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SEISMIC RESPONSE OF INELASTIC CONCRETE STRUCTURES WITH THE EQUIVALENT LINEAR METHOD

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Abstract: The equivalent linear method is widely used in Geotechnical Earthquake Engineering to calculate the accelerations at the surface of soil deposits subject to earthquakes of moderate intensity. The method approximately takes into account the nonlinear behavior of soils by using equivalent shear moduli and damping ratios that are function of effective shear strains. A series of linear analysis are performed each time using equivalent soil properties until their values at two consecutive steps are approximately equal. The effective strain is defined by reducing the peak strain retrieved from the time response. The reduction factor, usually 0.65, accounts for the fact that the peak strain only occurs at a single instant of time. This paper investigates the application of this technique to calculate the seismic response of reinforced concrete buildings with moderate nonlinear behavior. The method is tested using a 3-D finite element model of a building created in the program ANSYS. Nonlinear constitute relations in the form of stress vs. strain are used to calculate the full nonlinear response and also to define the equivalent modulus of elasticity and damping ratio by means of the Masing's rule. It was found that a key parameter affecting the accuracy of the results is the reduction factor. Considering a number of seismic records with different frequency contents an optimal reduction factor was defined that is a function of six parameters related to the intensity of the earthquake. The accuracy of the results obtained with the equivalent linear method depends on the response of interest sought (displacement, shear, moment, acceleration) but it proved to be quite acceptable for all the cases considered. The floor response spectra for the building with nonlinear behavior were also obtained and they compare very well with those computed with the exact nonlinear analysis.

1. INTRODUCTION

A most important problem in Soil Dynamics and Geotechnical Earthquake Engineering is the so called “site response analysis” in which the acceleration at the free surface of a stratified soil deposit is computed using as data the earthquake induced motion at the bedrock or at a rock outcrop. Soil materials undergo nonlinear deformations even when subjected to earthquakes of moderate intensity and thus it is important to account for their nonlinear behavior, even in an approximate way. Although the response of the soil deposit can be calculated via a rigorous nonlinear step-by-step dynamic analysis, this is not the approach followed in practical applications, except in research work or for special projects. Most commonly the “equivalent linear method” originally proposed by Seed and Idriss (1970) and later implemented in the well-known program SHAKE (Schnabel, 1972) is used to calculate the seismic response of the soil deposit. Basically the method consists in performing a series of linear analysis of the deposit by changing at each iteration step the material properties (the shear modulus G and the damping ratio ξ of each layer) so that they are consistent with the so-called “degradation curves”. These are graphs that depict the variation (i.e., the degradation) of G and ξ with the shear strain for each soil material.

The present paper presents a summary of an investigation that examined the feasibility of applying the equivalent linear method to calculate the approximate seismic response of reinforced concrete (RC) buildings with moment resistant frames. A baseline model of an RC three-story building was created in the finite element program ANSYS version 16 (ANSYS, Inc., 2016). The 3-D model is used to apply the equivalent linear method and to perform full nonlinear analyses to validate the proposed approach. Another objective of this study is to apply the equivalent linear method to calculate the floor response spectra (or in-structure response spectra) for a structure with inelastic behavior. The floor response spectrum is used to analyze non-structural components and equipment in a building but it is usually defined for structures with linear behavior.

It is known that the equivalent linear method has some limitations but nevertheless, as it was previously mentioned, it is extensively accepted in practical applications. One of the limitations of the method is that the nonlinear behavior of the soils must be moderate: it does not provide good results for soils undergoing strongly nonlinear deformations. It is reasonable to conclude that the same limitation will also apply to the intended application of this work, namely for building structures.

One of the reasons for using the equivalent linear method to calculate the seismic response of soil deposits is that the damping is accounted for by means of the complex modulus damping model. This damping model permits to assign different damping ratios to each of the soil layers of the deposit. In addition, it permits to model more accurately the real energy dissipation characteristics of soil materials. However, the complex modulus model requires an analysis in the frequency domain which is based on the Principle of Superposition and thus it cannot be applied to nonlinear systems. By iteratively replacing the nonlinear behaving soil deposit by a linear model with equivalent properties, one can apply a frequency domain analysis at each iteration step.

Because the original method was intended for soil dynamics applications where shear deformations govern the behavior, it needs to be adapted for frames undergoing bending deformations. Another difference is that the series of linear analyses required by the method will not be done in the frequency domain but rather in the time domain. In addition, the damping model used will be that available in the program ANSYS, namely the Rayleigh damping formulation.

2. THE TEST STRUCTURE

To evaluate the feasibility of applying the equivalent linear method to calculate the seismic response of a structure, a three-story building was designed following the provisions of the IBC 2015 code (International Code Council, 2014). A special moment resistant frame was used as a lateral force resistant system. The geometry of a typical frame of the building is displayed in Figure 1.

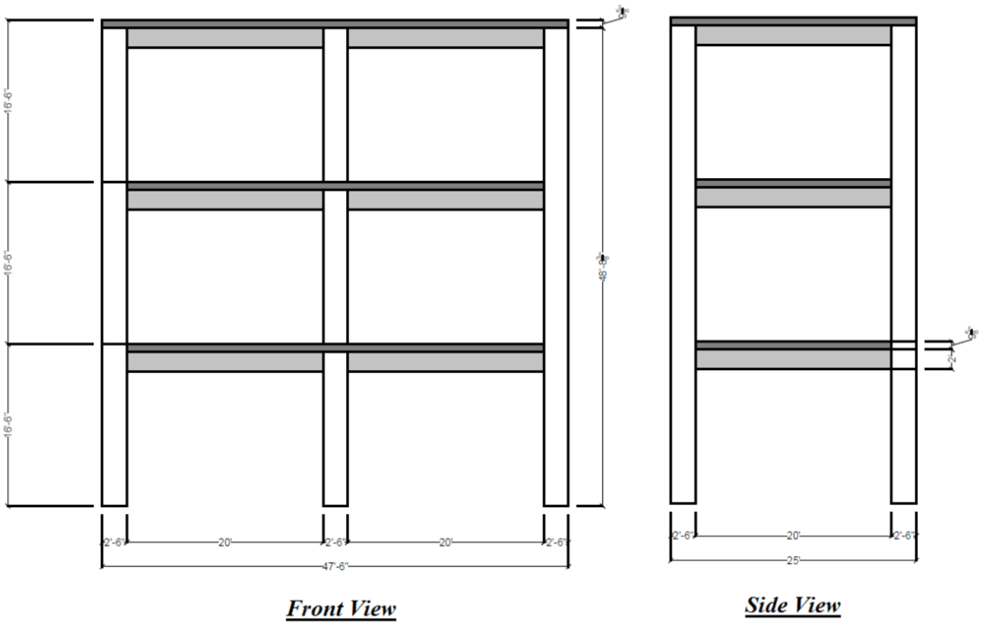

Figure 1: Geometry of a typical moment resistant frame.

A very detailed three-dimensional finite element model of the building was created in the computer program ANSYS version 16. The mesh consisted of 20-inches finite elements with hexahedral shape. Several tests were conducted to select a mesh that produced accurate results with an acceptable computational time. It was found that the element SOLID65 from the ANSYS library was the most appropriate for the present application due to its ability to predict the behavior of reinforced concrete in the nonlinear range.

To account for the nonlinear behavior of beams and columns, most structural analysis programs use moment-curvature or moment-rotation curves. However, this approach cannot be applied here because, as it was just mentioned, the structural elements were modeled with 3D finite elements. Therefore, similar curves but in terms of normal stresses and strains are required. For the combined concrete-reinforcing steel sections of the beams and columns, the nonlinear σ-ε constitutive relationships were obtained with the computer program SE::MC (Structure Express, 2015). Figure 2 displays the stress-strain curve calculated by this program for the beams. As shown in Figure 2, after the equivalent composite material reaches the yield state, it starts to lose capacity. This seriously complicated the nonlinear analysis with ANSYS and hence the stress capacity after yielding was assumed constant. This assumption does not introduce significant errors because in this study the nonlinear excursions will be limited; otherwise, the equivalent linear method cannot be applied for elements with strong nonlinear behavior. Similar constitutive relationships (not shown here) were derived for the columns cross sections and an analogous simplifying assumption regarding the post-yielding behavior was adopted.

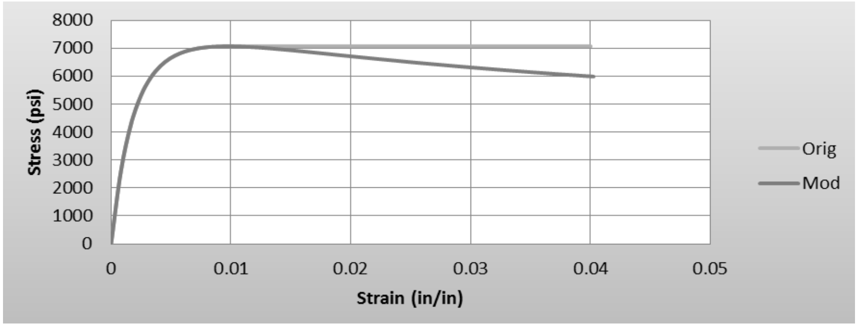

Figure 2: Stress-strain constitutive relationships for the beams.

3. SEISMIC EVENTS SELECTION

The selection of an acceleration time history to carry out linear or nonlinear dynamic analysis is an important issue. Usually a suite of accelerograms that represent the seismic hazard conditions at a site (the magnitude of the expected event, distance to the causative fault, faulting mechanism and local site geology) are selected. Because the objective of the present study is to assess if an approximate method can provide reasonably accurate results compared with a full nonlinear dynamic analysis, the selection of the accelerograms was not based on the aforementioned parameters. The intensity of the accelerograms was not an issue because they will be scaled so they generate a controlled nonlinear response. Therefore, it was decided to select records with different frequency content: for this purpose they are divided into “broad band” and “short band” records depending on whether they have a Fourier spectrum that is spread out through the frequency range or the dominant components are clustered in a narrow frequency band. The Pacific Earthquake Engineering Research Center (PEER) ground motion database which has a very large set of ground motions recorded worldwide of shallow crustal earthquakes was used to choose and pick the accelerograms (PEER, 2016). A total of eight recorded acceleration time series were retrieved.

Out of the eight seismic records, two were selected for a preliminary viability assessment of the application of the equivalent linear method to RC structures. The first accelerogram is representative of a broad band event: the El Centro earthquake of May 19, 1940 (officially known as the 1940 Imperial Valley earthquake). The acceleration time history with an original PGA of 0.313g and its (seudo) acceleration response spectrum for 5% damping are displayed in Figure 3. The vertical lines in the response spectrum indicate the first two natural periods of the building in the direction of the ground excitation.

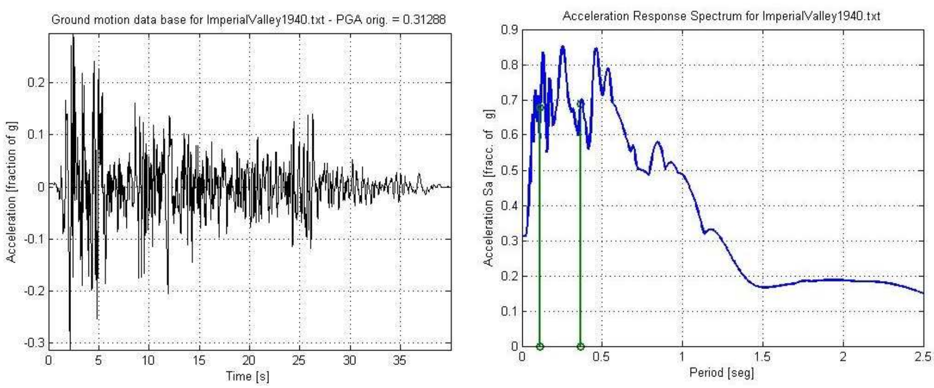

Figure 3: Accelerogram and response spectrum of the broad band event.

The short band ground motion record selected for a detailed preliminary study of the equivalent linear method corresponds to the 1986 San Salvador earthquake. The acceleration time history is shown in Figure 4 along with its 5% damping (seudo) acceleration response spectrum.

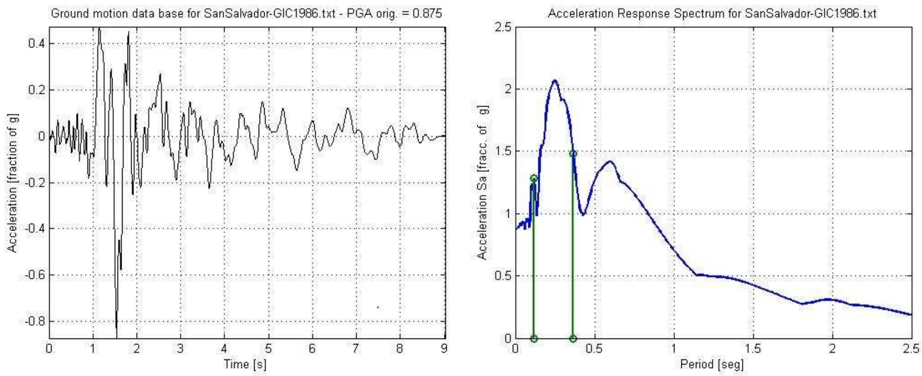

Figure 4: Accelerogram and response spectrum of the short band event.

4. THE EQUIVALENT LINEAR METHOD

The first step in the implementation of the equivalent linear method is to select a nonlinear stress-strain relationship. The relationship can be in the form:

$$\varepsilon = f(\sigma) \tag{1}$$

which defines the so called "Ramberg-Osgood models" (Suárez, 2008) or in the more common form:

$$\sigma = f(\varepsilon) \tag{2}$$

This expression defines the "Davidenkov models". In many cases both models are interchangeable, i.e. one can solve for one variable in terms of the other. There are, however, models which can only be defined in one of the two ways. In this thesis, the more common Davidenkov models will be adopted.

Although there are several well-known models for Soil Dynamics applications, such as the hyperbolic, the exponential, the Ramberg-Osgood model, etc., they were defined to represent the behavior of soil materials. Therefore, in this work the stress-strain relationship for reinforced concrete will be defined using a curve - fitting process, as it will be explained in the following section.

The relationships (1) or (2) define the so called "backbone curve" in the stress vs strain plane. This curve describes the stress generated in an element when it is monotonically deformed in the same direction (positive or negative) and it can be thought as the constitutive equation for a non-linear elastic element. Figure 5 displays a typical backbone curve.

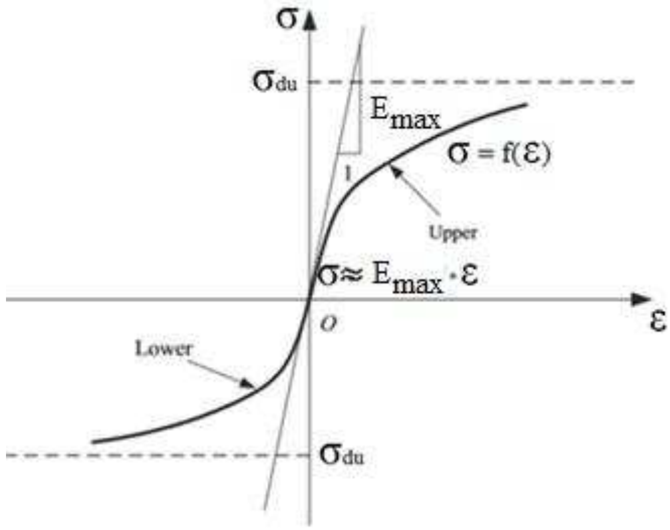

Figure 5: A typical backbone curve.

When the load reverses direction, i.e. when the element is subjected to a cyclic loading, the downloading path does not follow the same path as the backbone curve and a hysteresis loop is formed as the process continues. Figure 6 displays a typical hysteresis loop. To develop the equivalent linear method we need an explicit expression that defines the upper and lower branches of the hysteresis loop. In Soil Dynamics this is done be means of the so-called "Masing rule" (Suárez, 2008) explained in the following section.

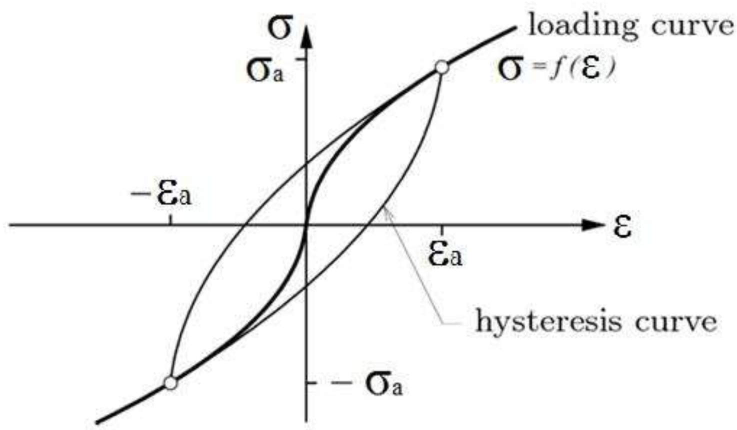

Figure 6: Hysteresis loop and the associated backbone curve.

4.1 **Masing's Rule Formulation**

To define the complete hysteresis loop by means of the Masing rule, the following variables will be used:

σ = *stress at a given point*
σ_a = *maximum value of the stress* (*at the point of cycle reverse*)
ε = *strain at a given point*
ε_a = *maximum value of strain* (*at the point of cycle reverse*)

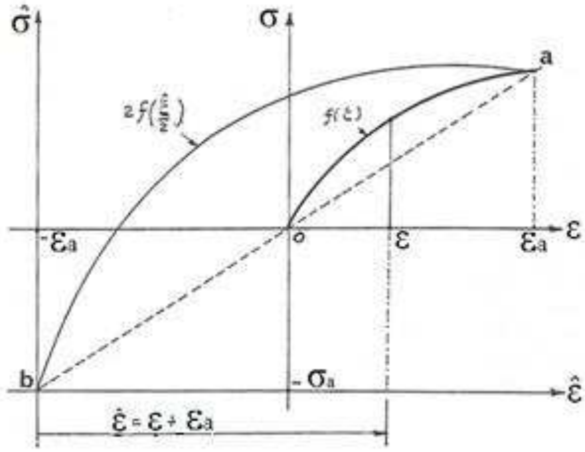

Figure 7: Generation of the hysteresis cycle according to the Masing rule.

We will begin defining the upper branch of the hysteresis loop. First the curve will be defined in terms of two auxiliary variables $\hat{\sigma}$ and $\hat{\varepsilon}$, as shown in Figure 7. The relation between the two set of variables is:

$$\begin{aligned}\hat{\sigma} &= \sigma + \sigma_a \\ \hat{\varepsilon} &= \varepsilon + \varepsilon_a\end{aligned} \tag{3}$$

The amplitude of the upper branch in the $\hat{\sigma}$ - $\hat{\varepsilon}$ plane is obtained by amplifying the original backbone curve by a factor of 2. To "stretch" the curve, i.e. to augment its range, the original argument of the function $f(\varepsilon)$ is divided by a factor of 2.

$$\hat{\sigma} = 2f\left(\frac{\hat{\varepsilon}}{2}\right) \tag{4}$$

In order to obtain the equation in terms of the original variables, one simply needs to replace them from equation (3):

$$\sigma = 2f\left(\frac{\varepsilon + \varepsilon_a}{2}\right) - \sigma_a \tag{5}$$

Proceeding in a similar fashion, it is straightforward to show that the lower branch of the hysteresis loop is defined by the following equation:

$$\sigma = 2f\left(\frac{\varepsilon - \varepsilon_a}{2}\right) + \sigma_a \tag{6}$$

The equations that define the backbone curve and the hysteresis cycle are not used directly in the equivalent linear method. Rather they are the basis to determine two essential parameters: an equivalent modulus of elasticity and an equivalent damping ratio.

Because the idea behind the method is to specify an equivalent linear system, the physical parameters that define this system are needed. For a homogeneous, isotropic and elastic material only two parameters are needed to uniquely define its constitutive equation. Commonly they are the pairs formed by the modulus of elasticity (or Young's modulus) *E* and the Poisson's ratio μ, or *E* and the shear modulus *G*, or another pair combination of these three. It can be is assumed that the Poisson's ratio is constant regardless of whether the structural system behaves in an elastic or inelastic fashion. Thus, the only parameter that needs to be defined is *E* or *G*. In Soil Dynamics the parameter selected is the shear modulus *G* of the soil because the shear deformation dominates the behavior of the material. For our purposes, it is more relevant to use the modulus of elasticity and, when it is needed, the shear modulus can be calculated using the well-known relationship:

$$G = \frac{E}{2(1+\mu)} \tag{7}$$

When the material is subjected to dynamic loads it is important to account for the energy dissipation, especially in the case of long duration excitations such as earthquakes. In this case, the typical constitutive relationship (Hooke's law) is usually replaced by the Kelvin-Voigt model. To define this model, in which the damping stresses are proportional to the time derivative of the strains, an additional parameter is required. In the Theory of Viscoelasticity the loss factor η is used to define the model, but in engineering applications, the damping ratio ξ is more commonly used.

In conclusion, we need to determine two material parameters: an equivalent modulus of elasticity and an equivalent damping ratio. The equivalent modulus of elasticity is the secant modulus E_{sec}. This modulus is the slope from the point of origin to the maximum point on the backbone curve, as shown in Figure 8.

The secant modulus of elasticity is defined as:

$$E_{\text{sec}} = \frac{\sigma_a}{\varepsilon_a} = \frac{f(\varepsilon_a)}{\varepsilon_a} \tag{8}$$

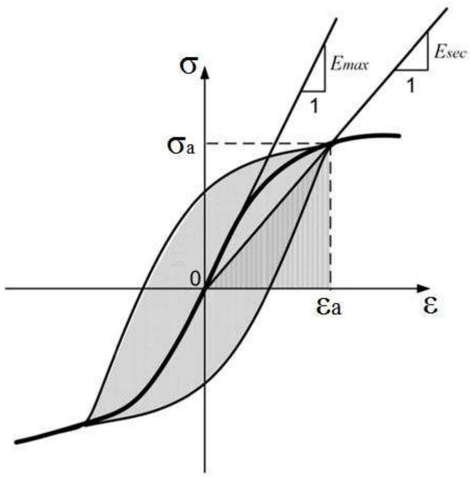

Figure 8: Low strain elastic modulus and secant modulus.

The next parameter that needs to be defined is the equivalent damping ratio. The area enclosed by the hysteresis loop is a measure of the energy dissipated per cycle of motion. This area is identified as ΔW and is the dotted area in Figure 9. To make this quantity independent of the maximum deformation, the area is normalized by the corresponding elastic energy stored up to the maximum deformation. This is the area identified by the vertical lines in Figure 9 and is denoted as W. To complete the definition of ξ, the ratio between the two areas is normalized by 4π:

$$\xi = \frac{1}{4\pi}\frac{\Delta W}{W} \tag{9}$$

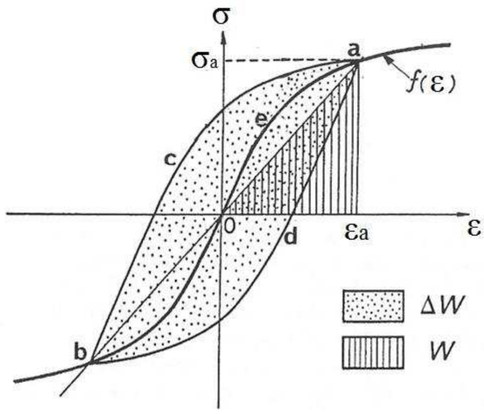

Figure 9: Definition of the equivalent damping ratio.

The area associated to the elastic energy stored W is simply the area of the triangle in Figure 9 and the energy of the hysteris loop ΔW is 8 times the area of the segment **o-e-a** in the same figure. It is straightforward to demonstrate that they can be calculated as follows:

$$W = \frac{1}{2}\sigma_a\varepsilon_a = \frac{1}{2} f(\varepsilon_a)\varepsilon_a \tag{10}$$

$$\Delta W = 8\int_0^{\varepsilon_a} f(\varepsilon)d\varepsilon - 4 f(\varepsilon_a)\varepsilon_a \tag{11}$$

Substituting ΔW and W in equation (9) the equivalent damping ratio becomes:

$$\xi = \frac{2}{\pi}\left(\frac{2\int_0^{\varepsilon_a} f(\varepsilon)d\varepsilon}{\varepsilon_a f(\varepsilon_a)} - 1\right) \tag{12}$$

4.2 Equivalent linear parameters for the beam elements

The first step required to apply the equivalent linear method to the building is to define the backbone curves. Due to the fact that the section properties are different for the columns and the beams, two curves $\sigma = f(\varepsilon)$ are needed. Moreover, because we need an analytical expression to define the secant modulus and to calculate the damping ratio, a polynomial equation was fitted to the actual stress-strain curve. The resulting polynomial equation is:

$$f\left(\varepsilon\right) = -1.43894\times10^{12}\varepsilon^4 + 5.05688\times10^{10}\varepsilon^3 - 6.31157\times10^8\varepsilon^2 + 3416520.0\varepsilon \tag{13}$$

Equation (13) can only be used to define the positive side of the curve; another equation is needed for the negative quadrant. To complete the definition of the backbone curve for negative values of the stresses and strain, the sign of the terms with even powers of ε needs to be switched. Therefore, the new equation is:

$$f\left(\varepsilon\right) = 1.43894\times10^{12}\varepsilon^4 + 5.05688\times10^{10}\varepsilon^3 + 6.31157\times10^8\varepsilon^2 + 3416520.0\varepsilon \tag{14}$$

Now that the information required to define the full backbone curve is available, the complete hysteresis loop can be drawn by combining equations (13) and (14) with those that define the upper and lower branches of the cycle, equations (5) and (6). The result is presented in Figure 10. Note that this was done only for illustrative purposes because neither the backbone curve nor the hysteresis loop are directly needed to implement the equivalent linear method.

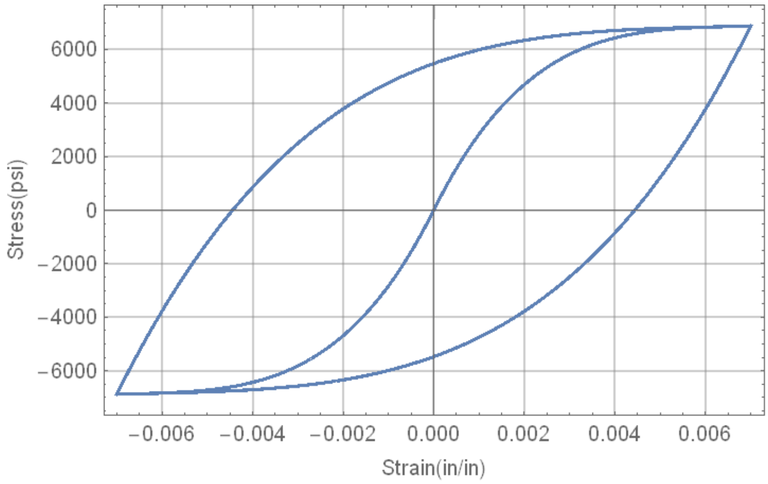

Figure 10: Backbone curve and hysteresis cycle for the beam elements.

The secant modulus E_{sec} is defined using equations (8) and (13):

$$E_{sec} = -1.43894\times10^{12}\varepsilon^3 + 5.05688\times10^{10}\varepsilon^2 - 6.31157\times10^8\varepsilon + 3416520.0 \qquad (15)$$

To simplify the notation, from now on the maximum strain will be denoted as ε instead of ε_a. Figure 11 displays the secant modulus for the beam elements.

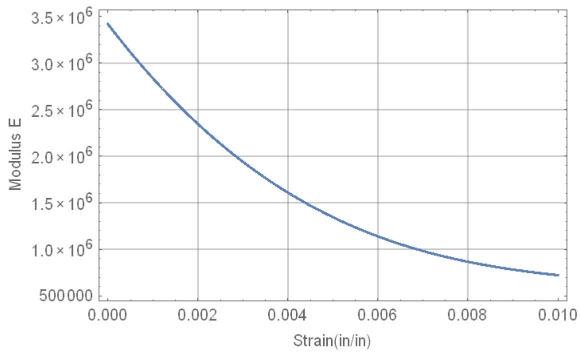

Figure 11: Degradation curve for the secant modulus of beams.

In a similar way, a closed form expression for the equivalent damping ratio can be obtained by substituting equation (11) in (10). The following equation was obtained with the symbolic manipulation computer software *Mathematica* version 10 (Wolfram Research, 2014).

$$\xi = \frac{2}{\pi}\left(\frac{3416520\varepsilon^2 - 4.208\times10^8\varepsilon^3 + 2.528\times10^{10}\varepsilon^4 - 5.776\times10^{11}\varepsilon^5}{3416520\varepsilon^2 - 6.312\times10^8\varepsilon^3 + 5.057\times10^{10}\varepsilon^4 - 1.439\times10^{12}\varepsilon^5} - 1\right) \qquad (16)$$

Figure 12 displays the degradation curve for the damping ratio, applicable to all the beam elements of the building model.

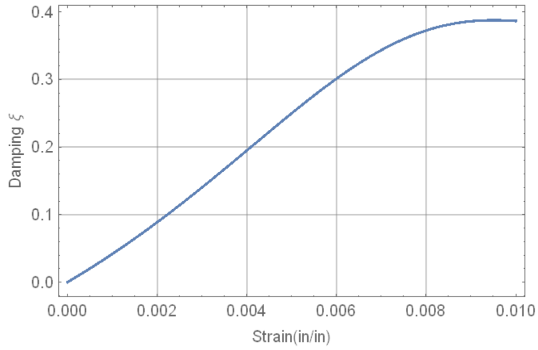

Figure 12: Degradation curve for the damping ratio of beams.

4.3 **Equivalent linear parameters for the column elements**

Because the nonlinear stress-strain relationship is different for the columns than for the beam elements, the process in the previous section must be repeated. Following the case of the beam elements, a fourth order polynomial was used. The resulting equation is:

$$f(\varepsilon) = -3.6886\times10^{11}\varepsilon^4 + 2.41744\times10^{10}\varepsilon^3 - 4.40088\times10^8\varepsilon^2 + 3124310.0\varepsilon \quad (17)$$

Equation (17) can only be used to define the backbone curve for the positive quadrant. To define it in the negative quadrant, the sign of the coefficients of the two even powers of ε is swapped:

$$f(\varepsilon) = 3.6886\times10^{11}\varepsilon^4 + 2.41744\times10^{10}\varepsilon^3 + 4.40088\times10^8\varepsilon^2 + 3124310.0\varepsilon \quad (18)$$

The backbone curve along with the hysteresis cycle for the column elements can be defined using equations (17), (18), (5) and (6). Both curves are displayed in Figure 13.

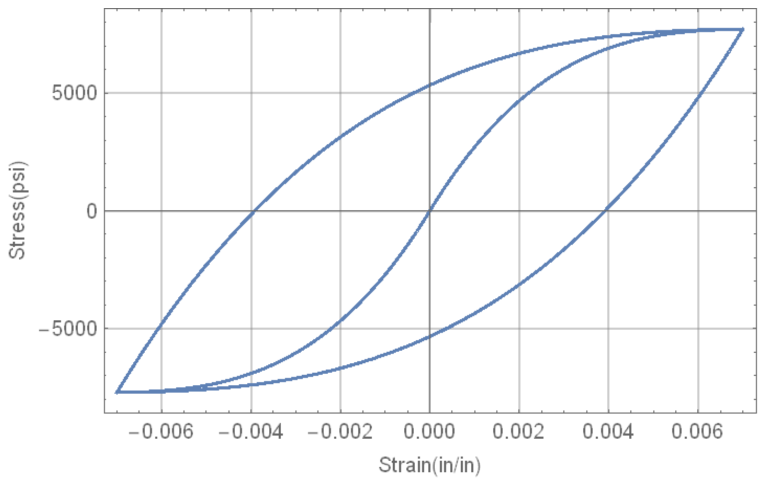

Figure 13: Backbone curve and hysteresis cycle for the column elements.

The next step is to obtain closed form expressions for the secant modulus and the damping ratio. First, the modulus of elasticity is obtained with equations (8) and (17):

$$E_{sec} = -3.6886\times10^{11}\varepsilon^3 + 2.41744\times10^{10}\varepsilon^2 - 4.40088\times10^8\varepsilon + 3124310.0 \quad (19)$$

Figure 14 displays the secant modulus for the column elements.

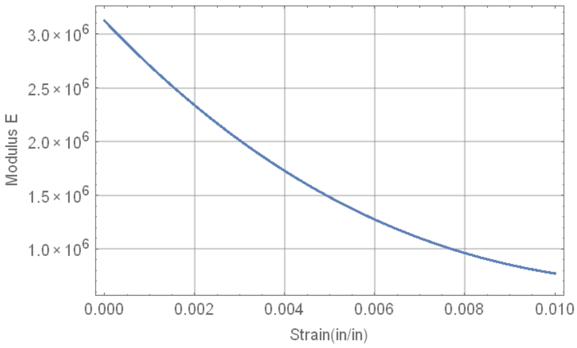

Figure 14: Degradation curve for the secant modulus of columns.

Next replacing $f(\varepsilon)$ from equation (17) in (19) and solving the integral, etc. and with the help of the program *Mathematica* (Wolfram Research, 2014), the following expression is obtained for the equivalent damping ratio:

$$\xi = \frac{2}{\pi}\left(\frac{3124310\varepsilon^2 - 2.934\times10^8\varepsilon^3 + 12.088\times10^9\varepsilon^4 - 14.754\times10^{10}\varepsilon^5}{3124310\varepsilon^2 - 4.401\times10^8\varepsilon^3 + 2.417\times10^{10}\varepsilon^4 - 3.689\times10^{11}\varepsilon^5} - 1\right) \tag{20}$$

The equivalent damping ratio for the columns is plotted in Figure 15.

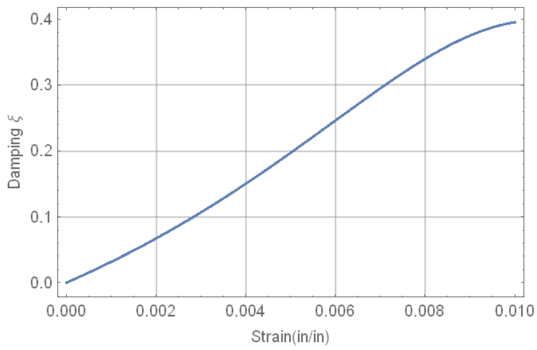

Figure 15: Degradation for the damping ratio of columns.

5. EQUIVALENT LINEAR DYNAMIC ANALYSES

The building was subjected to two types of loading: static forces due to the structure's self-weight and a dynamic excitation due to the earthquake ground acceleration. The gravitational load was applied by slowing increasing its magnitude for five seconds and then keeping it constant. The earthquake acceleration was applied one second after the gravitational load reached its final magnitude, i.e. at six seconds. The extra second was aded to dampen out any remaining oscillation.

5.1 **Broad-band seismic record**

The first study is a comparison of the response obtained with a nonlinear and a linear analysis of the structure. The building model created in ANSYS was subjected to a typical broad band event, namely the 1940 Imperial Valley record. This record is typical of a ground motion with a broad band frequency content. The record was scaled by a factor of 3.5 so that induced a nonlinear response in the structure. Four response quantities were calculated, namely the relative displacement (with respect to the base) of a point at the roof, the shear force and bending moment at a column at the base of the building and the absolute acceleration. The maximum response quantities were retrieved from the time histories and are shown in Table 1. It is evident that the linear analysis over predicts the true response.

Displacement (in)		
Non-linear result	***Linear result***	***%Diff***
5.3374	5.9308	11.12%
Shear force (kip)		
Non-linear result	***Linear result***	***%Diff***
407.1	484.0	18.93%
Bending moment (kip-in)		
Non-linear result	***Linear result***	***%Diff***
42442	50700	19.53%
Acceleration (in/s^2)		
Non-linear result	***Linear result***	***%Diff***
1436.8	1899.7	32.22%

Table 1: Non-linear and linear results due to the 3.5x scaled broad band record.

Next, the response will be calculated with the equivalent linear method. The method needs an initial estimate of the secant modulus and equivalent damping ratio due to the nonlinear behavior. An initial value of the normal strain ε equal to 0.001 is used to calculate the secant modulus E_{sec} and the equivalent damping ratio ξ. Equations (15) and (16) are used to compute them for the beam elements and equations (19) and (20) for the columns. These four quantities are used as input to the ANSYS program and the dynamic response to the Imperial Valley record is calculated. The values of E_{sec} and ξ for the beams and columns need to be updated using the newly calculated response. Here the maximum absolute values from the normal strain time histories in a selected beam and column are used for this purpose. Table 2 displays the intermediate and final results obtained during the iteration process.

Equivalent linear beam results using maximum strains					
Iteration #	***Assumed value of ε***	***Equivalent damping ξ***	***Secant modulus E (psi)***	***Computed value of ε***	***Difference in %***
1	0.001	0.0617672	2834490	0.0054803	448.03%
2	0.0054803	0.295136	1239520	0.0061969	13.08%
3	0.0061969	0.330147	1104800	0.0062896	1.50%
4	0.0062896	0.334345	1089230	0.0062759	0.22%
Equivalent linear column results using maximum strains					
Iteration #	***Assumed value of ε***	***Equivalent damping ξ***	***Secant modulus E (psi)***	***Computed value of ε***	***Difference in %***
1	0.001	0.0516967	2708030	0.0030056	200.56%
2	0.0030056	0.12697	2009950	0.0028699	4.51%
3	0.0028699	0.121366	2051690	0.0027923	2.70%
4	0.0027923	0.118194	2075910	0.0027648	0.98%

Table 2: Results of iterations with maximum strains for the broad-band earthquake.

Once the convergence criterion was reached, the maximum response was retrieved at the same points where it was calculated in the previous analysis. Table 3 displays the maximum relative displacement and absolute acceleration, and the maximum shear force and bending moment at the base. As it can be seen, in this case the equivalent linear method underestimates the true response (i.e., that obtained with a full nonlinear analysis). This means that the equivalent linear method produces a too stiff equivalent structural system. Actually, by comparing Tables 1 and 3 it is evident that the simple linear analysis yielded better results than the equivalent linear method. The most likely reason is that the iteration process was implemented using the maximum normal strains from the time histories. For a nonstationary excitation like an earthquake acceleration, this peak strain only occurs at a single instant of time and thus it is not reasonable to use the maximum values of the strains in the iteration process. In the following examples, an effective (reduced) strain will be used.

Displacement (in)		
Non-linear result	***Linear result***	***%Diff***
5.3374	3.2412	39.27%
Shear force (kip)		
Non-linear result	***Linear result***	***%Diff***
407.1	241.43	40.70%
Bending moment (kip-in)		
Non-linear result	***Linear result***	***%Diff***
42442	25655	39.55%
Acceleration (in/s^2)		
Non-linear result	***Linear result***	***%Diff***
1436.8	877.22	38.95%

Table 3: Final response with maximum strains for the broad-band earthquake.

Before continuing with the application of the equivalent linear method, it is pertinent to discuss how the equivalent damping ratios obtained at each iteration steps are used in the ANSYS building model. First, it is noticed that at each iteration step two equivalent damping ratios are used, one for the beams and another for the column elements. However, ANSYS (and most structural analysis programs) cannot assign different damping properties to specific parts of a structure. Rather, the damping is usually introduced as modal damping ratios, i.e. each vibration mode is assigned a specific value. Moreover, in ANSYS there is an additional restriction: because the program uses the Rayleigh damping model, only two modes can be assigned a specific value. To implement the equivalent linear method in ANSYS, the mean value of the two equivalent damping ratios obtained at each iteration step from the degradation curves was calculated. This value was assigned to two modes of the building through the damping matrix of the Rayleigh model. The damping ratio (along with the natural frequencies) was used to calculate the parameters α and β that multiply to the mass and stiffness matrix.

When the equivalent linear method is applied for site response analysis, i.e. to calculate the acceleration at the surface of a layered soil deposit, usually an effective shear strain γ_{efec} equal to 65% of the maximum absolute value γ_{max} is used in the process. Therefore to asses if this approach is viable to compute the seismic response of the building, the previous iterative process was repeated using $\varepsilon_{efec} = 0.65\ \varepsilon_{max}$. The partial results obtained with the iteration process are presented in Table 4. Since this was only a trial, the convergence criterion was set equal to 2%.

Equivalent linear beam results using 65% of strains						
Iteration #	***Assumed value of ε***	***Equivalent damping ξ***	***Secant modulus E (psi)***	***Computed value of ε***	***Effective ε (0.65*max.)***	***Difference in %***
1	0.001	0.0617672	2834490	0.0054803	0.003562195	256.22%
2	0.0035622	0.190614	1744850	0.0053088	0.00345072	3.13%
3	0.00345072	0.18451	1781600	0.005238	0.0034047	1.33%
Equivalent linear column results using 65% of strains						
Iteration #	***Assumed value of ε***	***Equivalent damping ξ***	***Secant modulus E (psi))***	***Computed value of ε***	***Effective ε (0.65*max.)***	***Difference in %***
1	0.001	0.0516967	2708030	0.0030056	0.00195364	95.36%
2	0.00195364	0.0854747	2354050	0.0025624	0.00166556	14.75%
3	0.00166556	0.0748915	2456670	0.0025094	0.00163111	2.07%

Table 4: Results of iterations with 65% strain reduction for the broad-band earthquake.

Displacement (in)		
Non-linear result	***Linear result***	***%Diff***
5.3374	3.7013	30.65%
Shear Force (kip)		
Non-linear result	***Linear result***	***%Diff***
407.1	262.93	35.41%
Bending moment (kip-in)		
Non-linear result	***Linear result***	***%Diff***
42442	27509	35.18%
Acceleration (in/s^2)		
Non-linear result	***Linear result***	***%Diff***
1436.8	891.69	37.94%

Table 5: Final response with 65% strain reduction for the broad-band earthquake.

Once convergence was achieved with the $0.65\varepsilon_{max}$ effective strain, the same four maximum response quantities previously used were obtained from the time histories. They are shown in Table 5. Although there is a slight improvement in the accuracy of the results compared to the previous case (i.e., using the maximum normal strains ε_{max}), the values of the four response quantities calculated are not satisfactory. The equivalent linear method continues to underestimate the true response. It is then preliminarily concluded that the typical approach to define the effective strain in Soil Dynamics is not applicable for calculating the seismic response of buildings.

Therefore, since there are no other guidelines to select a reduction factor, it was decided to vary this parameter beginning with 100% (i.e., no reduction) and decreasing it to zero. The building response was calculated by applying the equivalent linear method using each of the reduction factors to define the effective normal strain. The errors in the relative displacement, absolute acceleration, shear force and bending moments at the same selected points as before were calculated and are plotted in Figure 16. It is evident that there are optimal reduction factors but their values vary depending on the type of response.

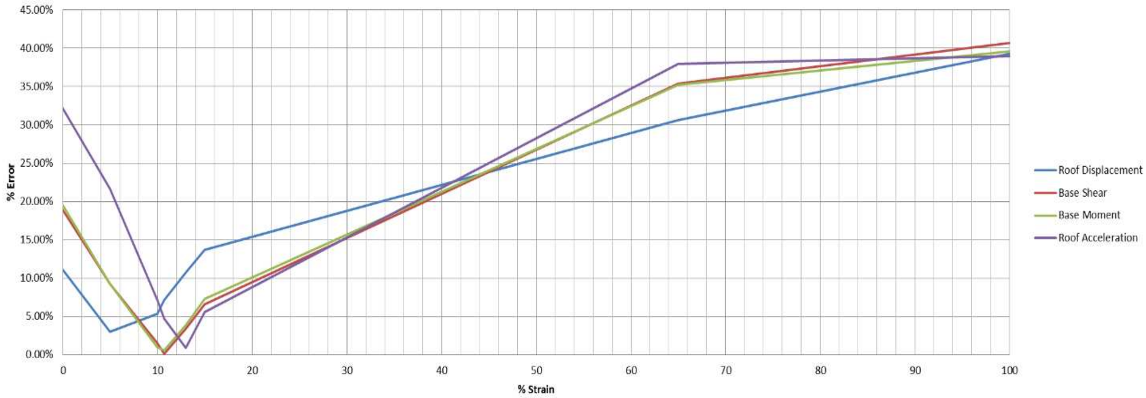

Figure 16: Error in the response for different strain reduction factors (broad band record).

The optimum values for each of the four response quantities are summarized in Table 6. The reduction factor varies from 6.8% for the displacement to 12.6% for the acceleration. Clearly, these values are much smaller than the 65% used in the Soil Dynamics applications.

For practical applications, it is not convenient to select different reduction factors depending on the response sought. A single value would be preferred, even though the error may be slightly higher for some of the response quantities. In this study it is recommended to use a weighted average value calculated by given more weight to the shear force and bending which are essential quantities for structural design. The final recommended reduction factor is presented in Table 7.

Error	***Strain %***
Displacement	6.796
Shear	10.756
Moment	10.432
Acceleration	12.641

Table 6: Optimum values of the reduction factor for the broad band earthquake.

Earthquake	**Scale Factor**	**Class**	**Factor**
Imperial	x3.5	Broad	0.10711

Table 7: Recommended reduction factor for the broad band earthquake.

To evaluate the effectiveness of the proposed reduction factor for a broad-band ground motion, the building is again subjected to the acceleration record of the 1940 Imperial Valley earthquake. The details of the iteration process are presented in Table 8. This time, the tolerance to check the convergence was set equal to 1% in order to get more precise results.

Equivalent linear beam results using 10.71% of strains						
Iteration #	***Assumed value of ε***	***Equivalent damping ξ***	***Secant modulus E (psi)***	***Computed value of ε***	***Effective ε (0.107*max.)***	***Difference in %***
1	0.001	0.061767	2834490	0.0054803	0.000586984	41.30%
2	0.000586984	0.043891	3063173	0.0062432	0.000668697	13.92%
3	0.000668697	0.047358	3016649	0.0061259	0.000656133	1.88%
4	0.000656133	0.046823	3023761	0.0061465	0.000658339	0.34%

Equivalent linear column results using 10.71% of strains						
Iteration #	***Assumed value of ε***	***Equivalent damping ξ***	***Secant modulus E (psi)***	***Computed value of ε***	***Effective ε (0.107*max.)***	***Difference in %***
1	0.001	0.051697	2708030	0.0030056	0.000321924	67.81%
2	0.000321924	0.029806	2985128	0.0032868	0.000352043	9.36%
3	0.000352043	0.030742	2972360	0.0032294	0.000345895	1.75%
4	0.000345895	0.030551	2974963	0.003239	0.000346923	0.30%

Table 8: Results of iterations with 10.71% strain reduction for the broad-band earthquake.

Table 9 compares the exact response with that predicted by the equivalent linear method once convergence was achieved. It can be seen that the results for the internal forces and moments are excellent; the errors for the displacement and acceleration are higher but still quite reasonable. The difference in the errors between these two types of quantities is due to the fact that, as explained before, the shear force and moment were given priority over the deformations quantities to define the optimal reduction factor for the strains.

Displacement (in)		
Non-linear result	***Linear result***	***%Diff***
5.3374	4.9858	6.59%
Shear Force (kip)		
Non-linear result	***Linear result***	***%Diff***
407.1	408.96	0.46%
Bending moment (kip-in)		
Non-linear result	***Linear result***	***%Diff***
42442	42377	0.15%
Acceleration (in/s^2)		
Non-linear result	***Linear result***	***%Diff***
1436.8	1508.7	5.00%

Table 9: Final response with 10.71% strain reduction for the broad-band earthquake.

5.2 Short-band seismic record

The previous study is repeated but using an earthquake ground motion with a typical short band frequency content, namely the 1986 San Salvador record registered at the CIG station. The first task is to calculate the linear response of the building and compare it with the results of a full nonlinear analysis. In order to force a nonlinear behavior, the earthquake record is scaled by a factor of 2. This factor was found by a trial and error process.

The differences in the maximum values of four response quantities calculated with nonlinear and linear analysis are shown in Table 10. As in the previous cases, the responses selected for comparison are the relative displacement and absolute acceleration at the top of the building and the shear force and bending moment at a column at the base of the structure. It can be seen that, similarly to the case of the broad band event, the linear analysis overestimates the four response quantities compared.

Displacement (in)		
Non-linear result	***Linear result***	***%Diff***
5.2808	5.7681	9.23%
Shear Force (kip)		
Non-linear result	***Linear result***	***%Diff***
411.43	4.95E+02	20.28%

Bending moment (kip-in)		
Non-linear result	***Linear result***	***%Diff***
42531	5.16E+04	21.40%
Acceleration (in/s^2)		
Non-linear result	***Linear result***	***%Diff***
1676.6	2202.2	31.35%

Table 10: Non-linear and linear results due to the 2.0x scaled short band record.

Observing the error obtained it is evident that in order to get a better approximation of the non-linear values another reduction factor must be implemented. As it was the case for the broad band record, using as effective strain either the full value or 0.65ε_{max} did not produce acceptable results. Therefore, a more appropriate reduction factor will have to be obtained by varying its value, calculating the responses and comparing them with the exact results. The reduction factor was varied from 0% to 100%: the zero value corresponds to a linear analysis and the 100% value is the case when ε_{max} is used. The errors in percent for the four different response quantities as a function of the reduction factor are presented in Figure 17. It is evident from the figure that there is no single optimum reduction factor that is applicable to the four responses.

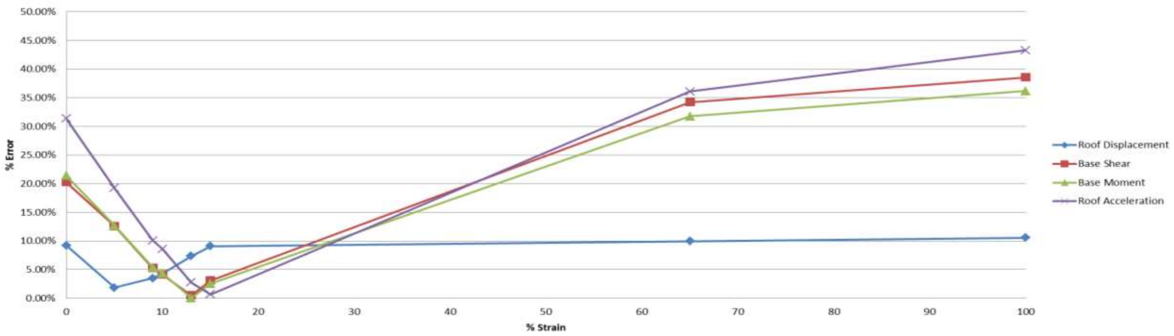

Figure 17: Error in the response for different strain reduction factors (short band record).

The optimum values for each response quantity are summarized in Table 11. The range of optimal values goes from 6.5% for the relative displacement to 14.6% for the absolute acceleration.

Error	***Strain %***
Displacement	6.494
Shear	12.674
Moment	13.002
Acceleration	14.601

Table 11: Optimum values of the reduction factor for the short band earthquake.

However, as it was mentioned before, for practical applications a single reduction factor that can be used for all the responses is desired. In order to obtain this parameter, a weighted mean was calculated where the shear force and bending moment parameters contribute more than the deformation quantities. The final factor recommended is shown in Table 12.

Earthquake	**Scale Factor**	**Class**	**Factor**
Salvador	x2	Short	0.12785

Table 12: Recommended reduction factor for the short band earthquake.

Using this factor to calculate the effective strain, the response of the building to the 1986 San Salvador earthquake was calculated again with the equivalent linear method. The normal strains, secant moduli and damping ratios at each iteration steps are provided in Table 13. Because these are final calculations, the convergence tolerance criterion was set equal to 1% in order to get more precise results.

Equivalent linear beam results using 12.78% of strains						
Iteration #	***Assumed value of ε***	***Equivalent damping ξ***	***Secant modulus E (psi)***	***Computed value of ε***	***Effective ε (0.1278*max.)***	***Difference in %***
1	0.00100	0.061767	2834490	0.0058474	0.000747561	25.24%
2	0.000748	0.050736	2972351	0.0063028	0.000805781	7.79%
3	0.000806	0.053251	2940026	0.0062604	0.000800361	0.67%
Equivalent linear column results using 12.78% of strains						
Iteration #	***Assumed value of ε***	***Equivalent damping ξ***	***Secant modulus E (psi)***	***Computed value of ε***	***Effective ε (0.1278*max.)***	***Difference in %***
1	0.00100	0.051697	2708030	0.0031288	0.000400001	60.00%
2	0.00040	0.032240	2952119	0.0033253	0.000425123	6.28%
3	0.000425	0.033028	2941559	0.0032936	0.00042107	0.95%

Table 13: Results of iterations with 12.78% strain reduction for the short-band earthquake

The final maximum relative displacement, absolute acceleration, shear force and bending moment obtained with the equivalent linear method and ε_{efec} = 0.128 ε_{max} are shown in Table 14 along with the exact responses and the relative errors. It is evident that the results are excellent for the internal forces and although the differences are higher for the deformation quantities, their accuracy is still very reasonable.

Displacement (in)		
Non-linear result	***Linear result***	***% Diff***
5.2808	4.9037	7.14%
Shear Force (kip)		
Non-linear result	***Linear result***	***% Diff***
411.43	410.61	0.20%
Bending moment (kip-in)		
Non-linear result	***Linear result***	***% Diff***
42531	42653	0.29%
Acceleration (in/s^2)		
Non-linear result	***Linear result***	***% Diff***
1676.6	1729.1	3.13%

Table 14: Final response with 12.78% strain reduction for the short band earthquake

6. SEISMIC INTENSITY PARAMETERS

There are many factors that affect the seismic response of a building, such as the intensity of the earthquake record, its duration and its frequency content. Each of these factors can be described by a parameter or index. A summary of these parameters is presented in a thesis by Miranda (2016) where their geographical distribution for the Island of Puerto Rico due to three low intensity seismic motions was presented. A few of these parameters that account for the main factors that affect the seismic demand imposed on buildings by an earthquake were selected; they are described in the next section. These indices will be computed for a number of historic ground motions with different characteristics and a reduction factor will be defined as a linear combination of them. The coefficients of the linear combination will be selected by a minimizing the difference between the exact nonlinear response and the approximate response calculated with the equivalent linear method.

6.1 The Peak Ground Acceleration

There are several factors associated with an earthquake record that has a distinct influence on the response of a structure. When these factors are measured by a quantifiable parameter they are usually referred to as "earthquake intensity indices". The most common parameter used to measure the intensity of an earthquake ground motion is the "Peak Ground Acceleration" (PGA). The PGA of a given seismic event is simply the maximum absolute value of the acceleration obtained from an accelerogram. For historical reasons and due to its simplicity the PGA was (and still is) widely used as an intensity index.

It is intuitive to assume that an earthquake with a higher PGA will cause a higher level of damage than another one with a smaller PGA. However, it is well known (e.g., Park et al., 1985) that reinforced concrete structures are generally damaged not only high stress excursions but also by a combination of repeated stress reversals. Since the PGA occurs at a specific instant of time, it may not be a proper measure of damage potential except for low-rise, short period buildings. Nevertheless, as it was mentioned the PGA is the most popular and extensively used intensity index and thus it is selected as the first parameter to be later applied to define the optimal reduction factor.

6.2 The Peak Ground Velocity

The "Peak Ground Velocity" (PGV) is another parameter commonly used to characterize the amplitude of a strong earthquake. The PGV is defined in a similar way as the PGA, but with the absolute value of the velocity. Usually, the velocity records show substantially less high frequencies than the acceleration. This is because the velocity is obtained by integration of the acceleration and this effectively results in a filtering of high frequencies. Due to the fact that the velocity is less sensitive to the high frequencies of strong motions, the PGV can better characterize the amplitude of strong motions at the intermediate frequencies. Moreover, several studies have found that the PGV correlates well with observed structural damage, especially in those structures with intermediate natural periods (Akkar and Bommer, 2007).

6.3 The Characteristic Intensity

The concept of "Characteristic Intensity" (Ic) emerged from the study of the seismic damage to reinforced concrete structures by Park et al. (1985). The rate of structural damage was defined as a linear combination of the damage caused by excessive deformation and the contribution of repeated load cycles. Their experiments consisted of analyzing two columns as a linear elastic structure with one degree of freedom, with one column being more ductile than other. With this, Park et al. determined the rate of damage for both columns, which they found to be proportional to the quantity defined in equation (21). The authors concluded that this would be viable representation of the destructive potential of a seismic event.

$$Ic = {A_{rms}}^{1.5}\, tf^{0.5} \tag{21}$$

where A_{rms} is the quadratic mean of the seismic acceleration and tf is the total duration of the seismic event.

6.4 The Arias Intensity

The “Arias Intensity” (AI) is a quantitative measure of the intensity of an earthquake that is based on instrumentation, and it can be regarded as the measurement of the total seismic energy absorbed by the soil. It correlates well with several commonly used demand measures of structural performance, liquefaction, and seismic slope stability. It is defined as:

$$AI = \frac{\pi}{2g}\int_0^{t_f}\left[\ddot{x}_g(t)\right]^2 dt \tag{22}$$

where $\ddot{x}_g(t)$ is the earthquake time history of acceleration, t_f is the total duration of the seismic event and g is the acceleration of gravity. It can be shown (using the Parseval's theorem) that the AI has a close relationship with the area under the squared amplitude of the Fourier spectrum calculated from the time history of acceleration.

6.5 The Cumulative Absolute Velocity

The “Cumulative Absolute Velocity” (CAV) is another parameter proposed as an index to quantify the potential earthquake damage to structures. One of its interesting characteristics is that it is proportional to load cycles causing low-cycle fatigue type damage (Katona, 2011). The CAV is defined as the area under the curve of the absolute value of the accelerogram. In mathematical terms, it is the integral of the absolute value of the acceleration time history over the duration of the earthquake. It is defined by equation (23) (EPRI, 1991):

$$CAV = \int_0^{t_f}\left|\ddot{x}_g(t)\right| dt \tag{23}$$

where $\ddot{x}_g(t)$ is the time history of the acceleration and t_f is the total duration of the seismic event.

6.6 Effective Design Acceleration

The idea of “Effective Design Acceleration” (EDA) was proposed by Benjamin and Associates, Inc. (Benjamin, 1988). They argued that the high frequency components of ground motions do not have a significant effect on the seismic responses of structures. However, their influence on the peak ground acceleration is important and therefore, they proposed a scaling parameter using the peak acceleration value. The approach consisted of only filtering out the peak accelerations that are above 8 - 9 Hz and using the remaining values as the EDA.

7. PROPOSED OPTIMAL REDUCTION FACTOR

It is proposed to define the optimal reduction factor *RD* as a linear combination of the six seismic demand indices described in the previous section, i.e. as:

$$RD = \alpha_1 + \alpha_2 PGA + \alpha_3 PGV + \alpha_4 Ic + \alpha_5 AI + \alpha_6 CAV + \alpha_7 EDA \tag{24}$$

A linear regression was implemented to find the seven coefficients α_i in equation (24). Five of the earthquake records were used to calculate the seismic response of the building: the 1940 Imperial Valley, the 1994 Northridge Sylmar, the 1966 Parkfield, the 1999 Hector Mine and the 1989 Loma Prieta ground motions. Next, the six seismic parameters for the each of the five earthquakes were calculated. The five weighted average errors along with the six indices for each of the seismic records were input into the program Microsoft Excel. Using the internal tools of this program a linear regression was performed from which the constants that multiply each seismic parameter in the linear combination were obtained.

The final formula to calculate the optimal reduction factor *RD* is provided in equation (25) for the fps system and in equation (26) for the SI system.

Using units of feet and seconds:

$$RD = -0.0395092 + 0.00209 AI + 0.0001658 CAV + 0.0234755 EDA - 0.0048305 Ic \\ - 0.0159971 PGA + 0.0332201 PGV \tag{25}$$

Using units of meters and seconds:

$$RD = -0.0395092 + 0.006857 AI + 0.000544 CAV + 0.077019 EDA - 0.0287059 Ic \\ - 0.0524837 PGA + 0.10899 PGV \tag{26}$$

8. VALIDATION OF THE RESULTS

In order to validate the proposed reduction factor, it was used to apply the equivalent linear method to calculate the response of the RC building model to eight seismic records. The objective was to determine the error in the nonlinear seismic response calculated with the approximate method for different earthquakes. The earthquake database used consisted of four broad-band and four short-band events that were selected to represent diverse seismic loadings that can be expected in a real case scenario. The seismic records were scaled up so that they cause a nonlinear behavior of the building. However, it is recalled that the equivalent linear method usually is not applicable to structures undergoing a highly nonlinear response and thus the scaling has its limits.

The results obtained for each of the eight seismic records is presented in Table 15. The table displays the seismic record applied, the scaling factor, the earthquake type in terms of its frequency content, the reduction factor calculated with equation (25), the relative errors in the relative displacement of the top floor, the shear force and bending moment in a first floor column, the absolute acceleration of the top floor and the average error.

As it can be seen, the maximum overall average error for all of the earthquake records is 9.7% and smaller for the other seven cases (around 3%).

Earthquake	Scale Factor	Class	Factor	Error Disp.	Error Shear	Error Mom.	Error Acc.	Average Error
Salvador	x2	Short	0.1278	7.1%	0.2%	0.3%	3.1%	2.7%
Imperial	x3.5	Broad	0.1071	6.6%	0.5%	0.2%	5.0%	3.1%
Loma Prieta	x2	Short	0.1804	12.8%	12.0%	12.3%	1.6%	9.7%
Northridge	x1	Short	0.1737	2.9%	0.1%	1.5%	1.0%	1.4%
Borrego	x80	Broad	0.0851	0.8%	4.0%	3.7%	3.4%	3.0%
Hector Mine	x18	Broad	0.0637	4.2%	0.0%	1.9%	5.8%	3.0%
Managua	x2	Broad	0.0461	5.8%	0.8%	0.5%	6.0%	3.3%
Parkfield	x6	Short	0.1222	1.2%	0.6%	0.4%	3.4%	1.4%

Table 15: Accuracy of the response predicted by the equivalent linear method with the proposed reduction factor for a collection of seismic records.

9. FLOOR RESPONSE SPECTRUM RESULTS

The nonstructural elements housed in a building consist of architectural components and other elements that do not contribute to the strength of the structure, and mechanical and electrical equipment. Especially in the nuclear industry, these are called secondary systems. When the building is subjected to a seismic ground motion, the components rigidly attached to a slab will experience the same acceleration of the floor. Most seismic codes provide formulas to estimate the forces acting on the component which has acceptable accuracy for non-critical and rigid systems. When the equipment itself is flexible or is not rigidly attached, the concept of floor response spectra is applied to calculate the seismic forces. This tool is widely used for equipment located in nuclear power plants and other important industrial facilities. It is basically a seismic response spectrum calculated using the absolute acceleration of a floor but for linear elastic structures, it can also be computed with closed form equations (Suárez and Singh, 1989).

9.1 Floor spectra for the broad-band earthquake

The time histories of the absolute accelerations of the three floors were obtained for the exact nonlinear case and for the equivalent linear system. The building was subjected to the typical seismic record with a broad band frequency content, namely the 1940 Imperial Valley earthquake. The damping ratio to calculate the floor response spectra was selected as 5%. Figure 18 displays the floor response spectra for the three floors of the building obtained with the two approaches considered. The results show that for the second and third floor the equivalent linear model was able to predict almost the exact response for the full range of periods considered. In the case of the first floor, for periods close to 0.1 sec (the 5th linear natural period of the building) the equivalent linear method underestimated the peak in the spectrum, but for the rest of the periods, it can be considered that it yielded an excellent approximation.

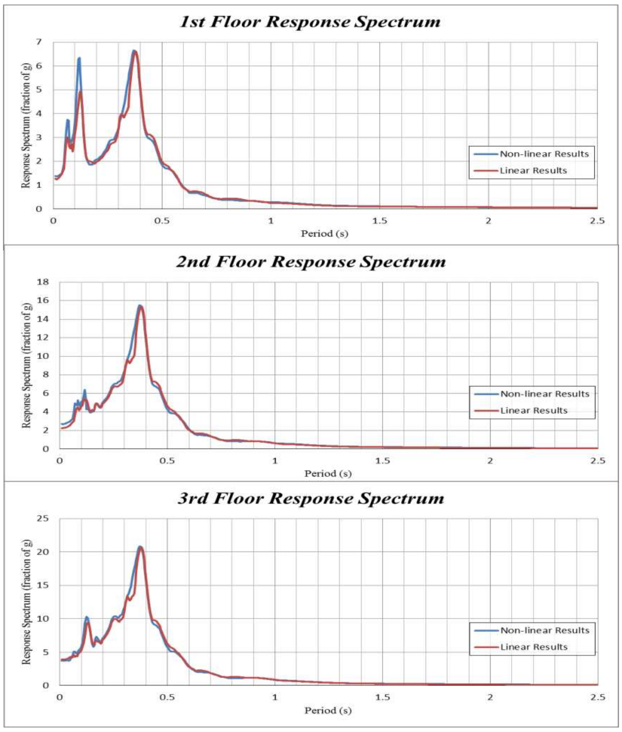

Figure 18: Broad-band event floor response spectra.

9.2 Floor spectra for the short-band earthquake

The procedure was repeated but this time subjecting the building to a typical seismic event with a short band frequency content, namely the 1986 San Salvador earthquake recorded at the CIG station. The acceleration of each floor obtained with the non-linear model and the equivalent linear method were retrieved and used to calculate the response spectra. The results are displayed in Figure 19. Examining the three graphs one can conclude that the equivalent linear method slightly underestimated the results. The differences are more pronounced at the first natural period of the building. However, in general, the equivalent linear method delivered a good approximation to the results of the full non-linear analysis for the short band event.

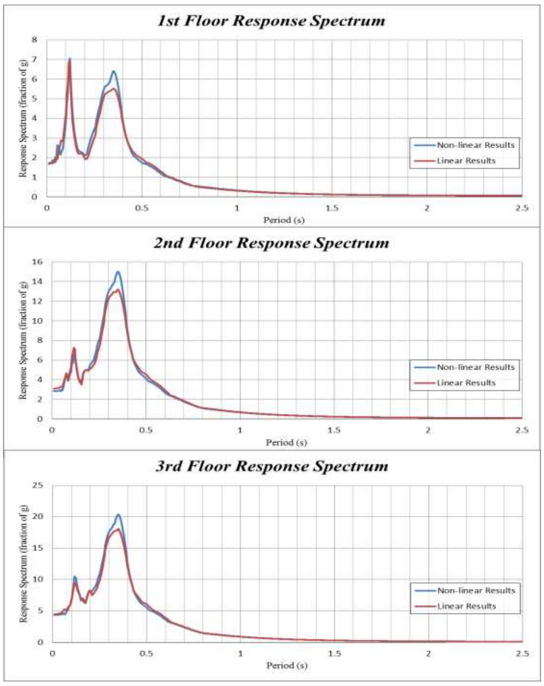

Figure 19: Short-band event floor response spectra

10. SUMMARY OF FINDINGS AND CONCLUSIONS

This paper presented the results of an investigation aimed at assessing whether the equivalent linear method, a method widely used in Soil Dynamics, can be applied to calculate the seismic response of reinforced concrete moment frames. To this end, a detailed 3D finite element model of a three-story reinforced concrete building was created in the program ANSYS. Constitutive relationships in the form of nonlinear σ-ε curves were obtained for the beams and columns sections that account for the concrete matrix and reinforcing steel bars. This information was input into the ANSYS program and the prototype model was subjected to the horizontal components of historic earthquakes. Two seismic records with different frequency content (broad band and short band) were selected for the first phase of the study. The structure was then analyzed with the equivalent linear properties using the elastic modulus and damping ratio obtained by applying the Masing rule.

The comparison between the full and the approximate nonlinear analyses showed that the 0.65 reduction factor commonly used in Soil Dynamics to define the effective strain from its peak value is not applicable. An optimal reduction factor that leads to an accurate estimation of the nonlinear response was found by trial and error for the broad and short band records. These results led us to generalize the reduction factor so that it can be applied to earthquakes with different characteristics. It was proposed to define the optimal reduction factor based on a linear combination of six parameters that account for the intensity of the earthquake records. Using eight historic records with different characteristics and a nonlinear regression, it was possible to obtain a more general reduction factor. Its accuracy was verified by comparing the full nonlinear response with the approximate response. In addition, the exact and approximate floor response spectra, a tool to calculate the seismic response of nonstructural components in buildings, were calculated and it was shown that they are quite similar.

It is acknowledged that a more comprehensive study is required to farther validate the application of the equivalent linear method to building structures. The methodology should be tested with more earthquake records and different moment resistant frames. However, the preliminary results presented in this work indicate that it is a promising approach and warrant the effort of carrying out further studies.

11. REFERENCES

Akkar, S. and Bommer, J. J. (2007). "Empirical prediction equations for peak ground velocity derived from strong-motion records from Europe and the Middle East", *Bulletin of the Seismological Society of America*, Vol. 97, No. 2, pp. 511-530.

ANSYS, Inc. (2015) ANSYS Version 16.0, computer software, Canonsburg, Pennsylvania.

Benjamin, J. R. & Associates (1988). "A criterion for determining exceedance of the Operating Basis Earthquake", EPRI Report NP 5930, Electric Power Research Institute, Palo Alto, California.

EPRI - Electrical Power Research Institute (1991). "Standardization of the Cumulative Absolute Velocity", Report EPRI TR-100082-T2, Palo Alto, California.

Katona, T.J. (2011). "Interpretation of the physical meaning of cumulative absolute velocity", *Pollack Periodica: An International Journal for Engineering and Information Science*, Vol. 6, No. 1, pp. 1-8.

International Code Council, Inc. (2014). 2015 International Building Code, Washington. DC.

Miranda, V. A. (2016) "Seismic parameters of earthquake records measured in Puerto Rico and its geographical distribution" (in Spanish), Master of Science Thesis, University of Puerto Rico, Mayagüez, PR.

Park, Y., Ang, A., and Wen, Y. (1985). Seismic damage analysis of reinforced concrete buildings, *Journal of Structural Engineering*, Vol. 111, No. 4, pp. 740-757.

PEER - Pacific Earthquake Engineering Research Center (2016). PEER Ground Motion Database, University of California at Berkeley, website: http://ngawest2.berkeley.edu/

Schnabel, P.B., Lysmer, J. and Seed, H.B. (1972). "SHAKE: a computer program for earthquake response analysis of horizontally layered sites", Report EERC-72-12, Earthquake Engineering Research Center, University of California, Berkeley.

Seed, H. B. and I. M. Idriss, (1970). "Soil moduli and damping factors for dynamic response analysis", Report EERC 70-10, Earthquake Engineering Research Center, University of California, Berkeley.

Suárez, L. E. and Singh, M. P. (1989). "Floor spectra with equipment-structure equipment interaction effects", *Journal of Engineering Mechanics*. Vol. 115, No. 2, pp. 247-264.

Suárez, L. E. (2008). "Non-linear dynamic analysis of soil systems", in: *Dynamics of Soils and Foundations,* Chapter 7, pp. 1-8 (in Spanish), University of Puerto Rico, Mayaguez, PR.

Wolfram Research, Inc. (2014) Mathematica, Version 10.0, computer software, Champaign, Illinois.