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Graciela Bertolino, Mariano Cantero, Mario Storti y Federico Teruel (Eds.)
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ROBUST DESIGN OF A VIBRO-IMPACT ELETRO-MECHANICAL SYSTEM

Roberta Lima[a,b], Christian Soize[a] and Rubens Sampaio[b]

[a] *Université Paris-Est, Laboratoire Modélisation et Simulation Multi Echelle, MSME UMR 8208 CNRS, 5 bd Descartes, 77454 Marne-la-Vallée, France*

[b] *Pontifícia Universidade Católica do Rio de Janeiro, Avenida Marquês de São Vicente, 255, Gávea, Rio de Janeiro, Brasil*

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Abstract. In this paper, the robust design with an uncertain model of a vibro-impact electro-mechanical system is done. The electro-mechanical system is composed of a cart, whose motion is excited by a DC motor, and an embarked hammer into this cart. The hammer is connected to the cart by a nonlinear spring component and by a linear damper, so that a relative motion exists between them. A linear flexible barrier, placed outside of the cart, constrains the hammer movements. Due to the relative movement between the hammer and the barrier, impacts can occur between these two elements. The developed model of the system takes into account the influence of the DC motor in the dynamic behavior of the system. Some system parameters are uncertain, such as the stiffness and the damping coefficients of the flexible barrier. The objective of the paper is to perform an optimization of this electro-mechanical system with respect to design parameters (spring component, and gap of the barrier) in order to maximize the impact power under the constraint that the electric power consumed by the DC motor is lower than a maximum value. This optimization is formulated in the framework of robust design due to the presence of uncertainties in the model.

1 INTRODUCTION

Electro-mechanical systems are common in actual technologies, and their design is of a great interest in many areas. Many works have been done in this topic, as Zhankui and Sun (2013) and lee (2006), for characterizing the mutual interaction between electrical and mechanical components. This interaction leads us to analyze very interesting nonlinear dynamical systems, in which the nonlinearities affects the two most important variables used to evaluate the performance of electro-mechanical systems, related to the power consumed by the electrical component, and the power used into the movement of the mechanical component. As the mutual interaction between electrical and mechanical components affects the two powers used to evaluate the system performance, the coupling effects must be analyzed in the design optimization problem for electro-mechanical systems.

The present work deals with the design optimization of a vibro-impact electric-mechanical system in order to improve its performance. The electrical component of the system is a DC motor, and the mechanical component is a vibro-impact system. It should be noted that, in Lima and Sampaio (2012), the equations and the numerical integration were presented for a similar electric-mechanical system for which the embarked mass was replaced by a pendulum and for which there was no impact. The vibro-impact dynamics can be affected by many factors, and the optimal design of vibro-impact dynamical systems requires to taken into account uncertainties in the computational models that are used (see for instance Sampaio and Soize (2007)).

This paper is organized as follows. In Section 2 , the elements (motor, cart, hammer, and barrier) of the electro-mechanical system are presented and the initial value problem is formulated for the vibro-impact electro-mechanical system. In Section 3, we define the variables of interest for the design optimization. The construction of the probabilistic models of the uncertain parameters, and the formulation of the robust design optimization problem are given in Section 4. The robust design optimization consists in finding the optimal design point using the computational model in presence of uncertainties. The numerical results of the robust design optimization problem are presented in Section 5.

2 DYNAMIC OF THE ELECTRO-MECHANICAL SYSTEM

2.1 Electrical component: DC motor

The modeling of DC motors is based on the Kirchhoff law (see Karnopp et al. (2006)), which is written as

$$\begin{aligned} l\,\dot{c}(t) + r\,c(t) + k_e\,\dot{\alpha}(t) &= \nu\,, \\ j_m\,\ddot{\alpha}(t) + b_m\,\dot{\alpha}(t) - k_e\,c(t) &= -\tau(t)\,, \end{aligned} \tag{1}$$

where t is time, ν is the source voltage, c is the electric current, $\dot{\alpha}$ is the angular speed of the motor, l is the electric inductance, j_m is the inertia moment of the motor, b_m is the damping ratio in the transmission of the torque generated by the motor to drive the coupled mechanical system, k_e is the constant of the motor electromagnetic force, and r is the electrical resistance. Figure 1 shows a sketch of the DC motor. The available torque delivered to the mechanical component, in the z-direction, is represented by τ (see Fig. 1).

2.2 Mechanical component: cart and hammer

As described in the introduction, the mechanical component is composed by a cart whose movement is driven by the DC motor, and by a hammer that is embarked into the cart. The

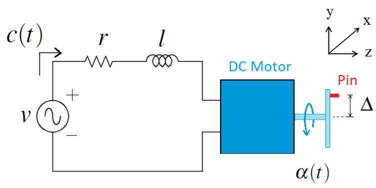


Figure 1: Sketch of the DC motor.

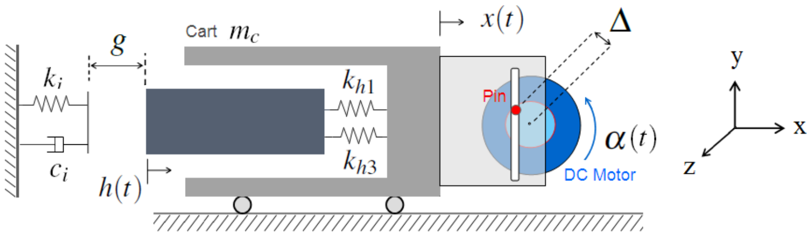


Figure 2: Vibro-impact electro-mechanical system. The nonlinear component spring is drawn as a linear spring with constant k_{h1} and a nonlinear cubic spring with constant k_{h3}.

motor is coupled to the cart through a pin that slides into a slot machined in an acrylic plate that is attached to the cart, as shown in Fig. 2. The off-center pin is fixed on the disc at distance Δ of the motor shaft, so that the motor rotational motion is transformed into a cart horizontal movement. The horizontal force that the DC motor exerts in the cart is f_x, and the vertical force is f_y (induced by the viscous friction between in the pin and the slot). The available torque τ and vertical force f_y are written as

$$\tau(t) = f_y(t)\ \Delta\ \cos\alpha(t) - f_x(t)\ \Delta\ \sin\alpha(t)\,, \tag{2}$$

$$f_y(t) = c_{pin}\ \Delta\ \dot{\alpha}(t)\ \cos\alpha(t)\,, \tag{3}$$

where c_{pin} is defined in Fig. 2. The embarked hammer is modeled as a rigid body of mass m_h and its relative displacement is h with respect to the cart. In the adopted model, the constitutive equation of the spring component between the hammer and the cart is written as $f_s(t) = k_{h1}\ h(t) + k_{h3}\ h(t)^3$. The rate of nonlinearity of the hammer stiffness is defined as $r_h = k_{h3}/k_{h1}$. The horizontal cart displacement is represented by x. Due to constraints, the cart is not allowed to move in the vertical direction. The spring-damper element modeling the medium on which the impacts occur, is constituted of a linear spring with stiffness coefficient k_i and a damper with damping coefficient c_i. The equations of the mechanical component are

$$\ddot{x}(t)\ (m_c + m_h) + \ddot{h}(t)\ m_h + c_{ext}\ \dot{x}(t) = -f_{\text{imp}}(t) + f_x(t)\,, \tag{4}$$

$$\ddot{x}(t)\ m_h + \ddot{h}(t)\ m_h + c_{int}\ \dot{h} + k_{h1}\ h(t) + k_{h3}\ h^3(t) = -f_{\text{imp}}(t)\,, \tag{5}$$

where, c_{ext} is the viscous friction coefficient between the cart and the rail and $c_{int} = 2\varsigma_{int}\sqrt{m_h k_{h1}}$ is the viscous friction coefficient between the cart and the hammer (ς_{int} is the damping ratio), and where f_{imp} is the impact force between the hammer and the barrier, which is written as

$$f_{\text{imp}}(t) = -\phi(t)\left(\ k_i\ (x(t) + h(t) + g) + \ c_i\ (\dot{x}(t) + \dot{h}(t))\right)\,, \tag{6}$$

where

$$\phi(t) = \begin{cases} 1, & \text{if} \quad x(t) + h(t) + g < 0 \quad \text{and} \quad \dot{h}(t) + \dot{x}(t) < 0\,, \\ 0, & \text{in all other cases}\,, \end{cases} \tag{7}$$

in which g is defined as the horizontal distance from the hammer (when $\alpha = \pi/2$ rad) to the equilibrium position of the barrier.

2.3 Coupled vibro-impact electro-machanical system

Due to the system geometry, we have the following constraint

$$x(t) = \Delta\, \cos(\alpha(t))\,. \tag{8}$$

Substituting Eqs. (2) to (8) into Eq. (1), we obtain the initial value problem for the vibro-impact electro-machanical system that is written as follows. Given a constant source voltage ν, find (α, c, h) such that, for all $t > 0$,

$$l\dot{c}(t) + rc(t) + k_e\dot{\alpha} = \nu\,, \tag{9}$$

$$\begin{aligned} &\ddot{\alpha}(t)\left[j_m + (m_c + m_h)\Delta^2 \sin(\alpha(t))^2\right] - \ddot{h}(t)\left[m_h\Delta\, \sin(\alpha(t))\right] - k_e c(t) \\ &+\dot{\alpha}(t)\left[b_m + \dot{\alpha}(t)(m_c + m_h)\Delta^2\, \cos(\alpha(t))\sin(\alpha(t)) + c_{pin}\Delta^2\cos(\alpha(t))^2 - c_{ext}\Delta^2\sin(\alpha(t))^2\right] \\ &= \phi\left(k_i(\Delta\, \cos(\alpha(t)) + h + g) + c_i(-d\dot{\alpha}(t)\sin(\alpha(t)) + \dot{h}(t))\right)\Delta\, \sin(\alpha(t))\,, \end{aligned} \tag{10}$$

$$\begin{aligned} &\ddot{h}(t)m_h - \ddot{\alpha}(t)\left[m_h\Delta\, \sin(\alpha(t))\right] - \dot{\alpha}(t)\left[m_h\Delta\,\dot{\alpha}(t)\cos(\alpha(t))\right] + \dot{h}(t)c_{int} + k_{h1}h(t) + k_{h3}h^3(t) \\ &= \phi(t)\left(k_i(\Delta\, \cos(\alpha(t)) + h + g) + c_i(-\Delta\,\dot{\alpha}(t)\sin(\alpha(t)) + \dot{h}(t))\right)\,, \end{aligned} \tag{11}$$

where

$$\phi(t) = \begin{cases} 1, & \text{if} \quad \Delta\, \cos\alpha(t) + h(t) + g < 0 \quad \text{and} \quad \dot{h}(t) - \Delta\,\dot{\alpha}(t)\cos(\alpha(t)) < 0 \\ 0, & \text{in all other cases}\,, \end{cases} \tag{12}$$

with the initial conditions: $\alpha(0) = 0\,,\ c(0) = \nu/r\,,\ h(0) = 0\,.$

3 DEFINING SOME POWERS OF THE SYSTEM

The average of the electric power consumed in an interval $[0, T]$ is written as

$$\pi_{\text{elec}} = \frac{1}{T}\int_0^T \nu\, c(t)\, \mathrm{d}t. \tag{13}$$

Let t_b^j and t_e^j be the instants of begin and end of the j-th impact, such that for all t belonging to $[t_b^j, t_e^j]$, we have $\dot{x}(t) + \dot{h}(t) < 0$. At time t, the impact power, $\pi_{\text{imp}}^j(t)$, is then written as

$$\pi_{\text{imp}}^j(t) = k_i\,(x(t) + h(t))\,(\dot{x}(t) + \dot{h}(t)), \qquad t_b^j \le t \le t_e^j\,. \tag{14}$$

The time average of the impact power during the j-th impact, $\underline{\pi}_{\text{imp}}^{j}$, is written as

$$\underline{\pi}_{\text{imp}}^{j} = \frac{1}{t_e^j - t_b^j} \int_{t_b^j}^{t_e^j} \pi_{\text{imp}}^{j}(t)\; dt\,. \tag{15}$$

Let N_{imp} be the total number of impacts that occur during time interval $[0, T]$. The sum of the averages of the impact power is

$$\pi_{\text{imp}} = \sum_{j=1}^{N_{\text{imp}}} \underline{\pi}_{\text{imp}}^{j}\,. \tag{16}$$

The variables π_{imp} and π_{elec} are considered for measuring the system performance. More π_{imp} is large and smaller is π_{elec}, better will be the system performance.

4 ROBUST DESIGN OPTIMIZATION PROBLEM

As explained in the introduction, this paper deals with the robust design of the vibro-impact electro-mechanical system in presence of uncertainties in the computational model. The three parameters that are assumed to be uncertain are k_{h1}, k_i and c_i, which are modeled by independent Gamma random variables K_{h1}, K_i and C_i, with mean values $\underline{K}_i$, $\underline{C}_i$, and $\underline{K}_{h1}$, and with the same coefficient variation, denoted by δ. In order to formulate the robust design optimization problem, the set of all the system parameters is divided into three subsets. The first subset is the family of the fixed parameters that is represented by the vector $\mathbf{p}_{\text{fix}} = \{\, \nu,\ l,\ r,\ j_m,\ k_e,\ b_m,\ c_{pin},\ c_{ext},\ \varsigma_{int},\ r_h,\ m_c,\ m_h,\ \Delta\,\}$. The second one is the family of the design parameters that is represented by the vector $\mathbf{p}_{\text{des}} = \{\underline{K}_{h1}/m_h,\ g\}$. The third one is the family of the uncertain parameters that is represented by the random vector $\mathbf{P}_{\text{unc}} = \{K_i,\ C_i,\ K_{h1}\}$. Since $\mathbf{P}_{\text{unc}}$ is a random vector, the outputs of the electro-mechanical system are stochastic processes and, consequently, $\pi_{\text{imp}}(\mathbf{p}_{\text{des}}, \mathbf{p}_{\text{unc}})$ and $\pi_{\text{elec}}(\mathbf{p}_{\text{des}}, \mathbf{p}_{\text{unc}})$, become random variables $\Pi_{\text{imp}}(\mathbf{p}_{\text{des}}) = \pi_{\text{imp}}(\mathbf{p}_{\text{des}}, \mathbf{P}_{\text{unc}})$ and $\Pi_{\text{elec}}(\mathbf{p}_{\text{des}}) = \pi_{\text{elec}}(\mathbf{p}_{\text{des}}, \mathbf{P}_{\text{unc}})$. The cost function is defined by

$$J(\mathbf{p}_{\text{des}}) = E\{\Pi_{\text{imp}}(\mathbf{p}_{\text{des}})\}\,. \tag{17}$$

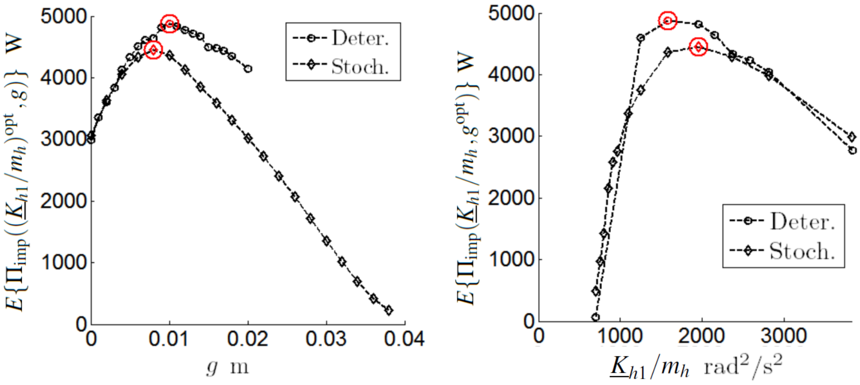
The robust design optimization problem is written as

$$\mathbf{p}_{\text{des}}^{\text{opt}} = \arg\max_{\mathbf{p}_{\text{des}} \in \mathcal{C}_{ad}} J(\mathbf{p}_{\text{des}})\,, \tag{18}$$

in which $\mathcal{C}_{ad} = \{\mathbf{p}_{\text{des}} \in \mathcal{P}_{\text{des}};\ E\{\Pi_{\text{elec}}(\mathbf{p}_{\text{des}})\} \le c_{\text{elec}}\}$, where $\mathcal{P}_{\text{des}}$ is the admissible set of the values of $\mathbf{p}_{\text{des}}$, and where c_{elec} is an upper bound.

5 RESULTS OF THE ROBUST OPTIMIZATION PROBLEM

For $\mathbf{p}_{\text{des}} \in \mathcal{C}_{ad}$, the cost function is estimated by the Monte Carlo simulation method using 100 independent realizations of random vector $\mathbf{P}_{\text{unc}}$ following its probability distribution. For solving the optimization problem defined by Eq. (18), admissible set $\mathcal{C}_{ad}$ is meshed as follows: for $\underline{K}_{h1}/m_h$, 13 values are nonuniformly selected in the interval $[703\,, 3, 830]$, and for g, 20 nonuniform values in $[0\,, 0.038]$. The hyperparameters δ_{K_i}, δ_{C_i} and $\delta_{k_{h1}}$, which control the level of uncertainties for K_i, C_i and K_{h1} are fixed to 0.1. The optimization problem is also considered whitout uncertainties in the systems parameters, that is, the deterministic case ($\delta_{K_{h1}} = \delta_{K_i} = \delta_{C_i} = 0$). For computation, the initial value problem defined by Eqs. (9) to (12) has been rewritten in a dimensionless form. Duration is chosen as $T = 10.0$ s. The 4th-order



(a) (b)

Figure 3: (a) Cost function as function of g with $(\underline{K}_{h1}/m_h)^{\text{opt}}$. (b) Cost function as function of $\underline{K}_{h1}/m_h$ with g^{opt}. In both graphs, the $E\{\Pi_{\text{imp}}(\mathbf{p}_{\text{des}}^{\text{opt}})\}$ is highlighted for the deterministic and stochastic cases with markers .

Runge-Kutta method is used for the time integration scheme for which we have implemented a varying time-step. The time-step is adapted to the state of the dynamical system according to the occurrence or the not occurrence of impacts. When the hammer is not impacting the barrier, the time step used is 10^{-4} s, but when the hammer is approaching the barrier and when it is impacting it, the time step is chosen as the value 10^{-5} s. The values used for the motor parameters, were obtained from the specifications of the motor Maxon DC brushless number $411,678$. The others elements of $\mathbf{p}_{\text{fix}}$ are: $\nu = 2.4$ V, $m_c = 0.30$ Kg, $m_h = 0.50$ Kg, $r_h = 0.30$ 1/m^2, $c_{pin} = c_{ext} = 5.00$ Ns/m, $\varsigma_{int} = 0.05$, and $\Delta = 0.01$ m. Upper bound c_{elec} is 6.00 W. For the deterministic case, the components of the optimal solution $\mathbf{p}_{\text{des}}^{\text{opt}}$ are $(\underline{K}_{h1}/m_h)^{\text{opt}} = 1,580$ rad^2/s^2 and $g^{\text{opt}} = 0.011$ m. For case with uncertainties, it is $\underline{K}_{h1}/m_h = 1,950$ rad^2/s^2 and $g = 0.008$ m. The role played by uncertainties on the optimal values of the design parameters can be analyzed through Fig. 3, which display the graphs $g \mapsto E\{\Pi_{\text{imp}}((\underline{K}_{h1}/m_h)^{\text{opt}}, g)\}$, and $\underline{K}_{h1}/m_h \mapsto E\{\Pi_{\text{imp}}(\underline{K}_{h1}/m_h, g^{\text{opt}})\}$. The robustness of the optimal design point, $\mathbf{p}_{\text{des}}^{\text{opt}}$, can be analyzed in studying the evolution of the coefficient variation, $\delta_{\Pi_{\text{imp}}}(\mathbf{p}_{\text{des}}^{\text{opt}})$, of random variable $\Pi_{\text{imp}}(\mathbf{p}_{\text{des}}^{\text{opt}})$ as a function of the uncertainty level. However, in order to better analyze the sensitivity of the responses with respect to the uncertainty level, we have constructed Fig. 4 that displays the graphs $g \mapsto \delta_{\Pi_{\text{imp}}}((\underline{K}_{h1}/m_h)^{\text{opt}}, g)$ and $\underline{K}_{h1}/m_h \mapsto \delta_{\Pi_{\text{imp}}}(\underline{K}_{h1}/m_h, g^{\text{opt}})$. It can be seen that the value $\delta_{\Pi_{\text{imp}}}(\mathbf{p}_{\text{des}}^{\text{opt}})$ occurs in a region for which the two following functions $g \mapsto \delta_{\Pi_{\text{imp}}}((\underline{K}_{h1}/m_h)^{\text{opt}}, g)$ and $\underline{K}_{h1}/m_h \mapsto \delta_{\Pi_{\text{imp}}}(\underline{K}_{h1}/m_h, g^{\text{opt}})$ are minima. This means the optimal design point is robust with respect to uncertainties.

6 CONCLUSIONS

In this paper, the formulation and the solution of a robust design optimization problem have been presented for a nonlinear vibro-impact electro-mechanical system in presence of uncertainties in the computational model. Since this nonlinear electro-mechanical system is devoted to the vibro-impact optimization, the time responses exhibit numerous shocks that have to be identified with accuracy, and consequently, a very small time step is required. We have thus chosen an explicit time-integration scheme and not an implicit one. Nevertheless, due to the presence of low-frequency contributions in the time responses, a long time duration is required, which will imply a huge number of integration time step if the time step were chosen constant. This is the reason why we have implemented an adaptive integration time step. It was one of

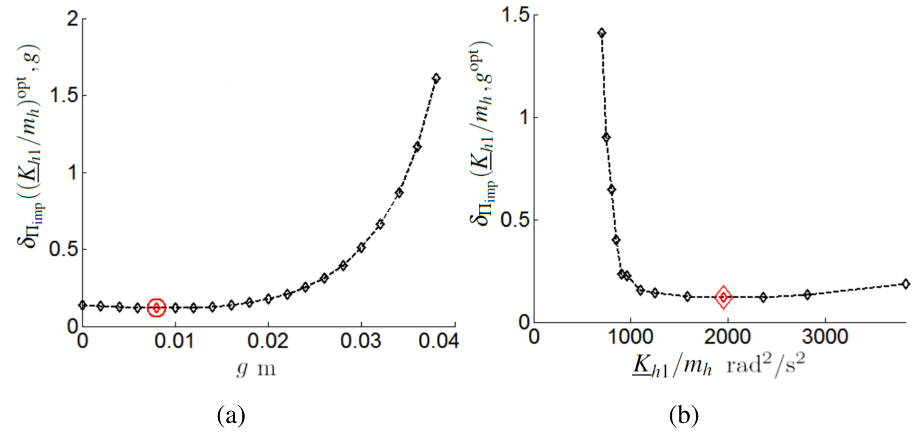


Figure 4: (a) Coefficient variation of $\Pi_{\rm imp}$ as function of g with $(\underline{K}_{h1}/m_h)^{\rm opt}$ (b) Coefficient variation of $\Pi_{\rm imp}$ as function of $\underline{K}_{h1}/m_h$ with $g^{\rm opt}$. In both graphs, the $\delta_{\Pi_{\rm imp}}(\mathbf{p}_{\rm des}^{\rm opt})$ is highlighted with markers.

the difficulties encountered for the solver implementation. The design optimization problem of the dynamical system without uncertainties yields an optimal design point that differs from the nominal values, and which can not be determined, *a priori*, without solving the design optimization problem. In addition, the robust analysis that has been presented demonstrates the interest that there is to take into account the uncertainties in the computational model. The optimal design point that has been identified in the robust design framework significantly differs from design point obtained with the computational model without uncertainties. For this electro-mechanical system, it has been seen that, the minimum value of the dispersion of the random output occurs in the region of the optimal design parameters, which means that the optimal design point is robust with respect to uncertainties.

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