Asociación Argentina
de Mecánica Computacional

Mecánica Computacional Vol XXXIV, págs. 1553-1564 (artículo completo)
Sebastián Giusti, Martín Pucheta y Mario Storti (Eds.)
Córdoba, 8-11 Noviembre 2016

OPTIMIZATION OF PARAMETERS FOR TUNED MASS DAMPER

Luciara Vellar Rossato[a], Letícia Fleck Fadel Miguel[b] and Leandro Fleck Fadel Miguel[c]

[a]*Master's degree student in Mechanical Engineering, Federal University of Rio Grande do Sul (UFRGS), Porto Alegre, RS, Brazil, luciaravellar@gmail.com*

[b]*Professor Dr., Graduate Program in Mechanical Engineering, Federal University of Rio Grande do Sul (UFRGS), Porto Alegre, RS, Brazil, letffm@ufrgs.br*

[c]*Professor Dr., Graduate Program in Civil Engineering, Federal University of Santa Catarina (UFSC), Florianópolis, SC, Brazil, leandro.miguel@ufsc.br*

Keywords: Structural dynamic, Vibration control, Tuned mass damper (TMD), Newmark method, Firefly algorithm.

Abstract. Currently, structures are being evaluated for a greater number of actions when compared to a few decades ago. This improvement in designing stage is happening because projects providing lightweight and slender structures, with lower implantation costs, are being more requested. As a result, evaluating structures not only subjected to static loads, but also to dynamic loads has become necessary. Dynamic loads acting on a structure are more damaging than static loads, if they are not well considered and dimensioned. Dynamic loads could occur from earthquakes, wind, equipment, movement of people or vehicles, among other sources, which cause vibrations in structures and may lead to a collapse. The tuned mass damper (TMD), a passive control device, emerges as a possibility to control and reduce vibration amplitudes. The TMD has several advantages, such as large capacity to reduce amplitude of vibration, easy installation, low maintenance, low cost, among others. This study aims at evaluating, through numerical simulation, the dynamic behavior of a ten-storey building subjected to seismic excitation, before and after the installation of TMD, which is optimized to get the best structural response towards the dynamic excitation. For this purpose, a computational routine is developed in Matlab using the Newmark method to determine the dynamic structural response in terms of displacement, velocity and acceleration. The seismic excitation accelerogram used regards to ground acceleration record occurred in 1940, under the Imperial Valley in southeastern California, called El Centro. First, structure is analyzed only with its own damping for comparison and reference. Second, TMD optimization is carried out, in which the objective function is to minimize the maximum displacement at the top of the building, and the design variable is the modal mass ratio (Structure-TMD). Firefly algorithm is used for optimization. The TMD is installed on top of the building and, after optimizing its parameters, new dynamic structural responses are determined. Finally, responses obtained after the optimized TMD installation are considerably reduced, minimizing the risk of damage and building collapse.

1 INTRODUCTION

Buildings, structures of equipment, bridges, transmission line towers; to name a few types of real structures that may be subjected to dynamic actions. Increased demand for lighter structures with lower deployment cost has made it necessary to evaluate structures not only under static loads, but also under dynamic loads. Dynamic loads can be more damaging than static loads, if they are not well considered and dimensioned. Dynamic loads could arise from earthquakes, wind, equipment, movement of people or vehicles, among other sources, which cause vibrations and may lead to a collapse. These actions may also cause damage and collapse when their frequencies match natural frequencies of structures.

To obtain structural amplitudes of vibration subjected to seismic events, we can proceed in two different ways. The first is to use accelerogram records of actual earthquakes near the deployment of the structure. However, there are no earthquakes records in several regions, which leads to the second alternative: simulate a seismic excitation record. One option of generating a seismic record would be acceleration across the spectrum proposed by Kanai and Tajimi, which considers site characteristics where the structure will be deployed.

Tuned mass damper (TMD), a passive control device, can be installed in the structure to control and reduce vibration amplitudes. A vibration control system aims at reducing vibration amplitudes by installing external devices, which increase structural damping. According to Ospina (2008), control systems can be classified into four categories: passive, active, semi-active and hybrid.

Active systems are used to adapt structures to different frequencies. According to the action to which the structure is subjected, the control system, using sensors installed on the structure, is connected to an external system, which processes and verifies the best force and damping to control this action. However, active systems require monitoring, power and permanent external control.

Passive control systems do not require external power to have a good performance. These devices are designed for one type of action, and use the structure movements to dissipate energy due to dynamic action. Therefore, they are advantageous in relation to active systems when it comes to cost, installation, maintenance and easiness of control. Examples of passive systems are tuned mass damper, tuned liquid dampers, among others.

Semi-active systems are an intermediate solution between passive and active control systems, requiring no external power supply to the system due to its ability to adjust to structural responses. Examples of this device are variable friction dampers, variable orifice dampers, among others, as mentioned by Chaves (2010).

Combinations of active and passive systems are hybrid devices, which make them more efficient and economic.

This paper develops a computational routine in Matlab to determine the system response before and after the installation of tuned mass dampers, aiming at controlling vibrations caused by seismic excitation. The excitation signal used for the seismic event relates to ground acceleration record occurred in 1940 under the Imperial Valley in southeastern California, called El Centro. This accelerogram is available for download at "http://www.vibrationdata.com/elcentro.htm". Dynamic response, in terms of displacement and acceleration amplitudes, is determined by the Newmark method. The determination of the parameters (mass, stiffness and damping) of the tuned mass damper (TMD) to be installed on top of the building is made in accordance with the method proposed by Villaverde (1980), Villaverde (1985). Firefly algorithm is used to optimize TMD parameters with different modal mass ratios (Structure-TMD). Responses before and after the implementation of TMD are compared, revealing optimal parameters, aiming for best cost-benefit in which an average with considerable reduction in amplitude of displacements and accelerations is achieved, minimizing risk of structural damage and collapse.

2 METHODOLOGY

2.1 Seismic excitation signal

Earthquake (also known as a quake, tremor or temblor) is the name given to the release of energy in the crust of planet Earth, which usually occurs due to the shock of tectonic plates, creating seismic waves. Seismic activity regards to frequency, type and size of earthquakes recorded in an area over a period of time. Earthquakes are recorded by seismometers with units of a moment magnitude scale (MMS), measuring magnitude of earthquakes in terms of energy released. Earthquakes records with magnitude below 5 occur more frequently, and are verified by observatories using local magnitude scale better known as Richter scale. Moment magnitude scale and Richter scale have numerical similarities. Moment magnitude scale was introduced by Thomas C. Haks and Hiroo Kanamori as an alternative to replace the Richter scale developed by Charles Francis Richter. Earthquakes with intensity below 3 on a local scale are almost imperceptible, whereas higher than 7 can cause damage to various structures, depending on their depth. Mercalli scale, developed by Giuseppe Mercalli, is used for greater scales. The shallower the earthquake, which is more superficial on Earth's crust, the greater magnitude and damage.

Every year, estimated 500,000 earthquakes are detected by existing instrumentation, and about 20% of those can be felt (Pressler, 2010). Most earthquakes often occur in places like California and Alaska, in the United States, and also in countries like El Salvador, Mexico, Guatemala, Chile, Peru, Indonesia, Iran, Pakistan, the Azores in Portugal, Turkey, New Zealand, Greece, Italy, India and Japan, according to the Earthquake Hazards Program.

In 1940, the earthquake called El Centro (or 1940 Imperial Valley earthquake) occurred at 21:35 Pacific Standard Time on May 18, in the Imperial Valley Southeast California, near the international border between the United States and Mexico. The magnitude of this earthquake was 6.9 and a maximum perceived intensity of X (Extreme) in Mercalli intensity scale. The first and strongest earthquake recorded in the Imperial Valley by a large seismograph located near a fault rupture (Hough, 2004). El Centro was characterized as a typical moderate-sized destructive event, causing widespread damage to irrigation systems, the death of 9 people and damage of 80% of buildings to some degree. On Brawley’s business area, 50% of the structures were condemned and all were hit. It was the first endurance test to public schools after the Long Beach earthquake in 1933. The earthquake’s seismic record indicates duration of about 54 seconds. Figure 1 shows El Centro’s record.

In order to validate the dynamic structural responses, the structure will be subjected to a seismic signal corresponding to El Centro’s record, available for download at "http://www.vibrationdata.com/elcentro.htm".

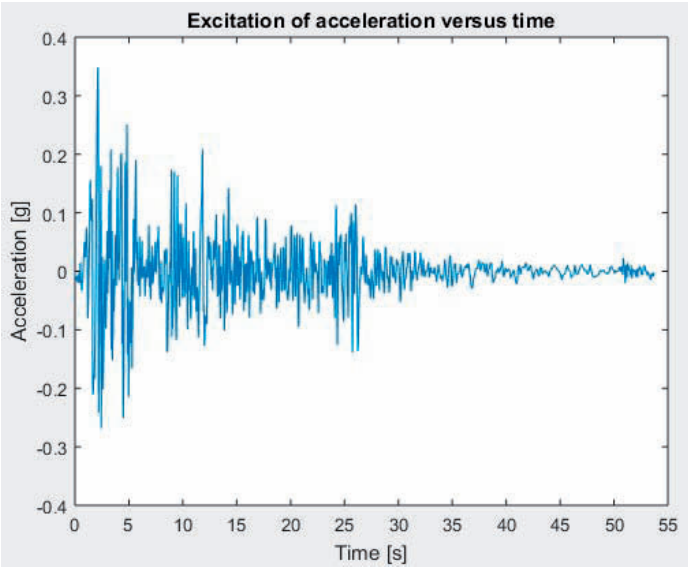

Figure 1: El Centro earthquake - acceleration versus time.

2.2 Dynamic structural response

The dynamic response of a structure with multiple degrees of freedom, subjected to an acceleration in the base, is given by the differential equations of dynamic motion system in Equation (1)

$$[M]\ddot{\vec{x}}(t)+ [C]\dot{\vec{x}}(t)+[K]\vec{x}(t)= -[M]\ddot{\vec{x}}_g(t) \tag{1}$$

in which:
[M] is the mass matrix of the system
[C] is the damping matrix of the system
[K] is the stiffness matrix of the system
t is the time
$\ddot{\vec{x}}(t)$ is the vector of system acceleration, as a function of time
$\dot{\vec{x}}(t)$ is the vector of system velocity in function of time
$\vec{x}(t)$ is the displacement vector of the system as a function of time
$\ddot{\vec{x}}_g(t)$ is the acceleration of the soil

From the structural characteristics, such as mass and stiffness, natural structural frequencies are calculated, being later used to determine optimal parameters of the tuned mass damper (TMD).

2.3 Newmark method

Direct numerical integration methods are classified into implicit and explicit. These methods consist of numerical procedures that do not require prior calculation of structural eigenvalues and eigenvectors. Functions representing variation of velocity and displacement within time interval Δt are obtained by integrating the function representing acceleration variation within the same time interval Δt. Convergence and accuracy of the solution depend on the capacity of the adopted function to represent acceleration accurately within each time interval Δt; also, depend on the interval size.

According to Groehs (2001), the Newmark method is the most effective of implicit methods, using the equation of motion at time t + Δt to calculate the response at this same time. Equations representing variation in displacement, velocity and acceleration at time t + Δt, are given by Equation (2), (3) and (4) respectively:

$$\vec{x}(t_{i+1})=(a_0[M]+a_5[C]+[K])^{-1}\left\{F(t_{I+1})+[M]\left[a_0\vec{x}(t_i)+a_1\dot{\vec{x}}(t_i)+a_2\ddot{\vec{x}}(t_{i)}\right]+[C]\left[a_5\vec{x}(t_i)+a_6\dot{\vec{x}}(t_i)+a_7\ddot{\vec{x}}(t_i)\right]\right\} \tag{2}$$

$$\dot{\vec{x}}(t_{i+1}) = a_5[\vec{x}(t_{i+1})-\vec{x}(t_i)]-a_6\dot{\vec{x}}(t_i)-a_7\ddot{\vec{x}}(t_i) \tag{3}$$

$$\ddot{\vec{x}}(t_{i+1}) = a_0[\vec{x}(t_{i+1})-\vec{x}(t_i)]-a_1\dot{\vec{x}}(t_i)-a_2\ddot{\vec{x}}(t_i) \tag{4}$$

Vectors $\vec{x}(t_{i+1})$, $\dot{\vec{x}}(t_{i+1})$, $\ddot{\vec{x}}(t_{i+1})$ represent displacement, velocity and acceleration at t+Δt time, respectively; and vectors $\vec{x}(t_i)$, $\dot{\vec{x}}(t_i)$, $\ddot{\vec{x}}(t_i)$ correspond to displacement, velocity and acceleration at time t, respectively.

Constant parameters a_i are given by Equation 5:

$$\begin{gathered} a_0=\frac{1}{\alpha\Delta t^2},\ a_1=\frac{1}{\alpha\Delta t},\ a_2=\frac{1}{2\alpha}-1,\ a_3=(1-\delta)\Delta t \\ a_4=\delta\Delta t,\ a_5=\frac{\delta}{\alpha\Delta t},\ a_6=\frac{\delta}{\alpha}-1,\ a_7=\frac{\Delta t}{2}\left(\frac{\delta}{\alpha}-2\right) \end{gathered} \tag{5}$$

Parameters α and δ are unconditionally stable for values $\alpha = 0.5$ and $\delta = 0.25$. For the first-time step, boundary conditions of displacement and velocity vectors must be known. The initial acceleration vector is given by Equation 6:

$$\ddot{\vec{x}}(t_0)=[M]^{-1}\left[\vec{F}(t_0)-[C]\dot{\vec{x}}(t_0)-[K]\vec{x}(t_0)\right] \tag{6}$$

2.4 TMD Dimensioning

Tuned mass damper (TMD) is a passive device to control vibration, consisting of a mass, a spring and a damper attached to a structure. This device is usually installed on top of a structure, and it is tuned to dampen the vibrations of the first vibration mode. The TMD is tuned to the same frequency of the first structural vibration mode, so that when the structure vibrates, part of the energy generated in the system is absorbed by the device. In case one wishes to control more than one vibration mode, additional TMDs will be required, so that each device will be tuned to the frequency of the corresponding mode.

Defining optimum parameters for TMD installation depends on the type of action to which the structure is subjected. The method proposed by Villaverde (1980), Villaverde (1985) is used for seismic action, in which dimensioning must satisfy Equation 7:

$$|\zeta_s-\zeta_T|=|\phi_m\sqrt{\mu}| \tag{7}$$

ζ_s is the structural damping; ζ_T is the damping of the TMD, which should be highest possible, being below the critical damping, so that energy is dissipated and TMD may oscillate; ϕ_m is the vibration mode to perform amplitude control; and μ is the mass ratio, defined by Equation 8:

$$\mu=\frac{m_T}{m_s} \tag{8}$$

m_T is the mass of the TMD, and m_s is the mass of the structure.

According to Paredes (2008), the TMD dimensioning is performed using a dynamic system having a single vibrational mode. However, actual structures have several DOF, i.e., several vibration modes. Thus, for the method to be valid, vibration modes should be normalized, having unitary participation factors. For seismic actions, normalization is made with participation factors (Fj), expressed as:

$$F_j = \frac{L_j}{M_j} \tag{9}$$

L_j is the modal participation factor in mode j:

$$L_j = \phi_j [M]\{1\} \tag{10}$$

ϕ_j is vibration mode already normalized; [M] is the mass matrix of the structure; and {1} is a column vector with dimension n x 1, n representing the total number of DOF, and 1 indicating all vector elements are equal to a unit (1).

The modal mass system is given in the following:

$$M_j = \phi_j^T [M] \phi_j \tag{11}$$

In order for the participation factor to be unitary, Equation 12 must be solved, finding out the value of constant β_j, which multiplied by ϕ_j will provide normalized vector of the unitary participation factor φ_j.

$$\frac{(\beta_j \phi_j)[M]\{1\}}{(\beta_j \phi_j^T)[M](\beta_j \phi_j)} = 1 \tag{12}$$

Isolating β_j:

$$\beta_j = \frac{(\phi_j)[M]\{1\}}{(\phi_j^T)[M](\phi_j)} \tag{13}$$

Resulting in solution:

$$\varphi_j = \beta_j \phi_j = F_j \phi_j \tag{14}$$

2.5 Firefly algorithm

In mathematical optimization, Firefly algorithm is a metaheuristic proposed by Xin-She Yang and inspired by social flashing behavior of fireflies.

Based on characteristics of the light of fireflies, the algorithm is developed. Yang states three rules in optimizing the Firefly algorithm:

- All fireflies are unisex, so that one firefly will be attracted to other regardless of its sex;
- Attractiveness is proportional to its brightness, thus for any two bright fireflies, the less brighter will move towards the brighter one. The attractiveness is proportional to the brightness and both decrease as their distance increases. However, the intensity diminishes as they are more distant;
- The brightness of a firefly is affected or determined by the landscape of the objective function.

Based on these three rules, the FA is presented briefly through pseudocode in steps shown in Figure 2:

```
Begin
   1) Objective function: f(x),   x = (x_1, x_2, ..., x_d);
   2) Generate an initial population of fireflies x_i   (i = 1, 2, ..., n);.
   3) Formulate light intensity I so that it is associated with f(x)
      (for example, for maximization problems, I ∝ f(x) or simply I = f(x);)
   4) Define absorption coefficient γ

   While (t < MaxGeneration)
      for i = 1 : n (all n fireflies)
         for j = 1 : n (n fireflies)
            if (I_j > I_i),
               move firefly i towards j;
               Vary attractiveness with distance r via exp(−γ r);
               Evaluate new solutions and update light intensity;
            end if
         end for j
      end for i
      Rank fireflies and find the current best;
   end while

   Post-processing the results and visualization;

end
```

Figure 2: Pseudocode Firefly Algorithm [reproduced from Xin-She Yang, 2009].

In Firefly Algorithm, there are two important issues: the change in light intensity and the formulation of attractiveness. Simply put, we can assume the attractiveness of a firefly is determined by its brightness, which is associated with the encoded objective function; the brightness *I* of a firefly at a certain location *x* can be defined as *I(x) α f(x)*.

However, the attractiveness β is relative, it should be seen in the eyes of the beholder or judged by other fireflies. Thus, it will vary with the distance r_{ij} between firefly *i* and firefly *j*. Additionally, light intensity decreases with distance from its source, and light is also absorbed in the media, then we should allow attractiveness to vary with the degree of absorption.

In the simplest form, the light intensity *I(r)* varies according to the inverse square law, expressed in Equation 15:

$$I(r) = \frac{I_s}{r^2} \tag{15}$$

I_s represents light intensity at the source. For a given medium with a fixed light absorption coefficient γ, light intensity *I* varies with distance *r*:

$$I = I_0 e^{-\gamma r} \tag{16}$$

I_0 is the original light intensity. To avoid singularity at $r = 0$, in Equation 15, the combined effect of both the inverse square law and absorption can be approximated using Gaussian form in Equation 17:

$$I(r) = I_0 e^{-\gamma r^2} \tag{17}$$

As a firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, we can define the attractiveness β of a firefly by

$$\beta = \beta_0 e^{-\gamma r^2} \tag{18}$$

β_0 is the attractiveness in r = 0. As it is generally faster to calculate 1/(1+ r²) than an exponential function, the above function (equation 18) can be conveniently approximated by Equation 19

$$\beta = \frac{\beta_0}{1+\gamma r^2} \tag{19}$$

Both Equations 18 and 19 define the characteristic distance

$$\Gamma = \frac{1}{\sqrt{\gamma}} \tag{20}$$

Over which the attractiveness changes significantly from β to $\beta_0 e^{-1}$ in Equation 18, or $\beta_0/2$ in Equation 19.

In the implementation, the form of attractiveness function $\beta_{(r)}$ can be any function decreasing monotonically, such as Equation 21

$$\beta(r) = \beta_0 e^{-\gamma r^m},\ (m \geq 1) \tag{21}$$

The movement of a firefly i that is attracted to another firefly j (brighter) is determined by Equation 22

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2}\left(x_j - x_i\right) + \alpha\epsilon_i \tag{22}$$

Second term regards to the attraction. Third term is the randomization, in which α represents the randomization parameter, and ϵ_i is a vector of random numbers from a Gaussian distribution or uniform distribution.

2.6 Additional data

The building model adopted to determine structural responses subjected to the seismic event El Centro is the 10-storey building shown in Figure 3. The values adopted for mass and stiffness of each floor are shown in Table 1. The structure's own damping is initially adopted for calculation, corresponding to 0.5%. For the integration method, Δt = 0.02 seconds.

In order for the reduction of vibration amplitudes to be the greatest possible, TMD will be installed on top of the building, as supported by literature.

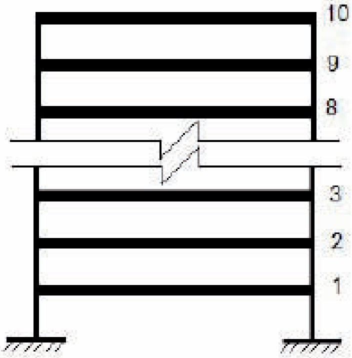

Figure 3: Structure with 10 degrees of freedom, from Mohebbi et al. (2012)

Floor	Mass [Kg]	Stiffness [N/m]
1	105000	990000000
2	105000	990000000
3	105000	792000000
4	105000	792000000
5	105000	792000000
6	105000	792000000
7	105000	404400000
8	105000	288600000
9	105000	185400000
10	105000	185400000

Table 1: Data for the building structure study

3 ANALYSIS OF RESULTS

This chapter presents the results, from the determination of the dynamic structural responses to the implementation of TMD and its optimization, obtaining best cost-benefit.

3.1 Dynamic analysis without TMD

First, ten natural frequencies of the structure without TMD are calculated, resulting in Hz: 1.93; 4.44; 7.61; 10.28; 12.04; 14.64; 17.54; 21.89; 25.60; 27.75.

Subjecting the structure of the building to the earthquake record El Centro shown in Figure 1, we obtain the amplitudes presented in Table 3.

Maximum values of displacement and acceleration occur on top floor of the building. Figure 4 shows structural responses on top floor, without the TMD, in terms of displacement and acceleration versus time.

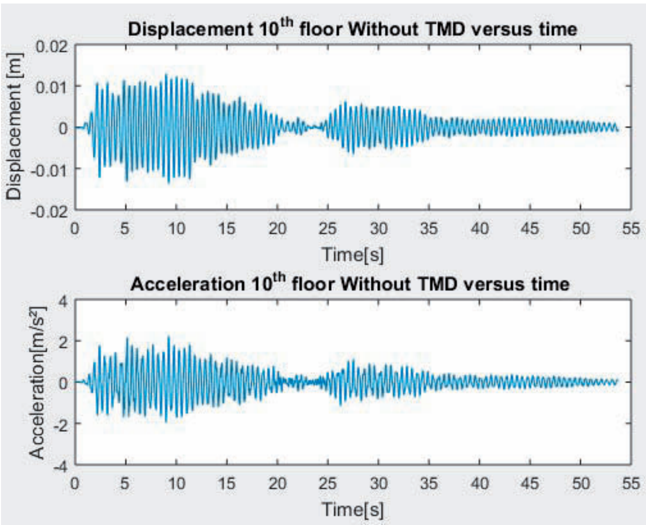

Figure 4: Structural response in terms of displacement and acceleration as a function of time

3.2 Dynamic analysis with TMD

Equations presented in Section 2.4 were used to dimension the TMD, along with the Firefly algorithm. For firefly parameters, lower bound and upper bound values were, respectively, 0.0

and 0.02; these limits were adopted by the user to define modal mass ratio (structure-TMD), which is the design variable. Population size adopted corresponded to 10 individuals, with 50 iterations. Thus, bringing the total number of 500 evaluations. Best result was found when modal mass ratio (μ) is 0.02; values adopted for TMD parameters as mass, stiffness, damping and damping ratio are presented in Table 2.

Mass [kg]	Damping [Ns/m]	Stiffness [N/m]	Damping ratio [ζT]
15466	79647	2265257	0.2128

Table 2: Values of TMD parameters for modal mass ratio of 0.02

The mass of TMD is not directed related to the total mass of the structure, but it is determined as established in section 2.4., i.e., the mass of TMD is defined using the modal mass ratio, and not 2% of total structural mass.

After attaching the TMD and setting values presented in Table 3, the dynamic analysis is redone in order to verify vibration amplitudes. A comparison between the structural responses, before and after using TMD, is given in Table 3.

	Without TMD		With TMD			
Floor	Displacement [m]	Acceleration [m/s²]	Displacement [m]	Acceleration [m/s²]	% Reduction of Displacements	% Reduction of Accelerations
1	0.00111	0.37770	0.00084	0.32836	24.32	13.06
2	0.00218	0.66025	0.00165	0.60147	24.31	8.9
3	0.00343	0.98042	0.00259	0.73857	24.49	24.67
4	0.00455	1.18824	0.00347	0.82008	23.74	30.98
5	0.00554	1.21014	0.00426	1.01244	23.1	16.34
6	0.00653	1.43001	0.00496	1.17302	24.04	17.97
7	0.00821	1.73907	0.00613	1.25532	25.33	27.82
8	0.01014	1.88588	0.00752	1.15675	25.84	38.66
9	0.01246	2.23697	0.00923	1.71536	25.92	23.32
10	0.01367	2.54138	0.01019	2.11332	25.46	16.84
TMD	-	-	0.02319	3.00268	-	-

Table 3: Displacements and acceleration per floor

As presented in Table 3, displacement and acceleration amplitudes reduced on all floors of the building. Significant reduction in displacement was observed, ranging from 23% to 26%. Reductions were even higher for acceleration, ranging from 9% to 39%.

Figure 5 shows structural responses on top floor, with and without the TMD, in terms of displacement and acceleration versus time.

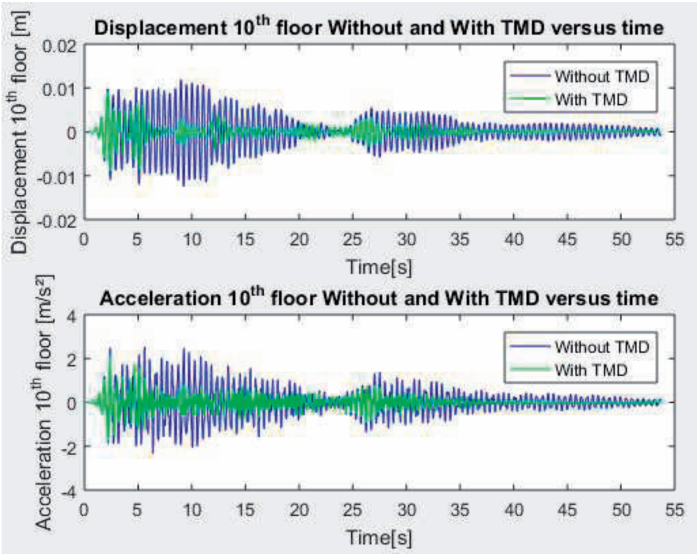

Figure 5: Response of the structure, with and without TMD, in terms of displacement and acceleration as a function of time

4 CONCLUSIONS

This paper presented a method to control vibration using TMD. A dynamic analysis of a 10-storey building subjected to the El Centro earthquake was performed, before and after the installation of TMD, showing a significant reduction in maximum displacement and acceleration after implementing TMD.

TMD's frequency was tuned to the natural frequency of the first mode, which is usually the most relevant in this case.

If the earthquake has another predominant frequency matching the natural frequency of the structure, it can lead to its resonance and collapse. However, each excitation frequency of the earthquake will result in a new tuning of TMD to the structure. Each tuning frequency will have a corresponding mass ratio to the reduction of amplitudes of structural response; in other words, for each frequency, a TMD with optimal parameters will be set for obtaining a good performance in reducing amplitudes caused by earthquakes.

The greater the total number of function evaluations on the algorithm adopted, the more refined the objective function resulted will be; but consequently, computational time required to perform optimization will also be greater. The objective function was set in 500 evaluations, in order to ensure a good resulting ratio mass and cost-benefit, since reductions in displacement and acceleration were not as significant from a certain mass increment and total number of evaluations.

Optimal parameters of the TMD were achieved when modal mass ratio (μ) corresponded to 0.02. Maximum displacements were reduced considerably.

In order to avoid costs and difficulty during implementation due to structural overload, mass ratio (μ) must not be too high.

After implementing the TMD, reductions ranged from 23% to 26% for the displacements, and from 9% to 39% for the maximum accelerations were reached.

5 ACKNOWLEDGMENT

The authors acknowledge the financial support provided by CNPq and CAPES through scholarships.

REFERENCES

Chaves, S. N. P. H., "Atenuação da resposta sísmica de estruturas de edifícios utilizando amortecedores viscosos", Dissertação de Mestrado, Universidade do Porto,2010.

Fadel Miguel, L. F.; Miguel, L. F. F.; Lopez, R. H., "Discussion of paper: Estimating optimum parameters of tuned mass dampers using harmony search". J. Eng. Struct, v.54, p.262-264, 2013.

Groehs, A. G. "Mecânica Vibratória", Editora Unisinos, 2001.

Hough, SE , "Encontrando falhas na Califórnia: um guia do turista terremoto". Mountain Press Publishing. p. 185. ISBN 978-0-87842-495-5, 2004.

Mohebbi, M. et al, "Designing optimal multiple tuned mass dampers using genetic algorithms (GAs) for mitigating the seismic response of structures." Journal of Vibration and Control; DOI: 10.1177/1077546311434520, 2012.

Ospina, C. S., "Otimização de amortecedores de massa sintonizados, múltiplos e interligados, para o controle de vibrações em edifícios altos", Dissertação de mestrado, Universidade de Brasília, 2008.

Paredes, M.M, "Utilização de amortecedores de massas sintonizadas no controlo de vibrações em estruturas". Dissertação de Mestrado, Universidade do Porto, Portugal, 2008.

Pressler, Margaret Webb, "More earthquakes than usual? Not really". KidsPost (Washington Post: Washington Post). pp. C10, 2010.

Villaverde, R., Newmark, N. M., "Seismic Response of Light Attachments to Buildings". University of Illinois, Urbana, 1980.

Villaverde, R., "Reduction in seismic response with heavily-damped vibration absorbers". Earthquake Engineering and Structural Dynamics Vol. 13, pp. 33-42, 1985.

Villaverde, R., "Earthquake response of systems with nonproportional damping by the conventional response spectrum method", Proc. 7th world conference on earthquake engineering. Istanbul, 5, pp. 467-474, 1980.

Yang, X.S, "Firefly Algorithms for Multimodal Optimization". Stochastic Algorithms: Foundations and Applications, pp 169-178, 2009.