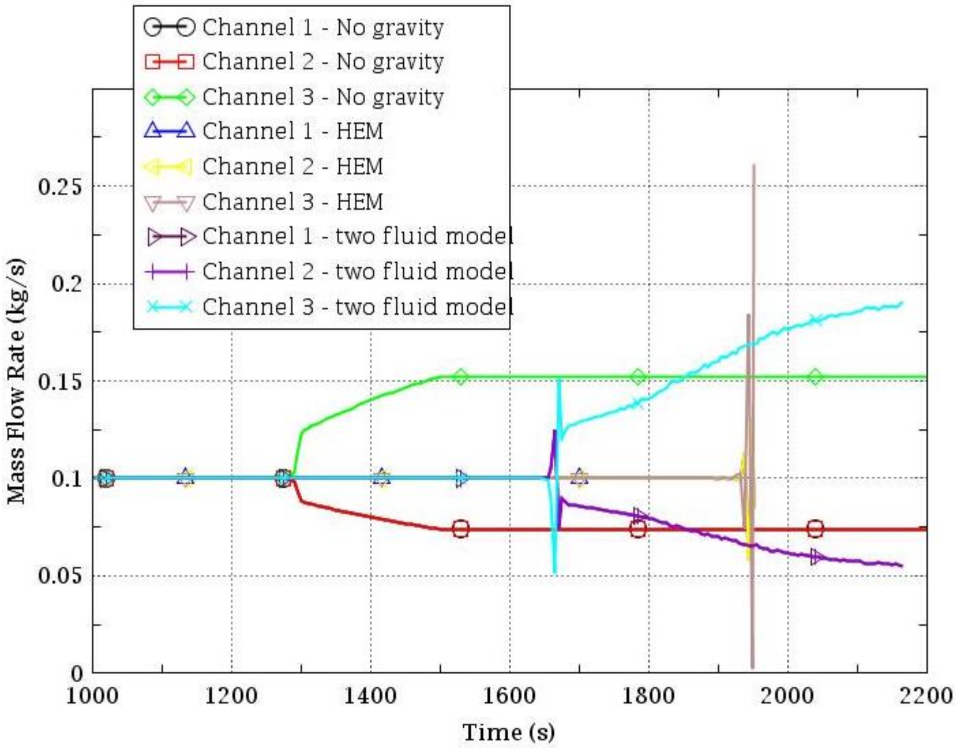

Figure 11 RELAP5 calculated mass flow rate through channels (named 1, 2 and 3). There is a stable flow splitting close for the HEM with vanishing gravity contribution and with the two fluid model. No splitting exists for HEM model with gravity contribution.

When gravity is taken into account, no flow splitting was observed at least at the expected power. Moreover, the RELAP5 code fails near to the 2000 s, corresponding to 200 kW. On the contrary, with two fluid model and gravity, flow splitting occurs at 1300 s, corresponding to 167 kW.

RELAP5 calculations for four identical channels are shown in figures 12 through 15 considering different modeling options. One again, when gravity is taken into account no flow splitting took place unless exit k's value is increased as the case in figures 13 and 14, respectively. Flow splitting for vanishing gravity or with gravity and k_e equal to 25 comes at 125 kW, whereas for two fluid model near to 170 kW. This is consistent with the three channel models showing that two fluid model requires more power to become unstable. This difference might be due to the nucleate subcooled boiling region before fluid saturation.

In addition, as in the three channels case, flow splitting appears in (1 and *N-1*) pairs. It should be noted that power required for flow splitting is independent of the number of channels, this means that for two, three or four parallel channels the needed power is ranged 120-130 kW. This support the idea that flow splitting or flow maldistribution comes up when a single tube becomes unstable.

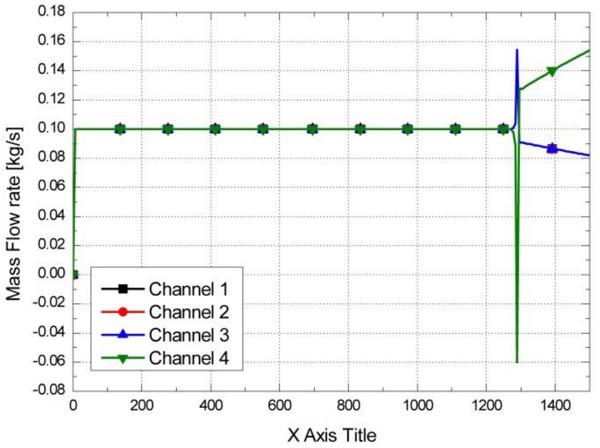

Figure 12 RELAP5 calculated mass flow rate through channels (named 1, 2, 3 and 4). There is a stable flow splitting close for the HEM model with vanishing gravity.

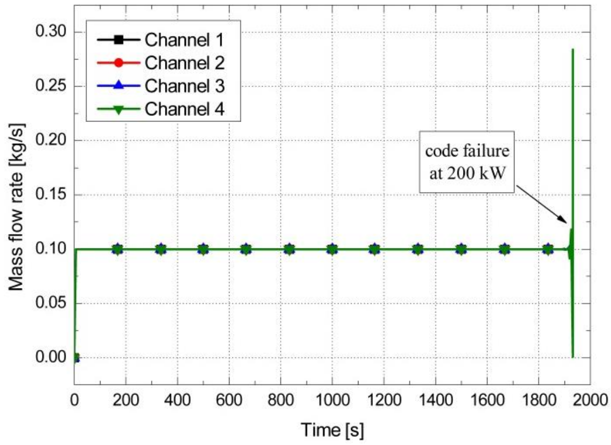

Figure13 RELAP5 calculated mass flow rate through channels (named 1, 2, 3 and 4). There is a stable flow splitting close for the HEM model with gravity.

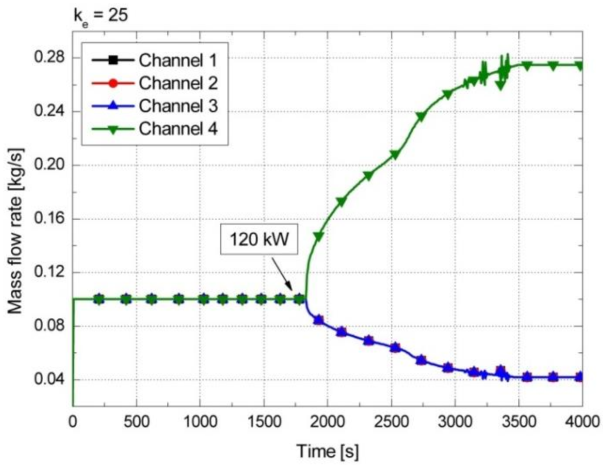

Figure 14 RELAP5 calculated mass flow rate for a four parallel pipe arrangement with HEM model and gravity contribution. The exit concentrated pressure losses coefficient was set to 25. Stable flow splitting is observed.

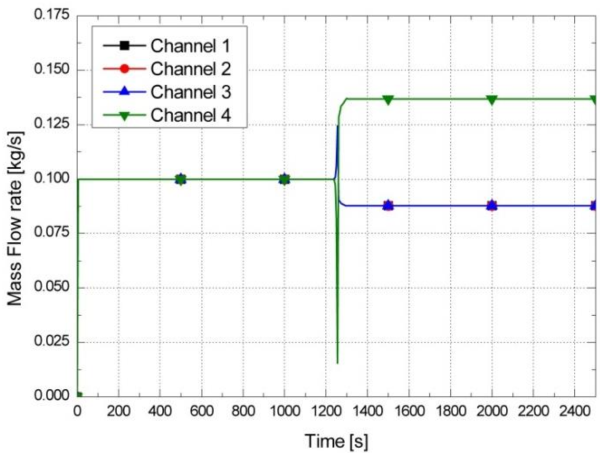

Figure 15 RELAP5 calculated mass flow rate for a four parallel pipe arrangement with two fluid model and gravity contribution. The exit concentrated pressure losses coefficient was set to 5 (Table 1). Stable flow splitting is observed.

Finally, some results for nine and twelve parallel identical tubes are shown in Figures 16 and 17, respectively. Flow oscillations may be observed in Figure 17 after flow splitting for twelve tubes by increasing power delivered to the fluid. This oscillation corresponds to a dynamic instability of density wave type. It must be noted that flow splitting comes, once again, in pairs (1 tube increase or decrease flow rate while the other eleven increase their flow rate proportionally).

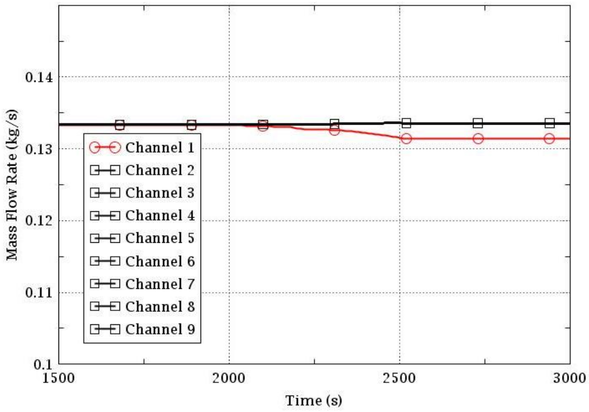

Figure 16 RELAP5 calculated mass flow rate for nine parallel pipes with two fluid model. Stable flow splitting is observed, eight channel to increase mass flow and the others decrease mass flow.

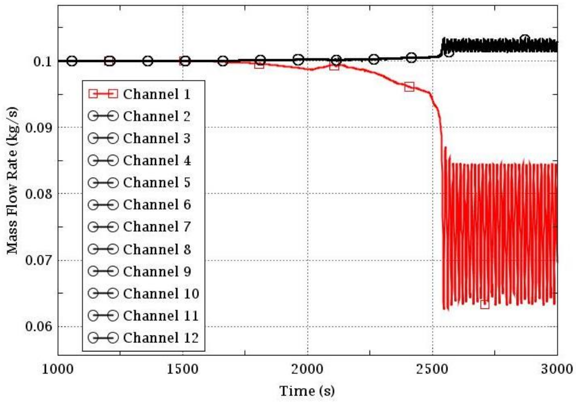

Figure 17 RELAP5 calculated mass flow rate for twelve parallel pipes with two fluid model. Stable flow splitting is observed, eleven channels to increase mass flow and the others decreases mass flow.

5. CONCLUSIONS

Static instabilities in boiling channels with fixed inlet mass flow rate and outlet pressure have been studied. An analytical simplified model using HEM was developed to study static instabilities in twin parallel channels. The latter analysis was extended to three and four parallel channels and, finally, for N parallel channels. Using a perturbation analysis upon the momentum equation for HEM model, a stability analysis was performed for two, three and four parallel channels and a necessary condition and the stability maps based on dimensionless numbers were obtained. It was shown that when the instability boundary is crossed the mass flow rate in the channels split, even though in twin channels in non-symmetric ways. When a larger number of channels are checked, it was found that splitting comes up in a particular way: one channel increases (decreases) mass flow whereas the other N-1 channels decrease (increase) mass flow rate. From necessary stability conditions, a system with several parallel channels could be stable while some of the channels fall individually in the unstable condition.

Concluding, stability maps for flow splitting in a system of three or four parallel channels were obtained. Systems consisting of up to twelve identical parallel pipes have also been considered. It was shown that gravity has a stabilization effect in the sense that unstable region becomes smaller. In addition, when the outlet concentrated pressure drop coefficient increases the system becomes more unstable, as is well known from density wave oscillation analysis. Both instabilities may come up separated or together as depicted in Figure 16 and Figure 17.

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NOMENCLATURE

Parameter	Description
ρ_m	Mean density (kg/m^3)
k_i	Concentrated pressure losses at pipe entrance
k_e	Concentrated pressure losses at pipe outlet
W_T	Total mass flow (kg/s)
A	Flow area (m^2)
D	Pipe diameter (m)
p	Pressure (Pa)
g	gravity acceleration
L	channel length
Z_B	boiling length
ρ_f	the liquid density
$\phi_{2\phi}^2$	Two-phase friction factor
f	Single-phase friction factor
$N_p = \frac{Q\ v_{fg}}{h_{fg}\ v_f\ W}$	Phase change number
$N_s = \frac{\Delta h_{in}\ v_{fg}}{h_{fg}\ v_f}$	Subcooling number
$x_e = x_{e_{in}} + \frac{Q}{h_{fg} W}$	Thermodynamic quality
v_{fg}	difference between the specific gas and liquid volumes $v_g - v_f$
$N_{fr} = \frac{fL}{2D}$	friction factor number
$\phi = \frac{W_i}{W_T}$	Mass flow rate fraction of channel i
$N_{p1} = \frac{N_{pM}}{\phi}$, $N_{p2} = \frac{N_{pM}}{1-\phi}$	Mass flow rate fraction of channel 1, 2