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SHARP ILL-POSEDNESS RESULTS FOR THE SCHRODINGER-DEBYE AND BENNEY SYSTEM

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Abstract. We establish ill-posedness results for the Initial Value Problem (IVP) associated to the Schrodinger-Debye system in one-dimensional case. This system is derived from the equations of Maxwell-Debye. The Maxwell-Debye system describes the non resonant delayed interaction of an electromagnetic wave with a media.

This model can also be viewed as a perturbation of the Schrodinger equation with a cubic nonlinearity and this view has great theoretical value.

We studied the system below:

i ∂_t u +1/2 ∂_x^2 u = uv, t ≥ 0, x ∈ R,τ ∂_t v + v = e|u|^2,u (x, 0) = u_0 (x), v (x,0) = v_0 (x) where u = u (x, t) is a complex valued function , v = v (x, t) is a real function , τ > 0, e= ± 1 and ∂ is the Differential operator in one dimension.

In [3] The authors obtained for this model, well-posednessresults for the Sobolev space H^k x H^s with sek satisfying |k|+ 1/2 < s <min {k+1,2k+ 1/2} and k>-1/4.

In particular, we show that some of the results obtained local well-posedness in [3] are sharp.

We show the solution flow associated with the system is not C^2 at the origin for certain relations Sobolev indices.

Our result implies the impossibility of the use of interactive numerical methods for obtaining numerical solutions to this problem.

Similar results were obtained for the IVP Benney associated with the system. This system appears on the general theory of wave interaction of water in a nonlinear medium and was introduced by Benney.

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