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WAVES IN A POROELASTIC SOLID SATURATED BY A THREE-PHASE FLUID

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Abstract. This work presents an analysis of the behavior of body waves in a multiphase system consisting of a poroelastic solid saturated by a three-phase fluid, taken to be oil, water and gas, with water being the wetting phase. The constitutive relations and the equations of motion include the effect of two capillary relations between the water and oil phases and the oil and gas phases, and three relative permeability functions. A plane wave analysis shows that four compressional waves and one shear wave can propagate in this multiphase system, all suffering dispersion and attenuation effects. The behavior of all waves as function of confining pressure, saturation of the fluid phases and frequency is analyzed in the numerical examples.

1 INTRODUCTION

The theory of propagation of waves in a poroelastic solid saturated by a single-phase fluid was presented by M. Biot in Biot (1956a,b, 1962), where he showed the existence of two compressional waves (Type I or fast wave and Type II or slow wave) and one shear wave. When the pore volume is occupied by more than one fluid phase a different treatment is required, depending on the behavior of the fluids and their distribution within the pore space. Among authors employing Biot's theory to treat cases of miscible, immiscible or segregated fluids, we mention White et al. (1975) and Dutta and Odé (1979), that analyzed attenuation and dispersion of waves in rocks saturated by brine and gas, and Berryman et al. (1982), that studied cases of segregated or mixed liquids and gas. None of these authors take into account capillary forces or relative permeability functions in their models. This work presents a model that describes the propagation of waves in a poroelastic solid saturated by three immiscible, compressible, viscous fluids, namely water, oil and gas. Capillary pressure effects, due to pressure differences between the oil and water and the oil and gas phases, are included in the model as restrictions by introducing Lagrange multipliers in the principle of virtual complementary work (Fung, 1965). Capillary pressures are assumed to be functions of the saturation of the non-wetting phases. The relative permeability of each phase is a function of its own saturation. The elastic constants in the constitutive relations and the mass and viscous coupling coefficients are determined in terms of the properties of the solid and fluid phases, the two capillary pressure functions and the three-phase relative permeability functions. A plane wave analysis shows the existence of four compressional waves, denoted as P1, P2, P3 and P4, and one shear wave. The model is applied to compute the phase velocities and attenuation coefficients for a sample of Nivelsteiner sandstone saturated by water, oil and gas, with water assumed to be the wetting phase.

2 CONSTITUTIVE RELATIONS AND EQUATIONS OF MOTION

Let us consider a poroelastic isotropic homogeneous medium Ω saturated by a three-phase fluid, taken here to be oil, water and gas. Let $S_o = S_o(x), S_w = S_w(x)$ and $S_g = S_g(x)$ denote the oil, water and gas saturations, respectively, and assume the fluids fully saturate the pore space, so that

$$S_o + S_w + S_g = 1.$$

Let $\phi = \phi(x)$ be the effective porosity in Ω and let $u^s = u^s(x)$, $\widetilde{u}^o = \widetilde{u}^o(x)$, $\widetilde{u}^w = \widetilde{u}^w(x)$ and $\widetilde{u}^g = \widetilde{u}^g(x)$ denote the locally averaged solid, oil, water and gas displacements, respectively. Also, let $\epsilon_{ij}(u^s)$ be the strain tensor and set

$$u_i^\theta = \phi(\widetilde{u}_i^\theta - u_i^s), \quad \xi^\theta = -\nabla \cdot u^\theta, \quad \theta = o, w, g. \tag{1}$$

Let $\tau_{ij} = \overline{\tau}_{ij} + \Delta\,\tau_{ij}$ and $\sigma_{ij} = \overline{\sigma}_{ij} + \Delta\,\sigma_{ij}$ be the total stress tensor in the bulk material and the stress tensor in the solid part of Ω, respectively, where $\Delta\,\tau_{ij}$ and $\Delta\,\sigma_{ij}$ denote changes with respect to reference stresses $\overline{\tau}_{ij}$ and $\overline{\sigma}_{ij}$ associated with the initial equilibrium state.

Let $p_\theta = \overline{p}_\theta + \Delta p_\theta,\ \theta = o, w, g$, be the θ-fluid pressure, with Δp_θ being increment in the θ-fluid pressure with respect to given reference pressures $\overline{p}_\theta$ in the initial equilibrium state. The two capillary pressure functions are chosen to be strictly increasing functions of a single saturation as follows:

$$Pc_{ow} = Pc_{ow}(S_o) = p_o - p_w, \quad Pc_{go} = Pc_{go}(S_g) = p_g - p_o.$$

Next set

$$
\begin{aligned}
&\beta_{ow} = Pc_{ow}(S_o)/Pc'_{ow}(S_o),\ \beta^w_{ow} = \overline{p}_w/Pc'_{ow}(S_o),\\
&\beta^w_{go} = \overline{p}_w/Pc'_{go}(S_o),\ \beta_{go} = Pc_{go}(S_g)/Pc'_{go}(S_g),\\
&\beta^{ow}_{go} = Pc_{ow}(S_o)/Pc'_{go}(S_g),
\end{aligned}
$$

and define the generalized forces

$$
\begin{aligned}
\Delta\mathcal{F}^o &\equiv (S_o + \beta_{ow} + \beta^w_{ow})\Delta p_o - (\beta_{ow} + \beta^w_{ow})\Delta p_w,\\
\Delta\mathcal{F}^w &\equiv (S_w + \beta^w_{ow})\Delta p_w + (\beta^w_{go} - \beta^w_{ow})\Delta p_o - \beta^w_{go}\Delta p_g,\\
\Delta\mathcal{F}^g &\equiv (S_g + \beta_{go} + \beta^{ow}_{go} + \beta^w_{go})\Delta p_g\\
&\qquad -(\beta_{go} + \beta^{ow}_{go} + \beta^w_{go})\Delta p_o.
\end{aligned}
$$

Then, if $e^s = e_{ii}$, the stress-strain relations can be stated as

$$\Delta\tau_{ij} = 2N\,\epsilon_{ij} + \delta_{ij}(\lambda_c e^s - B_1\,\xi^o - B_2\,\xi^w - B_3\xi^g), \tag{2}$$
$$\Delta\mathcal{F}^o = -B_1\,e^s + M_1\,\xi^o + M_4\,\xi^w + M_5\,\xi^g, \tag{3}$$
$$\Delta\mathcal{F}^w = -B_2\,e^s + M_4\,\xi^o + M_2\,\xi^w + M_6\,\xi^g, \tag{4}$$
$$\Delta\mathcal{F}^g = -B_3\,e^s + M_5\,\xi^o + M_6\,\xi^w + M_3\,\xi^g. \tag{5}$$

The elastic constants in (2)-(5) can be determined employing a set of *gedanken* experiments (Santos and Savioli, 2016)

Let ρ_θ, $\theta = s, o, w, g$, be the mass densities of the θ-phase and let $\rho = (1-\phi)\rho_s + \phi(\sum_{\theta=o,w,g}\rho_\theta S_\theta)$. Let g_θ, a_θ, $\theta = o, w, g$, and g_{lt}, a_{lt}, $lt = ow, og, wg$, be the mass and viscous coupling coefficients. Also, let $k_{r\theta}(S_\theta), \theta = o, w, g$ be the three-phase relative permeability functions and k the absolute permeability (Peaceman, 1977). Then

$$g_\theta = S_\theta\rho_\theta T/\phi,\ \ a_\theta = \frac{S^2_\theta}{k_{r\theta}},\ \ b_\theta = \frac{a_\theta\eta_\theta}{k},\ \theta = o, w, g, \tag{6}$$
$$g_{st} = \epsilon(g_o g_w g_g)^{1/3},\ \ a_{st} = \epsilon(a_o a_w a_g)^{1/3},\ \ b_{st} = \frac{a_{st}(\eta_s\eta_t)^{1/2}}{k},\ s \neq t,$$

with T being the tortuosity factor, η_θ the viscosity of the θ-phase and ϵ a small number.

In the isotropic case the equations of motion are given by (Santos and Savioli, 2015, 2016)

$$\rho\ddot{\mathbf{u}}^s + \rho_o S_o\ddot{\mathbf{u}}^o + \rho_w S_w\ddot{\mathbf{u}}^w + \rho_g S_g\ddot{\mathbf{u}}^g - \nabla\cdot\Delta\tau(\vec{u}) = \mathbf{f}^{\,s}, \tag{7}$$

$$\begin{aligned}\rho_o S_o\ddot{\mathbf{u}}^s + g_o\ddot{\mathbf{u}}^o + g_{ow}\ddot{\mathbf{u}}^w + g_{og}\ddot{\mathbf{u}}^g + b_o\dot{\mathbf{u}}^o + b_{ow}\dot{\mathbf{u}}^w&\\ + b_{og}\dot{\mathbf{u}}^g + \nabla\Delta\mathcal{F}_o(\vec{u}) = \mathbf{f}^{\,o},&\end{aligned} \tag{8}$$

$$\begin{aligned}\rho_w S_w\ddot{\mathbf{u}}^s + g_{ow}\ddot{\mathbf{u}}^o + g_w\ddot{\mathbf{u}}^w + g_{og}\ddot{\mathbf{u}}^g + b_{ow}\dot{\mathbf{u}}^o + b_w\dot{\mathbf{u}}^w&\\ + b_{wg}\dot{\mathbf{u}}^g + \nabla\Delta\mathcal{F}_w(\vec{u}) = \mathbf{f}^{\,w},&\end{aligned} \tag{9}$$

$$\begin{aligned}\rho_g S_g\ddot{\mathbf{u}}^s + g_{og}\ddot{\mathbf{u}}^o + g_{wg}\ddot{\mathbf{u}}^w + g_g\ddot{\mathbf{u}}^g + b_{og}\dot{\mathbf{u}}^o + b_{wg}\dot{\mathbf{u}}^w&\\ + b_g\dot{\mathbf{u}}^g + \nabla\Delta\mathcal{F}_g(\vec{u}) = \mathbf{f}^{\,g},&\end{aligned} \tag{10}$$

where $\mathbf{f}^{\,s}, \mathbf{f}^{\,o}, \mathbf{f}^{\,w}$ and $\mathbf{f}^{\,g}$ indicate external forces in the solid, oil, water and gas phases, respectively.

3 PLANE WAVE ANALYSIS

Assuming constant coefficients and absence of external sources, (7)-(10) become

$$\rho \ddot{\mathbf{u}}^s + \rho_o S_o \ddot{\mathbf{u}}^o + \rho_w S_w \ddot{\mathbf{u}}^w + \rho_g S_g \ddot{\mathbf{u}}^g \tag{11}$$
$$= (E_u \nabla e^s - \mu \nabla \times (\nabla \times \vec{u}^s) + B_1 \nabla e^o + B_2 \nabla e^w + B_3 \nabla e^g,$$
$$\rho_o S_o \ddot{\mathbf{u}}^s + g_o \ddot{\mathbf{u}}^o + g_{ow} \ddot{\mathbf{u}}^w + g_{og} \ddot{\mathbf{u}}^g + b_o \dot{\mathbf{u}}^o + b_{ow} \dot{\mathbf{u}}^w + b_{og} \dot{\mathbf{u}}^g \tag{12}$$
$$= B_1 \nabla e^s + M_1 \nabla e^o + M_4 \nabla e^w + M_5 \nabla e^g,$$
$$\rho_w S_w \ddot{\mathbf{u}}^s + g_{ow} \ddot{\mathbf{u}}^o + g_w \ddot{\mathbf{u}}^w + g_{og} \ddot{\mathbf{u}}^g + b_{ow} \dot{\mathbf{u}}^o + b_w \dot{\mathbf{u}}^w + b_{wg} \dot{\mathbf{u}}^g \tag{13}$$
$$= B_2 \nabla e^s + M_4 \nabla e^o + M_2 \nabla e^w + M_6 \nabla e^g,$$
$$\rho_g S_g \ddot{\mathbf{u}}^s + g_{og} \ddot{\mathbf{u}}^o + g_{wg} \ddot{\mathbf{u}}^w + g_g \ddot{\mathbf{u}}^g + b_{og} \dot{\mathbf{u}}^o + b_{wg} \dot{\mathbf{u}}^w + b_g \dot{\mathbf{u}}^g \tag{14}$$
$$= B_3 \nabla e^s + M_5 \nabla e^o + M_6 \nabla e^w + M_3 \nabla e^g,$$

where

$$e^\theta = \nabla \cdot u^\theta,\ \theta = s, o, w, g,$$

and

$$E_u = \lambda_u + 2\,\mu. \tag{15}$$

To obtain the equations determining the propagation of compressional waves, we apply the divergence operator in (11)-(14) and replace in the resulting equations a plane compressional wave of angular frequency ω and wave number $\ell = \ell_r + i\,\ell_i$ travelling in the x_1-direction in the form

$$\begin{aligned} e^s &= C_s^{(\ell)} e^{i(\ell x_1 - \omega t)} = C_s^{(\ell)} e^{-\ell_i x_1} e^{i\ell_r (x_1 - \frac{\omega}{\ell_r} t)}, \\ e^o &= C_o^{(\ell)} e^{i(\ell x_1 - \omega t)} = C_o^{(\ell)} e^{-\ell_i x_1} e^{i\ell_r (x_1 - \frac{\omega}{\ell_r} t)}, \\ e^w &= C_w^{(\ell)} e^{i(\ell x_1 - \omega t)} = C_o^{(\ell)} e^{-\ell_i x_1} e^{i\ell_r (x_1 - \frac{\omega}{\ell_r} t)}, \\ e^g &= C_g^{(\ell)} e^{i(\ell x_1 - \omega t)} = C_g^{(\ell)} e^{-\ell_i x_1} e^{i\ell_r (x_1 - \frac{\omega}{\ell_r} t)}. \end{aligned} \tag{16}$$

The equations are,

$$-\omega^2 \rho e^s - \omega^2 \rho_o S_o e^o - \omega^2 \rho_w S_w e^w - \omega^2 \rho_g S_g e^g \tag{17}$$
$$= (E_u \nabla^2 e^s + B_1 \nabla^2 e^o + B_2 \nabla^2 e^w + B_3 \nabla^2 e^g,$$
$$-\omega^2 \rho_o S_o e^s - \omega^2 g_o\, e^o - \omega^2 g_{ow}\, e^w - \omega^2 g_{og}\, e^g + \mathrm{i}\omega b_o e^o \tag{18}$$
$$+ \mathrm{i}\omega b_{ow} e^w + \mathrm{i}\omega b_{og} e^g$$
$$= B_1 \Delta\, e^s + M_1 \nabla^2 e^o + M_4 \nabla^2 e^w + M_5 \nabla^2 e^g,$$
$$-\omega^2 \rho_w S_w e^s - \omega^2 g_{ow}\, e^o - \omega^2 g_w\, e^w - \omega^2 g_{wg}\, e^g + \mathrm{i}\omega b_{ow} e^o \tag{19}$$
$$+ \mathrm{i}\omega b_w e^w + \mathrm{i}\omega b_{wg} e^g$$
$$= B_2 \nabla^2\, e^s + M_4 \nabla^2 e^o + M_2 \nabla^2 e^w + M_6 \nabla^2 e^g,$$
$$-\omega^2 \rho_g S_g e^s - \omega^2 g_{og}\, e^o - \omega^2 g_{wg}\, e^w - \omega^2 g_g\, e^g + \mathrm{i}\omega b_{og} e^o \tag{20}$$
$$+ \mathrm{i}\omega b_{wg} e^w + \mathrm{i}\omega b_g e^g$$
$$= B_3 \nabla^2\, e^s + M_5 \nabla^2 e^o + M_6 \nabla^2 e^w + M_3 \nabla^2 e^g.$$

Setting

$$\gamma = \frac{\omega}{\ell} \tag{21}$$

(17)-(20) leads to the following eigenvalue problem

$$\gamma^2 \mathcal{A} C^{(\gamma)} = \mathcal{E} C^{(\gamma)}. \tag{22}$$

where

$$\mathcal{A} = \begin{pmatrix} \rho & \rho_o S_o & \rho_w S_w & \rho_g S_g \\ \rho_o S_o & \widetilde{g}_w & \widetilde{g}_{ow} & \widetilde{g}_{og} \\ \rho_w S_w & \widetilde{g}_{ow} & \widetilde{g}_w & \widetilde{g}_{wg} \\ \rho_g S_g & \widetilde{g}_{og} & \widetilde{g}_{wg} & \widetilde{g}_g \end{pmatrix}, \quad \mathcal{E} = \begin{pmatrix} E_u & B_1 & B_2 & B_3 \\ B_1 & M_1 & M_4 & M_5 \\ B_2 & M_4 & M_2 & M_6 \\ B_3 & M_5 & M_6 & M_3 \end{pmatrix}, \quad C^{\gamma} = \begin{pmatrix} C_s^{\gamma} \\ C_o^{\gamma} \\ C_w^{\gamma} \\ C_g^{\gamma} \end{pmatrix},$$

and

$$\widetilde{g}_o = g_o + \mathrm{i}\frac{b_o}{\omega}, \quad \widetilde{g}_w = g_w + \mathrm{i}\frac{b_w}{\omega}, \quad \widetilde{g}_g = g_g + \mathrm{i}\frac{b_g}{\omega},$$
$$\widetilde{g}_{ow} = g_{ow} + \mathrm{i}\frac{b_{ow}}{\omega} \quad \widetilde{g}_{og} = g_{og} + \mathrm{i}\frac{b_{og}}{\omega} \quad \widetilde{g}_{wg} = g_{wg} + \mathrm{i}\frac{b_{wg}}{\omega}.$$

Hence, to determine the complex wave-numbers $\ell = \ell_r + \mathrm{i}\ell_i$ it is sufficient to solve the eigenvalue problem

$$\det(\mathcal{S} - \gamma^2 I) = 0, \tag{23}$$

where

$$\mathcal{S} = \mathcal{A}^{-1}\mathcal{E}. \tag{24}$$

The four physical meaningful solutions (*i.e.* $\ell_i > 0$) $(\gamma^{(j)})^2, j = 1, 2, 3, 4$ of (23) determine four compressional phase velocities $v^{(j)}$ and attenuation coefficients $b_i^{(j)}$ of the P1, P2, P3 and P4 modes of propagation from the relations

$$v_{p_j} = \frac{\omega}{|\ell_{r_j}|} \qquad b_{p_j} = 2\pi .8.655588 \frac{|\ell_{i_j}|}{|\ell_{r_j}|}. \tag{25}$$

The P1 wave is the analogue of the classical P1 wave in Biot theory. The P2, P3 and P4 waves are slow waves associated with the motion out of phase of the four phases.

Applying the curl operator in (11)-(14) and replacing plane wave in the resulting equations we can determine the phase velocities v_s and attenuation coefficients b_s of the rotational waves:

$$v_s = \frac{\omega}{|\ell_r|} \qquad b_s = 2\pi .8.655588 \frac{|\ell_i|}{|\ell_r|}. \tag{26}$$

4 NUMERICAL EXAMPLES

In this section we compute phase velocities and attenuation coefficients for a sample of Teapot sandstone. Its material properties, taken from Rosenbaum (1974), and those of the saturant fluids, water, oil and gas, are given in Tables 1 and 2.

The gas properties correspond to a dry gas at reference pressures 5, 10 and 20 MPa using the correlations given in Standing (1977) and McCoy (1983).

Figures 1, 2 and 3 display phase velocities of all waves for the purely elastic case (*i.e.*, zero viscosities) as function of gas saturation at water saturation $S_w = 0.25$ and water reference pressures $\overline{p}_w = 5$, 10 and 20 MPa.

Table 1: Material properties of the Teapot sandstone

Solid grains	bulk modulus, K_s	37.9 GPa
	density, ρ_s	2650 kg/m^3
Dry matrix	bulk modulus, K_m	8.6676 GPa
	shear modulus, μ_m	6.4798 GPa
	porosity, ϕ	0.297
	permeability κ	1.9 10^{-12} m^2

Table 2: Material properties of the saturant fluids

Water	bulk modulus, K_w	2.25 GPa
	density, ρ_w	1000 kg/cm^3
	viscosity, η_w	0.001 Pa · s
Oil	bulk modulus, K_o	0.57 GPa
	density, ρ_o	700 kg/cm^3
	viscosity, η_o	0.01 Pa · s
Gas at pressure 5 MPa	bulk modulus, K_g	44515183.855 $\times 10^{-10}$ GPa
	density, ρ_g	42.3156366 kg/m^3
	viscosity, η_g	1.1186139 $\times 10^{-5}$ Pa · s
Gas at pressure 10 MPa	bulk modulus, K_g	89314762.7 $\times 10^{-10}$ GPa
	density, ρ_g	86.5156181 kg/m^3
	viscosity, η_g	1.17348206 $\times 10^{-5}$ Pa · s
Gas at pressure 20 MPa	bulk modulus, K_g	229138783.0 $\times 10^{-10}$ GPa
	density, ρ_g	151.545384 kg/m^3
	viscosity, η_g	1.28131716 $\times 10^{-5}$ Pa · s

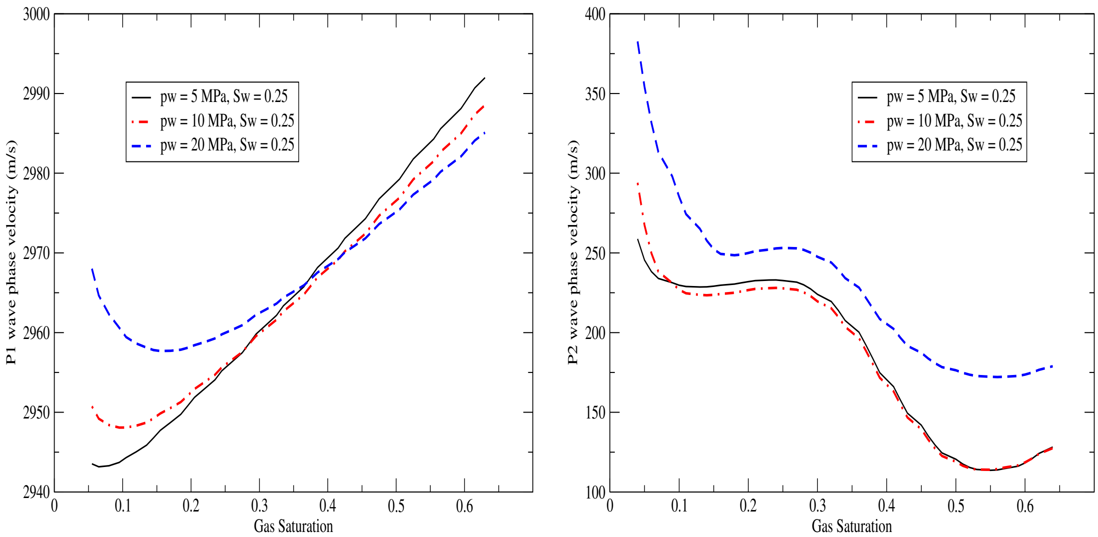

Figure 1: P1 (left) and P2 (right) wave phase velocity as function of gas saturation at S_w = 0.25 and $\overline{p}_w$ = 5, 10 and 20 MPa

In Figure 1 (left) it can be observed that P1 wave phase velocities decrease until a threshold gas saturation value is reached. For saturations greater than the threshold value, velocities exhibit a continuous increase. Besides, while at low gas saturations velocities increase with $\overline{p}_w$, the opposite behavior is observed at high gas saturations.

Figure 1 (right) and Figure 2 (left) show that P2 and P3 wave phase velocities have a general decreasing behavior as gas saturation increases.

On the other hand, P4 phase velocities in Figure 2 (right) are almost independent of gas saturation, and increase with increasing values of pressure $\overline{p}_w$. Besides phase velocities of P3 and P4 waves increase with increasing $\overline{p}_w$.

Shear waves in Figure 3 are increasing functions of gas saturation and decreasing functions of $\overline{p}_w$.

Figures 4, 5 and 6 show phase velocities of all waves as function of frequency at saturations $S_w = 0.25$ and $S_g = 0.1$ and reference water pressures equal to 5, 10 and 20 MPa.

Phase velocities for P1 and Shear waves in Figure 4 show very little dispersion over the whole range of frequencies. On the other hand, phase velocities of P2, P3 and P4 in Figures 5 and 6, are increasing functions of frequency. They vanish at low frequencies and stabilize at high frequencies. P2 waves are little sensitive to changes in reference water pressure while P3 and P4 waves increase as water pressure increases.

Figure 7 (left) show P1 and Shear wave attenuation at reference water pressure 10 MPa. It can be observed very similar (low) attenuation values for both waves with equal location of the attenuation peak.

Figure 7 (right) show P2, P3 and P4 wave attenuation at reference water pressure 10 MPa. It can be observed very similar attenuation values for the three waves. The attenuation is very high at low frequencies, showing their diffusive type behavior. At high frequencies, attenuation decays and these waves become truly propagating waves.

The corresponding attenuation Figures at $\overline{p}_w$ = 5 and 20 MPa are almost identical and are omitted.

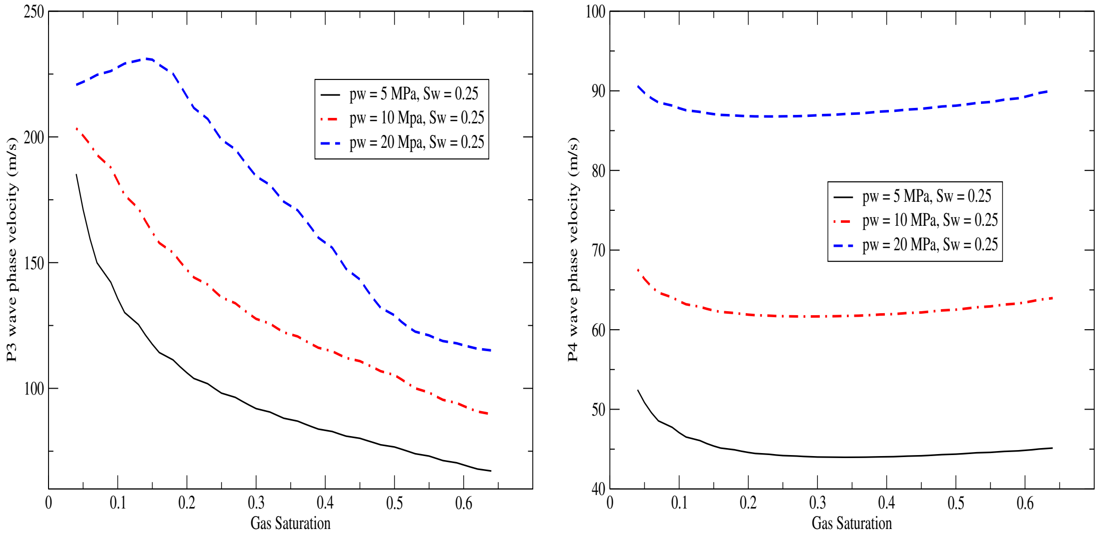

Figure 2: P3 (left) and P4 (right) wave phase velocity as function of gas saturation at $S_w = 0.25$ and $\overline{p}_w = 5$, 10 and 20 MPa

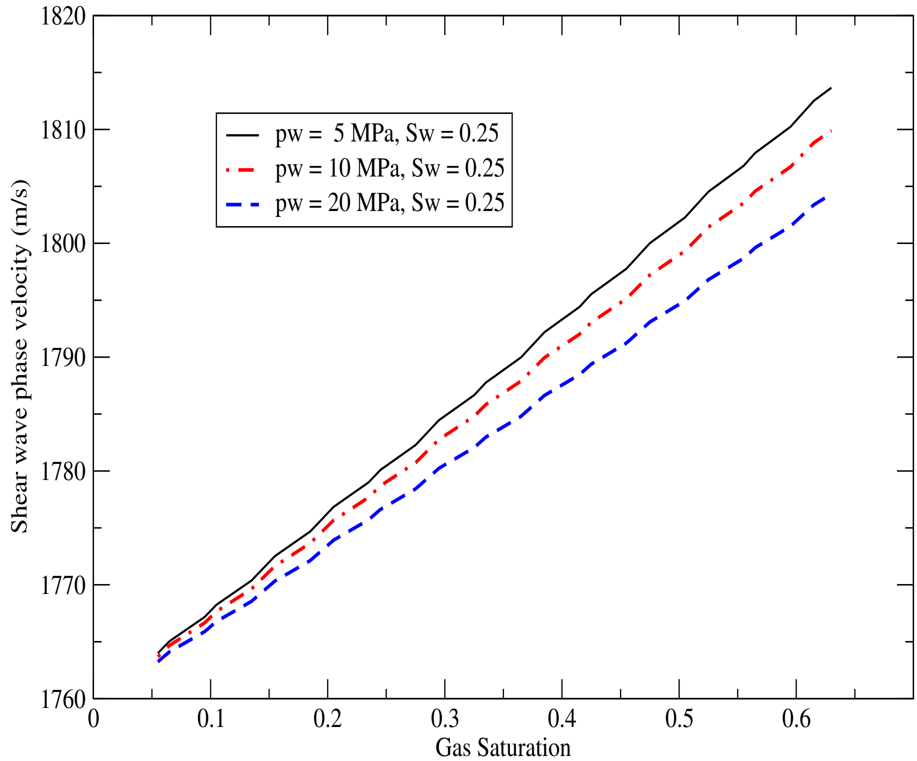

Figure 3: Shear wave phase velocity as function of gas saturation at $S_w = 0.25$ and $\overline{p}_w = 5$, 10 and 20 MPa.

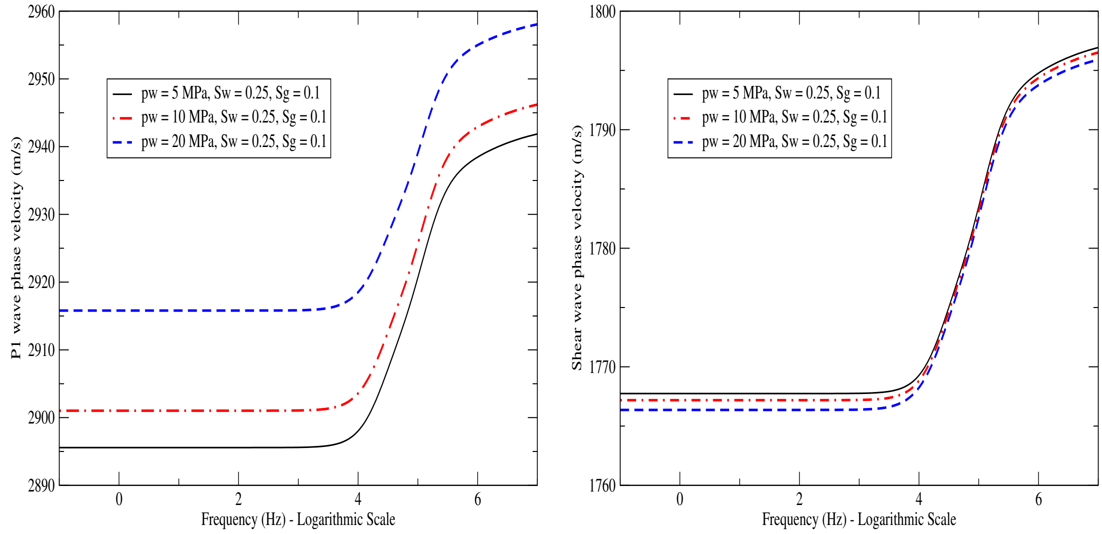

Figure 4: P1 (left) and Shear (right) wave phase velocity as function of frequency at $S_w = 0.25$, $S_g = 0.1$ and $\overline{p}_w = 5$, 10 and 20 MPa

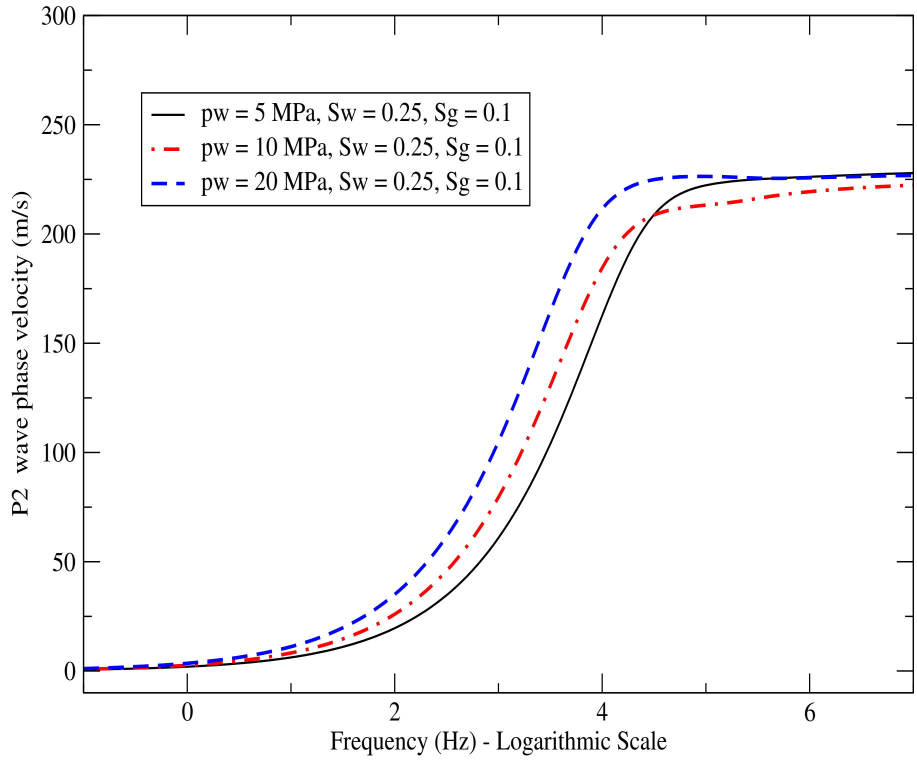

Figure 5: P2 wave phase velocity as function of frequency at $S_w = 0.25$, $S_g = 0.1$ and $\overline{p}_w = 5$, 10 and 20 MPa

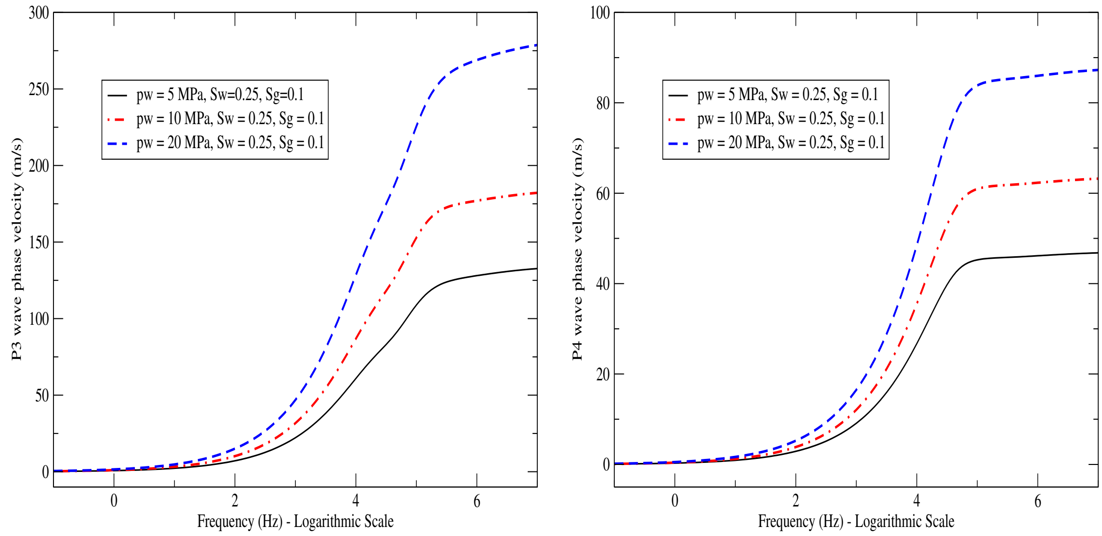

Figure 6: P3 (left) and P4 (right) wave phase velocity as function of frequency at $S_w = 0.25$, $S_g = 0.1$ and $\overline{p}_w = 5$, 10 and 20 MPa

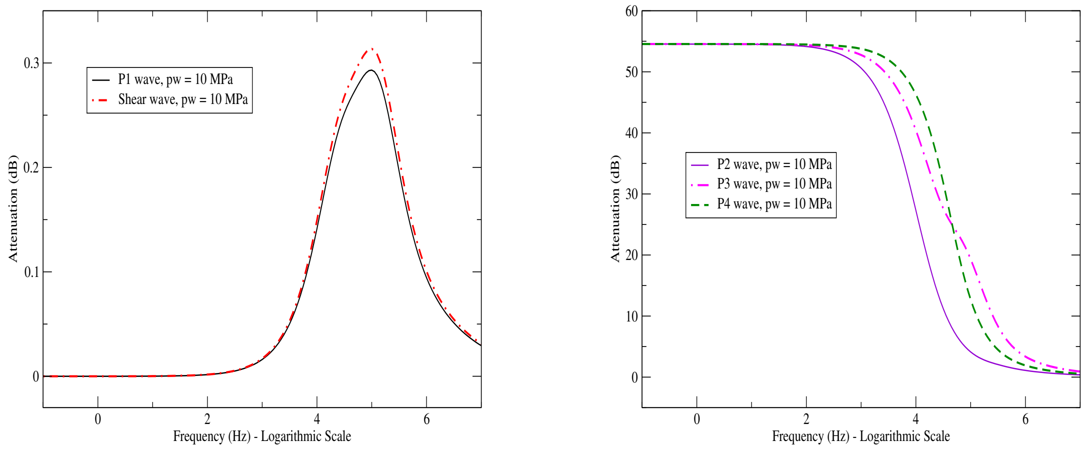

Figure 7: P1, Shear (left) and P2, P3,P4 (right) wave attenuation (dB) as function of frequency at $S_w = 0.25$, $S_g = 0.1$ and $\overline{p}_w = 10$ MPa

5 CONCLUSIONS

We presented a model to describe wave propagation in a poroelastic solid saturated by a three-phase fluid. The model predicts the existence of four compressional waves (P1, P2, P3, P4) and one shear (S) wave. In the elastic case, at low gas saturations, P1 velocities increase with water pressure and the opposite behavior is observed at high gas saturations while S-waves velocities decrease as water pressure increases. Besides, P2, P3 and P4 velocities have a general increasing behavior with water pressure.

In the general dissipative case, P1 and S waves show very little dispersion over the whole range of frequency and very similar attenuation with similar peaks in the sonic range. Concerning slow waves velocities vs. water pressure, P2 is little sensitive and P3, P4 are increasing functions. Furthermore, attenuation of slow waves is almost independent of water pressure.

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