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STRAIN LOCALIZATION ANALYSIS IN MATERIAL NONLINEAR MODELS

Lucas A. F. Fioresi, Roque L. S. Pitangueira, Samuel S. Penna and Humberto A. S. Monteiro

Department of Structural Engineering, Federal University of Minas Gerais (UFMG)
Avenida Antônio Carlos, 6627, Pampulha, CEP: 31270-901, Belo Horizonte, MG, Brasil
lucasaleksander@gmail.com, roque@dees.ufmg.br, spenna@dees.ufmg.br,
humbertomonteiro@gmail.com https://www.insane.dees.ufmg.br/en/home/

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Abstract. This paper presents a strain localization analysis in nonlinear models detached from constitutive model selection. The proposed implementation was performed on the INteractive Structural ANalysis Environment (INSANE) platform, an open source project developed by the Structural Engineering Department of the Federal University of Minas Gerais. Strain localization is associated with weak discontinuities and materials instabilities that occur during physically nonlinear structural analysis. Singularity of the acoustic tensor is considered the classical condition for strain localization and it can be calculated regardless of constitutive model selection. Localization analysis consists in searching for a unit vector which defines the direction at which the acoustic tensor becomes singular. This unit vector points towards the normal direction of the discontinuity surface in the body, which is created by the localization phenomenon. Localization analysis can be approached as a minimization problem of the acoustic tensor determinant. With parametrization techniques, it is possible to obtain the unit vector associated with the singularity. Strain localization analysis should be performed in each material or integration point, at each step of an incremental nonlinear analysis. At the end of localization analysis, one expects to detect points of material instabilities and use them as references for multiscale structural analysis.

1 INTRODUCTION

Strain localization is defined by de Borst et al. (1993) as the emergence of narrow regions in a structure where all further deformation tends to concentrate, despite a monotonic loading program from external actions.

From a kinematic sense, the region of localized deformation can be represented by strong discontinuities, which incorporate a jump across the displacement field and a singularity at the strain field, or by two weak discontinuities, which form a band of small but finite thickness that separate the failure zone from the remaining part of the body. In this case, there is a jump across the strain field, but the displacement fields remain continuous (Jirásek, 2001).

Modeling and interpreting localized failure is of utmost importance, since strain localization can preclude structural collapse at conditions not considered during structural design. The localization phenomenon is observed experimentally in a broad class of materials, including quasi-brittle media (see, e.g. Cedolin et al. (1987)), metals (see, e.g. Nadai (1931) and Duszek-Perzyna and Perzyna (1993)) and geomaterials (see, e.g. Alshibli and Sture (2000) and Desrues and Viggiani (2004)).

In numerical simulations, strain localization is associated with loss of ellipticity of the differential equations governing the boundary value problem. As a result, the boundary value problem may become ill-posed and allows an infinite number of solutions to occur, including those which involve discontinuities (de Borst, 2004). Thus, numerical solutions of an ill-posed boundary value problem are dependent of the adopted discretization.

Mesh refinement for Finite Element Analysis causes a more brittle structural behavior. Width of the localization region depends on the smallest material volume admissible, i.e. the smallest finite element of the mesh. Physically, the amount of energy dissipated during the fracture process tends to zero with mesh refinement (de Borst, 2004). Thus, these solutions are not compatible with Fracture Mechanics principles and do not have physical significance.

Constitutive models have been extensively developed in the last decades and many models describing concrete, metals, rocks and soils can be found in the literature. They usually consist of a set of analytical formulations, which are responsible for describing material behavior in a reliable manner.

In the context of physically nonlinear structural analysis, constitutive models can be formulated based on different fundamental concepts, such as Damage Mechanics, Fracture Mechanics or Theory of Plasticity. The use of distinct theoretical foundations can impact on the occurrence of strain localization in numerical analysis and on its detection.

Considering the scenario previously described, this paper presents a strain localization analysis in material nonlinear models detached from constitutive model selection.

2 STRAIN LOCALIZATION ANALYSIS

2.1 Material Stability

Schreyer and Neilsen (1996) defined two different types of instabilities. Structural stability is associated with the domain of the problem. It depends on factors such as geometry, boundary conditions and constitutive equations. On the other hand, material instability is only associated with constitutive equations.

Considering geometric linearity, Drucker (1959) proposed the following criteria for material stability:

$$\dot{\varepsilon}_{ij}\dot{\sigma}_{ij} > 0 \tag{1}$$

where,
$\dot{\varepsilon}_{ij}$ represents the strain rate tensor;
$\dot{\sigma}_{ij}$ represents the stress rate tensor.

For incrementally linear constitutive relations, one has the following relation between the stress and strain rate tensors:

$$\dot{\sigma}_{ij} = D_{ijkl}\dot{\varepsilon}_{kl} \tag{2}$$

where,
D_{ijkl} represents the fourth-order tangent stiffness tensor.

In this context, Drucker (1959) criteria can be expressed as:

$$\dot{\varepsilon}_{ij}D_{ijkl}\dot{\varepsilon}_{kl} > 0 \tag{3}$$

Since $\dot{\varepsilon}_{ij}$ can be interpreted as any virtual strain rate tensor in Equation 3, material stability depends only on the tangent stiffness tensor that is associated with material properties and strain history of the material/integration point considered (Chen and Baker, 2003).

For an uniaxial loading case, Drucker (1959) criteria for material stability is lost when the stress-strain relationship slope is negative. According to Schreyer and Neilsen (1996), specific boundary constraints, such as prescribed displacements, allow a body to be loaded well beyond the point at which all material points are stable. This additional loading may result in the appearance of softening and inhomogeneous deformations in the structure.

2.2 Localization Condition

The early work of Rice (1976) established the conditions at which localization of plastic deformation occurs. The author studied the development of a localized band of orientation **n** separated from the rest of the structure. Considering the same constitutive relation within and outside of the localization band, Rice (1976) obtained:

$$det(n_i D_{ijkl} n_l) = 0 \tag{4}$$

The second-order tensor $n_i D_{ijkl} n_l$ was named localization or acoustic tensor and denoted as Q_{jk}. Singularity of the acoustic tensor, as represented in Equation 4, is considered the classical condition for an incipient weak discontinuity and consequently strain localization.

From Equation 4, it is clear that localization analysis depends on the tangent stiffness tensor (D_{ijkl}) and the unit vector (**n**) normal to the discontinuity surface. In most cases, the tangent stiffness tensor is computed directly from constitutive models. Therefore, localization analysis consists in searching for a unit vector which defines the direction at which the acoustic tensor becomes singular (Jirásek, 2007).

2.3 Localization Analysis Techniques

Several authors have provided closed-form expressions for the localization problem (see, e.g. Ortiz et al. (1987) (for two-dimensional models), Ottosen and Runesson (1991), Runesson et al. (1991), Rizzi et al. (1995) and Oliver and Huespe (2004)).

Analytical solutions for localization analysis are robust and require little computational effort. However, they are formulated for specific loading conditions and constitutive models. To overcome this drawback, numerical procedures are used to provide a technique applicable to general loading conditions and any constitutive model (see, e.g. Ortiz et al. (1987) (for three-dimensional models), Mosler (2005) and Oliver et al. (2010)).

In numerical solutions, the localization analysis, also referred as discontinuous bifurcation, can be addressed as a constrained optimization problem of an objective function f, namely the acoustic tensor determinant.

The restrictions of the optimization problem is determined with the choice of a parametrization technique for the unit vector **n**. A two-step Newton's method is the most common method to numerically solve the optimization problem. The first step consists of a sweep on the parameterized space to find the best initial guess. Then, the iterative procedure is performed.

Of particular interest is the numerical technique proposed by Mota et al. (2016). The authors propose a Cartesian parametrization for the unit vector that defines the discontinuity surface. In contrast to traditional techniques (spherical, stereographic and tangent parametrizations), the Cartesian parametrization relaxes the restriction that the normal vector be of unit length. By doing so, the authors set a parameterized space defined by a cube centered at the origin and with side length of two, as represented in Figure 1.

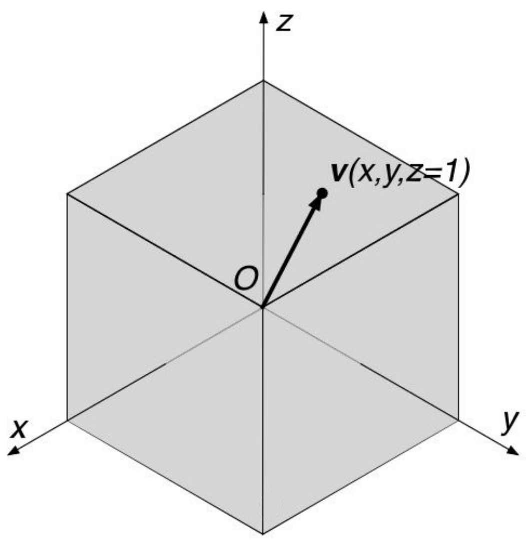

Figure 1: Cartesian parametrization (Mota et al., 2016).

Considering the symmetry of the bifurcation condition, only three faces of the cube have to be considered in the numerical procedure proposed by Mota et al. (2016). The vector **v** is, then, defined as:

$$\mathbf{v}(x,y,z) := \begin{cases} [x, y, 1]^T, \text{ if } x \in [-1,1] \text{ and } y \in [-1,1); \\ [1, y, z]^T, \text{ if } y \in [-1,1] \text{ and } z \in [-1,1); \\ [x, 1, z]^T, \text{ if } z \in [-1,1] \text{ and } x \in [-1,1); \\ [1,1,1]^T, \text{ otherwise.} \end{cases} \tag{5}$$

The unit vector **n** is obtained directly from **v**, as shown by Mota et al. (2016):

$$\mathbf{n} := \frac{\mathbf{v}}{||\mathbf{v}||} \tag{6}$$

where,
$||\mathbf{v}||$ denotes the norm of **v**.

When compared with traditional techniques, the Cartesian parametrization exhibited a simpler landscape for the objective function f, resulting in less dependence of the initial guess and more efficiency.

3 NUMERICAL IMPLEMENTATION

The numerical implementation of this work was performed on the **INSANE** (INteractive Structural ANalysis Environment) platform, an open source project developed by the Structural Engineering Department of the Federal University of Minas Gerais.

A crucial aspect of the localization analysis is the effective implementation of constitutive models. **INSANE** has a computational framework for constitutive modeling that is highly modular, easy to expand, independent on the adopted numerical method and on the peculiar analysis model as described by Gori et al. (2017). This framework is capable of providing the tangent stiffness tensor for a broad class of materials, including elasto-plastic and damage models.

The localization analysis implemented considered the classic criterion for strain localization (Equation 4). It also used the Cartesian parametrization proposed by Mota et al. (2016). Newton's method was utilized to solve the nonlinear constrained optimization problem arising from the localization analysis. Localization analysis was performed in each material/integration point, at each step of an incremental nonlinear analysis.

4 NUMERICAL SIMULATIONS

4.1 Uniaxial Tensile Loading

The example of a uniaxial tensile loading is presented to validate the implemented localization analysis technique. A coarse mesh with only three 4-node quadrilateral element is used, as shown in Figure 2.

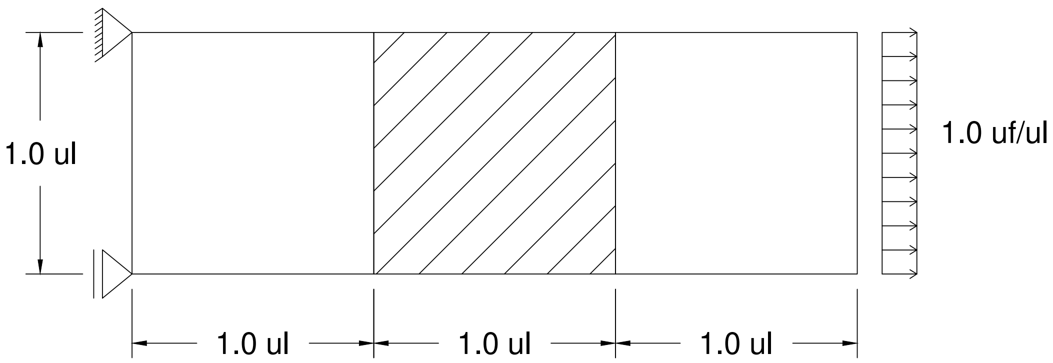

Figure 2: Uniaxial tensile test - Structural scheme.

Three different constitutive models were considered in this example, namely Simo and Ju (1987), Mazars and Lemaitre (1984) and de Vree et al. (1995) damage models. The hatched element in Figure 2 is weakened in order to provoke localization in this element. Localization analysis in this case should provide the unit vector $\mathbf{n} = (1, 0, 0)$ as the normal direction to the discontinuity surface. It also should detect the localization phenomenon in all four material/integration points of the weakened element at the same time.

Figure 3 shows the static equilibrium path for all constitutive models considered. Figure 4 shows the localization analysis outcome in a step during softening regime. The red line and arrow in Figure 4 are the geometrical representation of the numerically obtained vector **n** at each integration point. The green line represents the crack direction in a qualitative manner.

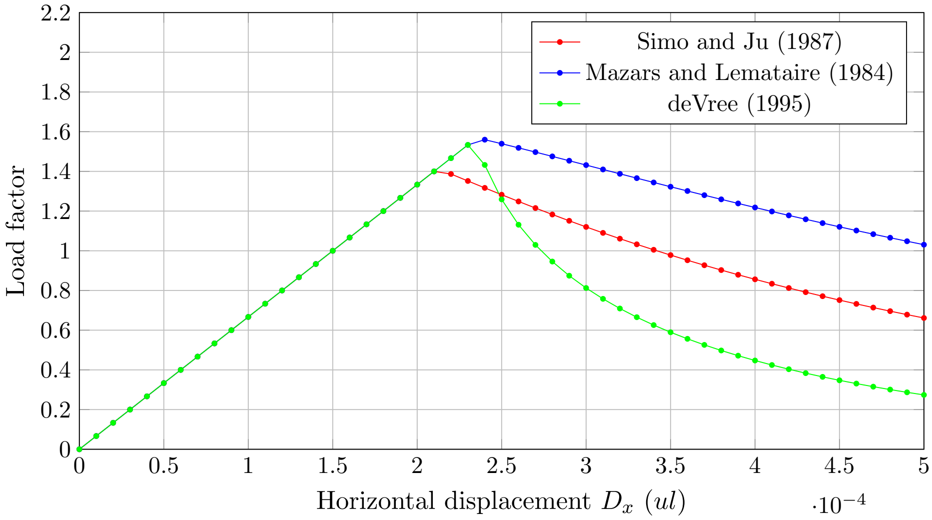

Figure 3: Uniaxial tensile test - Static equilibrium path.

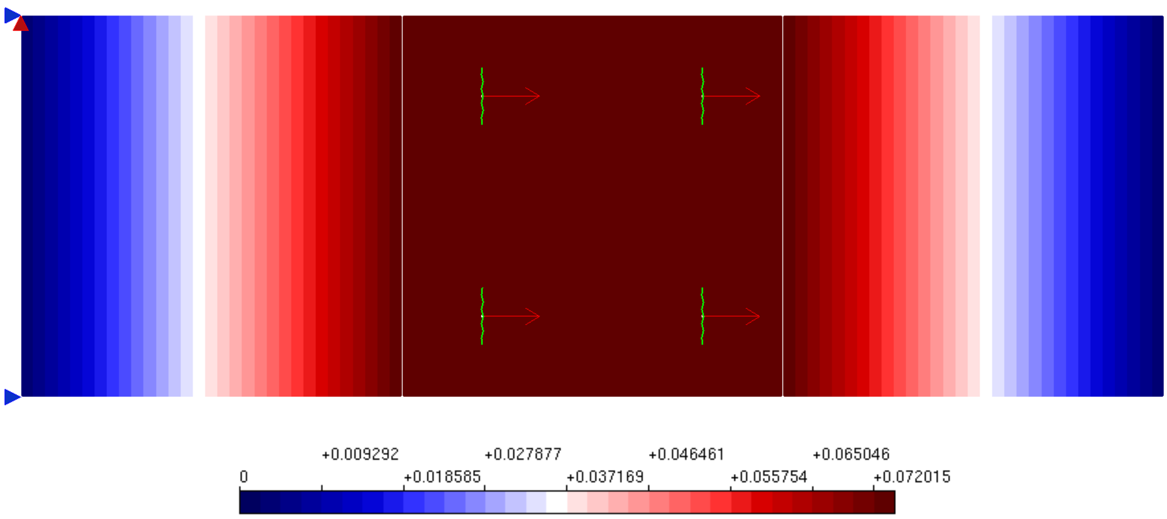

Figure 4: Uniaxial tensile test - Localization analysis.

4.2 L-Shaped Panel

A L-Shaped panel, extracted from Winkler et al. (2004), is now considered. This example provides a more generic set of geometry, mesh discretization and boundary conditions. A volumetric damage constitutive model proposed by Penna (2011) was considered. Figure 5 illustrates the structural scheme of this model. The mesh for Finite Element Analysis was defined using 3-node triangular elements.

Figures 6a) and 6b) represent the localization analysis results at two steps. As a result of

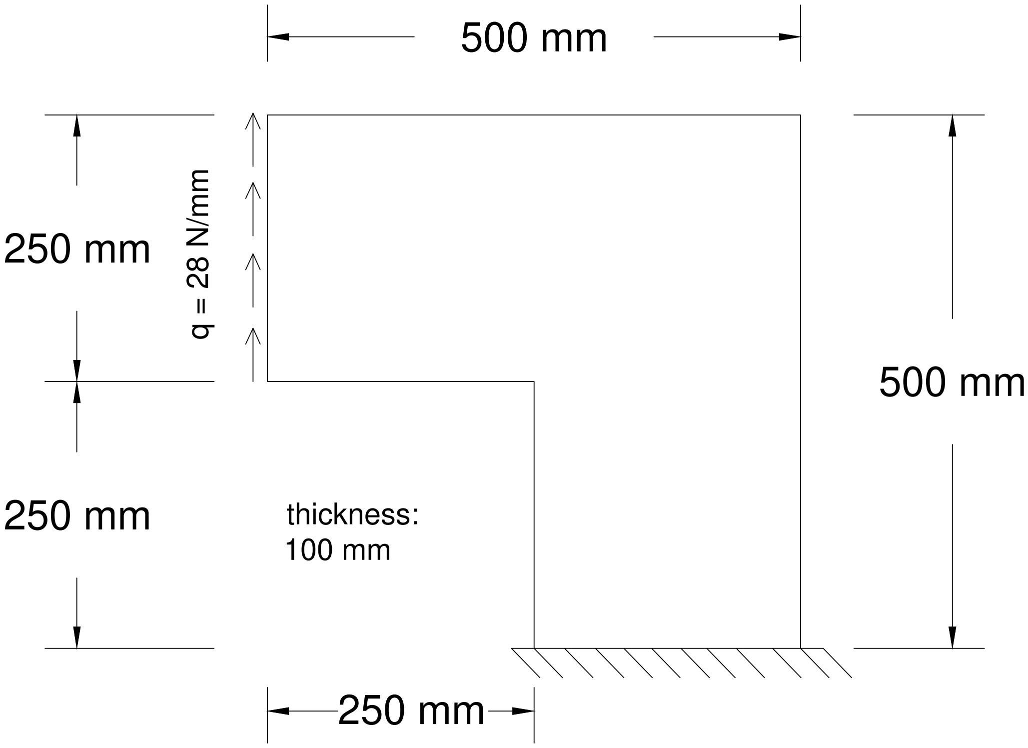

Figure 5: L-Shaped panel - Structural scheme.

material degradation, strain localization is detected in more material/integration points at a late step of a physically nonlinear analysis.

5 CONCLUSIONS

This paper presented a strain localization analysis that is dissociated from constitutive model selection. This independence was achieved considering the singularity of the acoustic tensor as a condition for strain localization and using a numerical procedure to solve the optimization problem. The vector defining the discontinuity surface was parameterized using the Cartesian parametrization technique proposed by Mota et al. (2016).

Strain localization analysis provide valuable input to regularization methods, such as enhanced continuum descriptions, cohesive-zone models and discrete failure methods. The unit vector defining the discontinuity surface **n** can be used to simulate crack propagation in quasi-brittle media. In a multiscale structural analysis, points of material instabilities could be employed to define local domains.

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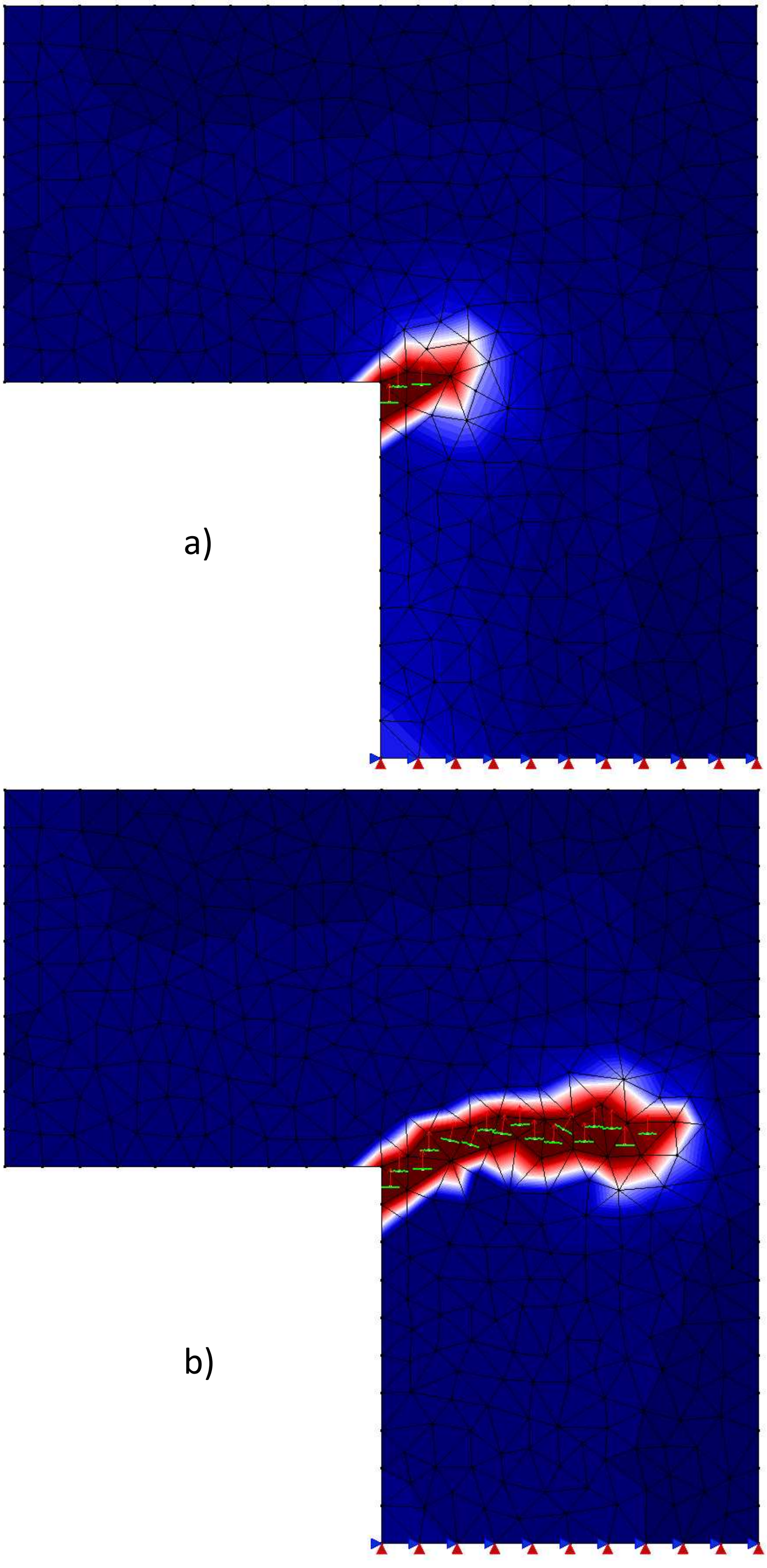

Figure 6: L-Shaped panel - a) Localization analysis at early step, b) Localization analysis at late step

REFERENCES

Alshibli K.A. and Sture S. Shear band formation in plane strain experiments of sand. *Journal of Geotechnical and Geoenvironmental Engineering*, 126(6):495–503, 2000.

Cedolin L., Poli S.D., and Iori I. Tensile behavior of concrete. *Journal of Engineering Mechanics*, 113(3):431–449, 1987.

Chen G. and Baker G. Material softening and structural instability. *Advances in Structural Engineering*, 6(4):353–357, 2003.

de Borst R. Damage, material instabilities, and failure. *Encyclopedia of Computational Mechanics*, 2:335–373, 2004.

de Borst R., Sluys L., Muhlhaus H., and Pamin J. Fundamental issues in finite element analyses of localization of deformation. *Engineering Computations*, 10:99–121, 1993.

de Vree J.H., Brekelmans W.A.M., and van Gils M.A.J. Comparison of nonlocal approaches in continuum damage mechanics. *Computers e Structures 55 (4)*, pages 581–588, 1995.

Desrues J. and Viggiani G. Strain localization in sand: An overview of the experimental results obtained in grenoble using stereophotogrammetry. *International Journal for Numerical and Analytical Methods in Geomechanics*, 28(4):279–321, 2004.

Drucker D.C. A definition of stable inelastic material. *Journal of Applied Mechanics*, 26:101–106, 1959.

Duszek-Perzyna M.K. and Perzyna P. Adiabatic shear band localization in elastic-plastic single crystals. *International Journal of Solids and Structures*, 30(1):61–89, 1993.

Gori L., Penna S.S., and Pitangueira R.L.S. A computational framework for constitutive modelling. *Computers & Structures*, 187:1–23, 2017.

Jirásek M. Modeling of localized damage and fracture in quasibrittle materials. *Lecture Notes in Physics*, 568:17–29, 2001.

Jirásek M. Mathematical analysis of strain localization. *Revue Européenne de Génie Civil*, 11(7-8):977–991, 2007.

Mazars J. and Lemaitre J. Application of continuous damage mechanics to strain and fracture behavior of concrete. *Shah, S. P. (Ed.), Application Of Fracture Mechanics to Cementitious Composites. NATO Advanced Research Workshop, 4-7 Setembro. Northwestern University*, pages 375–378, 1984.

Mosler J. Numerical analyses of discontinuous material bifurcation: Strong and weak discontinuities. *Computer Methods in Applied Mechanics and Engineering*, 194(9-11):979–1000, 2005.

Mota A., Chen Q., Foulk III J.W., Ostien J.T., and Lai Z. A cartesian parametrization for the numerical analysis of material instability. *International Journal for Numerical Methods in Engineering*, 108:156–180, 2016.

Nadai A. *Plasticity: A mechanics of the plastic state of matter*. McGraw-Hill, 1931.

Oliver J. and Huespe A.E. Theoretical and computational issues in modelling material failure in strong discontinuity scenarios. *Computer Methods in Applied Mechanics and Engineering*, 193(27):2987–3014, 2004.

Oliver J., Huespe A.E., Cante J.C., and Diaz G. On the numerical resolution of the discontinuous material bifurcation problem. *International Journal for Numerical Methods in Engineering*, 83:786–804, 2010.

Ortiz M., Leroy Y., and Needleman A. A finite element method for localized failure analysis. *Computer Methods in Applied Mechanics and Engineering*, 61(2):189–214, 1987.

Ottosen N.S. and Runesson K. Properties of discontinuous bifurcation solutions in elastoplasticity. *International Journal of Solids and Structures*, 27(4):401–421, 1991.

Penna S.S. *Formulação Multipotencial para Modelos de Degradação Elástica - Unificação Teórica, Proposta de Novo Modelo, Implementação Computacional e Modelagem de Estruturas de Concreto*. Ph.D. thesis, Universidade Federal de Minas Gerais, Belo Horizonte, MG, Brasil, 2011.

Rice J.R. The localization of plastic deformation. In *14th International Congress on Theoratical and Applied Mechanics - Delft, Netherlands*, pages 207–220. 1976.

Rizzi E., Carol I., and Willam K. Localization analysis of elastic degradation with application to scalar damage. *Journal of Engineering Mechanics*, 121(4):541–554, 1995.

Runesson K., Ottosen N.S., and Perić D. Discontinuous bifurcations of elastic-plastic solutions at plane stress and plane strain. *International Journal of Plasticity*, 7(1-2):99–121, 1991.

Schreyer H. and Neilsen M. Analytical and numerical tests for loss of material stability. *International Journal for Numerical Methods in Engineering*, 39(10):1721–1736, 1996.

Simo J.C. and Ju J.W. Strain- and stress-based continuum damage models - i. formulation. *International Journal of Solids and Structures 23 (7)*, pages 821–840, 1987.

Winkler B., Hofstetter G., and Lehar H. Application of a constitutive model for concrete to the analysis of a precast segmental tunnel lining. *International Journal for Numerical and Analytical Methods in Geomechanics*, 28(7-8):797–819, 2004.