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MODE SHAPE EXPANSION FROM DATA-BASED SYSTEM IDENTIFICATION PROCEDURES

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Abstract. In almost all cases, the experimental data set of a dynamical test is incomplete as the measurements are taken at selected locations in selected coordinate directions. This lack of measured degrees of freedom can be solved in two ways, either by reducing the finite element model to the size of experimental one or by expanding the experimental data to include the unmeasured degrees of freedom in the finite element model. In this sense, mode shape expansion techniques deal with the spatial incompatibility linked to the measurement of mode shapes through a limited set of physical sensors and their analytical prediction at a (larger) number of finite element (FE) degrees of freedom (DOFs). In other words, expansion methods seek to estimate the motion at all DOFs of a finite element model based on measured information (mode shapes or frequency response functions) and prior information about the structure under test in the form of a reference finite element model. In essence, the finite element model is used as a high order polynomial curve fitter to estimate the experimental mode shapes at the deleted DOF, and in general, more confidence can be placed in the expanded results by increasing the number of measurement points. This paper presents the behavior of two mode shape expansion techniques when modal data are obtained from system identification procedures. A dynamic test of a transmission line tower numerically, simulated by Fadel Miguel et al. (2006), provided the initial values and it is shown that accuracy results are achieved if the identified parameters are well correlated with the theoretical model.

1 INTRODUCTION

Unavoidably in almost all real practical situations, there will be an incompatibility in the size of the compared finite element and test models which mainly appear in two different ways: firstly not all the modes are measured and secondly just an analytical degrees of freedom subset may be measured.

The first of these difficulties arises because the available transducers and data acquisition hardware limit the frequency range that can be measured. Also, the increased modal density at higher frequencies makes the modal extraction considerable difficulty beyond moderately low order modes. There is little to be done to overcome this after the test execution, besides to use the more accurate modal identification methods.

In other way, the second incompatibility deals with the lack of the measured degrees of freedom. Generally, the finite element models can be very large, extending in many cases to several thousand degrees of freedom. However, for practical and economical situations in experimental modal analysis just a sensors subset is available, rarely exceeding more than 10% of these degrees of freedom. In addition, measurements difficulties are associated with the internal nodes and with the rotational degrees of freedom. As a clear consequence not all the degrees of freedom in the analytical model will be measured. Thus, this inherent difficulty may be solved in two different forms: reducing the analytical model order or expanding the measured mode shapes.

In a model updating procedure point of view, which the main goal is to improve the correlation between experimental and theoretical data, it is obviously advantageous that the final model remains in this coordinate system. For this situation and generally for damage studies, it is desirable to expand experimental data onto the corresponding finite element coordinate set because a reduction process destroys the original sparse pattern and propagates modeling errors all over the reduced matrices. Meanwhile expansion rate (number of finite element degrees of freedom in relation to the measured degrees of freedom) must be not so large that the experimental information is too diluted to provide localized information for updating.

In this context, this paper presents the behavior of two mode shape expansion techniques when modal data are obtained from system identification procedures. Firstly, the so-called Guyan Expansion/Reduction process, which is the oldest method presented for this kind of study, is introduced. Next, the more accurate System Equivalent Reduction Expansion Process (SEREP) (O'Callahan et al., 1989) is illustrated. This method uses theoretical eigenvectors to produce the transformation between master and slave coordinates and is showed that this may be a reliable tool for a mode shape expansion problem. A dynamic test of a transmission line tower is numerically simulated showing that accuracy results are achieved if the identified parameters are well correlated with the theoretical model.

2 EXPANSION OF MEASURED MODAL DATA

A grater variety of expansion techniques when compared to the reduction methods has been developed recently. Perhaps this reflects the relative value of having modal completeness for model reconciliation. In addition the effectiveness of the expansion methods has a significant influence on the overall location and updating procedure.

Four different schools of expansion method exist, and they have a varying dependency on the finite element model (Maia et al., 1999):

(i) *Analytical eigenvector substitution*. This method directly substitutes the analytical slave degrees of freedom for the unmeasured data, making no use of the available

measurements;

(ii) Direct data generation using the finite element model and experimental data. These methods create a transformation matrix from the master degrees of freedom to complete the finite element coordinate system;

(iii) Indirect data generation using the finite element model. A geometric fit of the experimental data is enhanced by knowledge acquired from the finite element model; and

(iv) Expansion of experimental data in isolation. These methods interpolate the experimental data by fitting a continuous function through the data by means of cubic splines, surfaces splines or polynomial fits.

The most reliable expansion techniques involve the use of the finite element model as a mechanism to complete the unmeasured degrees of freedom from the experimental modal model. In essence, the finite element model is used as a high order polynomial curve fitter to estimate the experimental mode shapes at the deleted degrees of freedom. In addition, the majority of the expansion techniques use model reduction transformation matrix as an expansion mechanism.

Next, it will be presented two very important finite element based mode shape expansion methods, which were chosen by their historical importance and because they are widespread in the literature.

2.1 Guyan Expansion

Possibly the most popular and simplest method was introduced by Guyan (1965). State and force vectors, $\vec{\boldsymbol{x}}$ and $\vec{\boldsymbol{f}}$, and mass and stiffness matrices, $\boldsymbol{M}$ and $\boldsymbol{K}$, are split into sub vectors and matrices relating to the master degrees of freedom, which are retained, and the slave degrees of freedom, which are eliminated. If no force is applied to the slave degrees of freedom, one can obtain:

$$\begin{bmatrix} [\boldsymbol{M}_{mm}] & [\boldsymbol{M}_{ms}] \\ [\boldsymbol{M}_{sm}] & [\boldsymbol{M}_{ss}] \end{bmatrix} \begin{Bmatrix} \ddot{\vec{\boldsymbol{x}}}_m \\ \ddot{\vec{\boldsymbol{x}}}_s \end{Bmatrix} + \begin{bmatrix} [\boldsymbol{K}_{mm}] & [\boldsymbol{K}_{ms}] \\ [\boldsymbol{K}_{sm}] & [\boldsymbol{K}_{ss}] \end{bmatrix} \begin{Bmatrix} \vec{\boldsymbol{x}}_m \\ \vec{\boldsymbol{x}}_s \end{Bmatrix} = \begin{Bmatrix} \vec{\boldsymbol{f}}_m \\ 0 \end{Bmatrix} \tag{1}$$

The subscripts m and s relate to the master and slave coordinates respectively. Neglecting the inertia terms for the second set of equations it can be obtained:

$$\boldsymbol{K}_{sm}\vec{\boldsymbol{x}}_m + \boldsymbol{K}_{ss}\vec{\boldsymbol{x}}_s = 0 \tag{2}$$

which may be used to eliminate the slave degree of freedom so that:

$$\begin{bmatrix} \vec{\boldsymbol{x}}_m \\ \vec{\boldsymbol{x}}_s \end{bmatrix} = \begin{bmatrix} [I] \\ -[\boldsymbol{K}_{ss}]^{-1}[\boldsymbol{K}_{sm}] \end{bmatrix} \vec{\boldsymbol{x}}_m = [\boldsymbol{T}_s]\vec{\boldsymbol{x}}_m \tag{3}$$

where $[\boldsymbol{T}_s]$ denotes the static transformation between the full state vector and the master coordinates, so the expanded mode shapes are:

$$\begin{bmatrix} \boldsymbol{\Phi}_m \\ \boldsymbol{\Phi}_s \end{bmatrix} = \begin{bmatrix} [I] \\ -[\boldsymbol{K}_{ss}]^{-1}[\boldsymbol{K}_{sm}] \end{bmatrix} [\boldsymbol{\Phi}_m] = [\boldsymbol{T}_s][\boldsymbol{\Phi}_m] \tag{4}$$

Notice that the masters' degrees of freedom remain unchanged as seen by the upper partition of this equation:

$$[\boldsymbol{\Phi}_m] = [I][\boldsymbol{\Phi}_m] \tag{5}$$

and that the deleted DOF are estimated by

$$[\boldsymbol{\Phi}_s] = \left[-[\boldsymbol{K}_{ss}]^{-1}[\boldsymbol{K}_{sm}]\right][\boldsymbol{\Phi}_m] \tag{6}$$

However, since this technique is solely based on the static stiffness of the system, there is no guarantee that the mode shape expansion will be accurate. Of course, the Guyan expansion process will not produce acceptable results unless there are sufficient degrees of freedom to describe the mass inertia of the system. If sufficient degrees of freedom are available, then the Guyan process will produce reasonably good results but will never produce exact results since the inherent formulation of the transformation matrix is approximate.

2.2 System Equivalent Reduction Expansion Process (SEREP)

The System Equivalent Reduction Expansion Process (SEREP) uses the theoretical eigenvectors to produce the transformation between master and slave coordinates. The analytical modal matrix $\boldsymbol{\Phi}_n$ is partitioned into master and slave coordinates so that:

$$\boldsymbol{\Phi}_n = \begin{bmatrix} \boldsymbol{\Phi}_m \\ \boldsymbol{\Phi}_s \end{bmatrix} \tag{7}$$

in which subscripts m and s relate to the master and slave coordinates respectively. The generalized or pseudo inverse of the master mode shapes is used to give the transformation $\boldsymbol{T}_u$, when the number of master coordinates is greater than the number of modes, as:

$$\boldsymbol{T}_u = \begin{bmatrix} \boldsymbol{\Phi}_m \\ \boldsymbol{\Phi}_s \end{bmatrix} \boldsymbol{\Phi}_m^{\dagger} \tag{8}$$

in which $\boldsymbol{\Phi}_m^{\dagger}$ is the Moore-Penrose pseudo-inverse of this matrix, and may be obtained as:

$$\boldsymbol{\Phi}_m^{\dagger} = \left(\boldsymbol{\Phi}_m^{\mathrm{T}} \boldsymbol{\Phi}_m\right)^{-1} \boldsymbol{\Phi}_m^{\mathrm{T}} \tag{9}$$

so the expanded mode shapes are:

$$\boldsymbol{\Phi}_n = \boldsymbol{\Phi}_n [\boldsymbol{\Phi}_m]^{\dagger} [\boldsymbol{\Phi}_m] = \begin{bmatrix} \boldsymbol{\Phi}_m \\ \boldsymbol{\Phi}_s \end{bmatrix} [\boldsymbol{\Phi}_m]^{\dagger} [\boldsymbol{\Phi}_m] \tag{10}$$

When the master degrees of freedom are expanded, there is the possibility that the initial measured degrees of freedom may be modified by the expansion process; this is referred to as smoothing of the measured degrees of freedom. This occurs since the SEREP process is based on a generalized inverse using a least squares error minimization. Therefore, the measured data are smoothed as part of the process. While much controversy exists over where or not to smooth the actual measured data, this is the most proper way to process the data, from a mathematical standpoint.

The SEREP expansion technique is extremely accurate in regards to the expanded mode shapes - actually it is exact due to its inherent formulation. However, if the experimental mode shapes are not correlated well with regards to the analytical mode shapes, then the results can produce inaccurate expanded mode shapes.

3 APPLICATION: TRANSMISSION LINE TOWER

The explained mode shape expansion techniques are verified analyzing a transmission line tower collapsed in Japan during a typhoon in 1991, presented by Murotsu et al. (1994). Fadel Miguel et al. (2006) numerically simulating an output-only system identification procedure studied this structure and extracted its modal parameters in different excitation conditions through the stochastic subspace system identification method (SSI). The mode shapes obtained in that paper are used here for the expansion procedures.

The transmission line tower has 82m high, 80 nodes and 163 bars (Figure 1), being analyzed as a plane truss structure. The structural physical and geometrical properties are available in Murotsu et al. (1994).

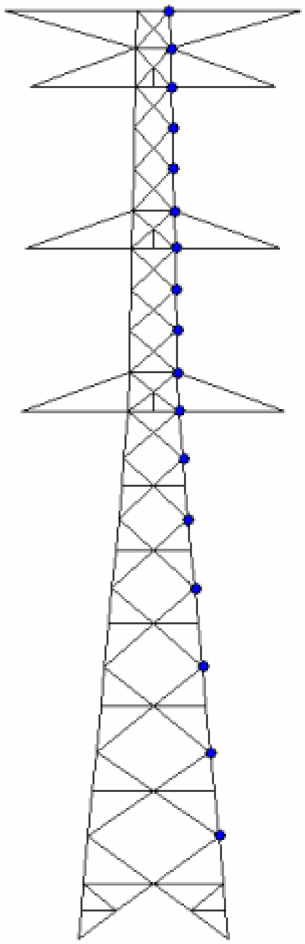


Figure 1: Transmission line tower.

Seeking to better reproduce an experimental situation Fadel Miguel et al. (2006) just considered a sensors' subset for the identification process. These nodes are represented with blue marks in Figure 1. In this way, mode shapes at these 17 degrees of freedom can be established, while the structure has a total of 80 nodes.

With the mode shape values at the measured nodes the Guyan Expansion and System Equivalent Reduction Expansion Process (SEREP), for the first six modes, are conducted and the results accuracy is evaluated.

For mode shape correlation analysis between theoretical and identified-expanded values is used the modal assurance criterion or MAC index. This index provides a correlation among two different groups of mode shapes, being used to verify general differences among them. The MAC value for two different modes shapes $\vec{\varphi}_a$ and $\vec{\varphi}_e$ (i.e., analytical and experimental mode shapes) can be defined as:

$$MAC(\vec{\varphi}_a, \vec{\varphi}_e)_i = \frac{\left| \{\vec{\varphi}_a\}_i^T \{\vec{\varphi}_e\}_i \right|^2}{\left| \{\vec{\varphi}_a\}_i^T \{\vec{\varphi}_a\}_i \right| \left| \{\vec{\varphi}_e\}_i^T \{\vec{\varphi}_e\}_i \right|} \tag{11}$$

in which i is *ith* analyzed mode. MAC index varies between 0 and 1, being 0 for not correlated and 1 when they are equal. In Table 1, Guyan Expansion and System Equivalent Reduction Expansion Process (SEREP) results are presented.

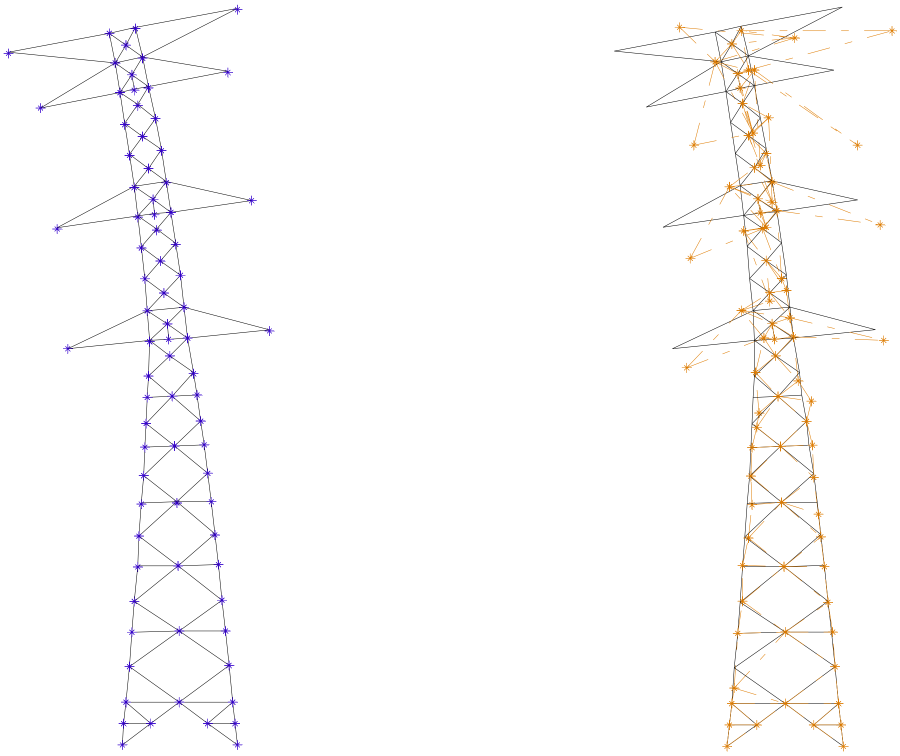
	Guyan	SEREP
1st Bending	0.3714	0.9996
2nd Bending	0.2137	1.0000
3rd Bending	0.1662	1.0000
1st Axial	2.8e-04	1.0000
4th Bending	0.2385	1.0000
5th Bending	0.0413	1.0000

Table 1: MAC results.

As was already pointed out in Section 2, Guyan procedure is not exact because the system mass inertia was not considered in its demonstration. So it is clear in this example that the measured degrees of freedom are not enough to describe this system mass inertia. An evident conclusion is that this technique cannot be surely used in a blind fashion to expand the mode shapes and particular attention must be given in the results reliability.

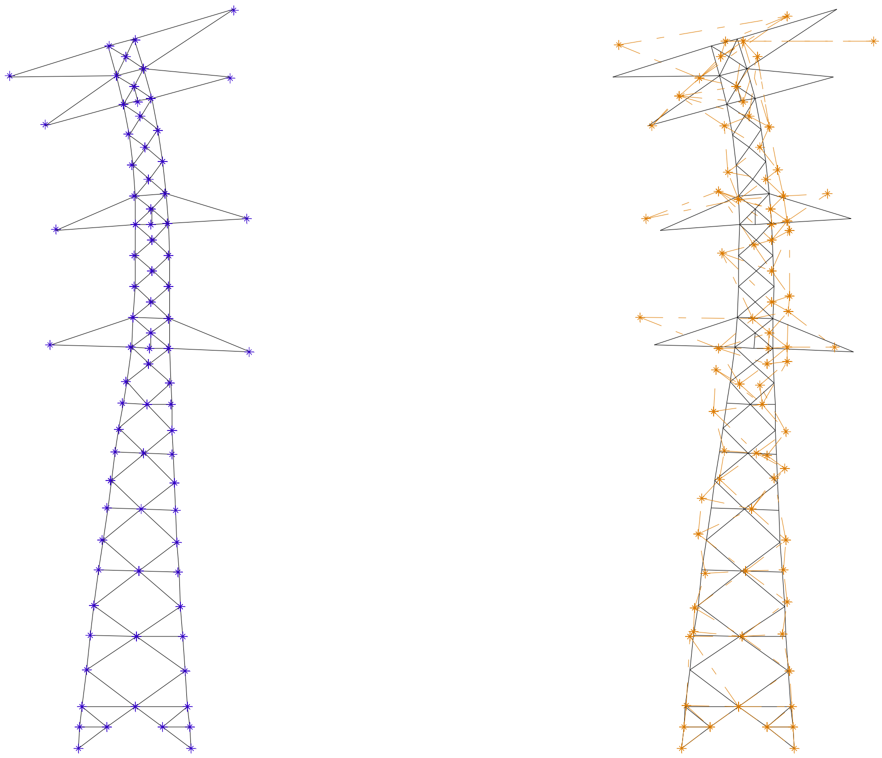
On the other hand, System Equivalent Reduction Expansion Process (SEREP) provided very accurate results. The procedure showed to be able to expand correctly the mode shapes, even considering the low ratio between measured nodes and total nodes in the example.

Figures 2 to 7 show graphically the expanded mode shapes behavior when compared with theoretical values, represented by the continuous black lines. On left side the blue marks illustrate the SEREP results, while on the right side the orange dashed lines show the Guyan results.



(a) (b)

Figure 2: First Mode Expansion: (a) SEREP, (b) Guyan.



(a) (b)

Figure 3: Second Mode Expansion: (a) SEREP, (b) Guyan.

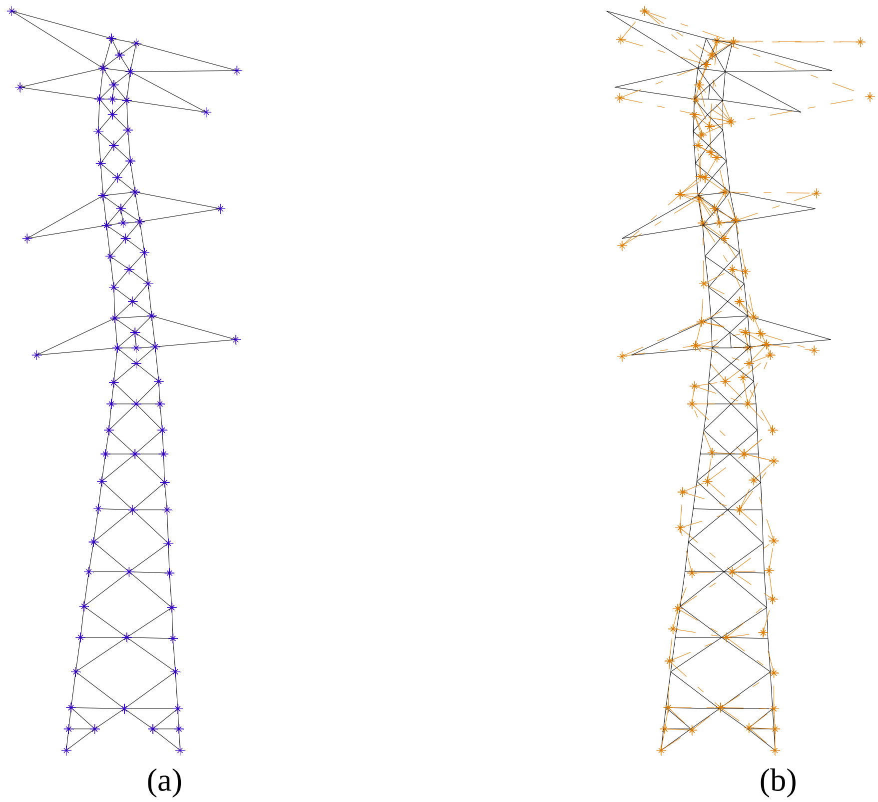


Figure 4: Third Mode Expansion: (a) SEREP, (b) Guyan.

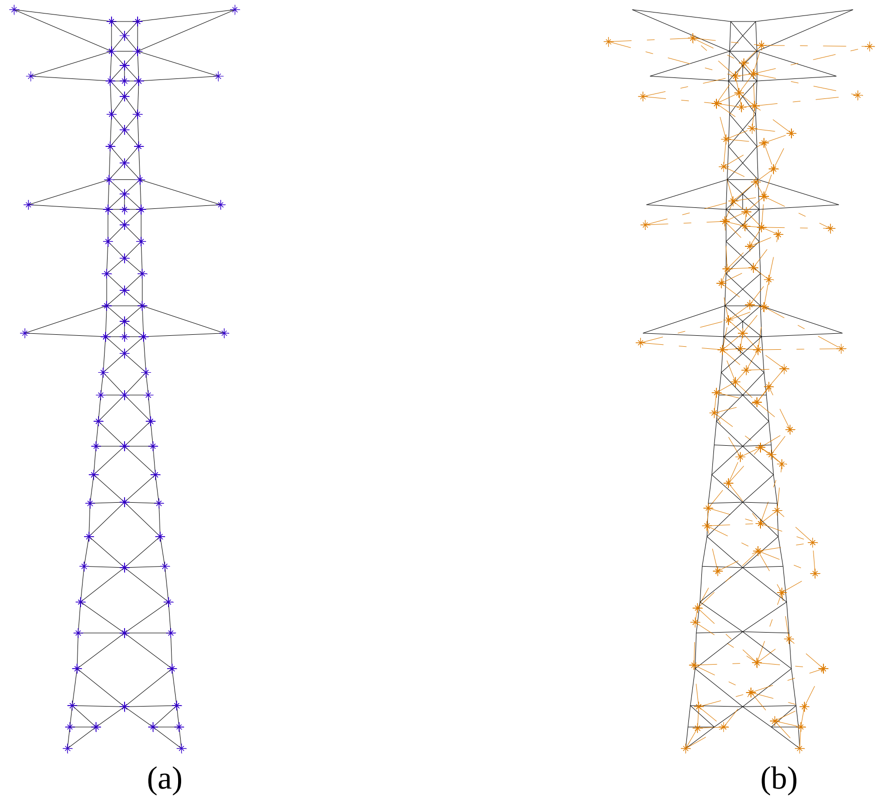
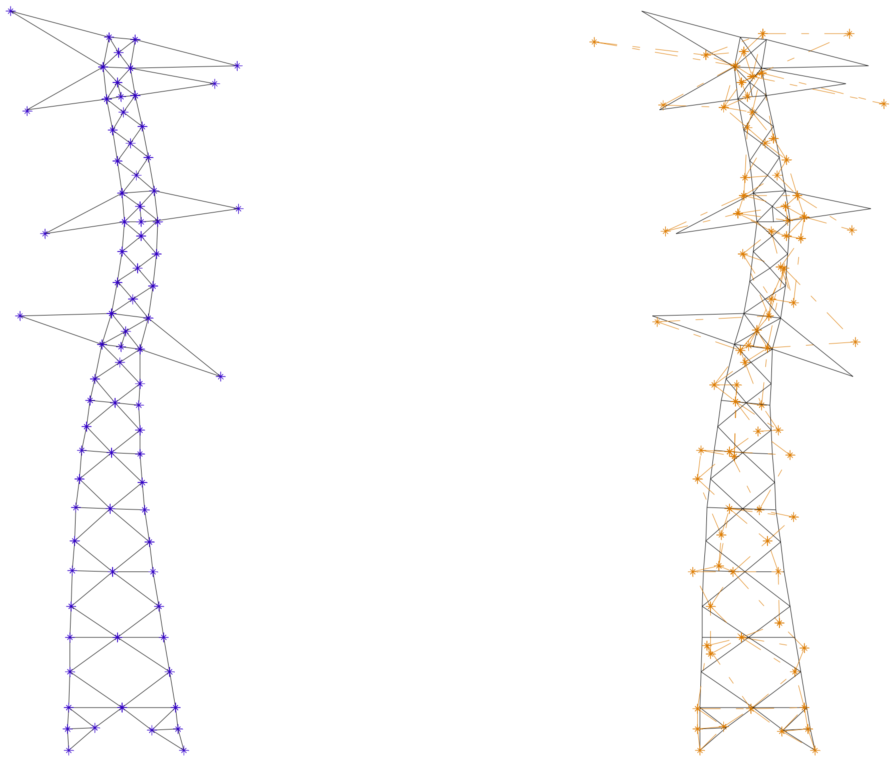
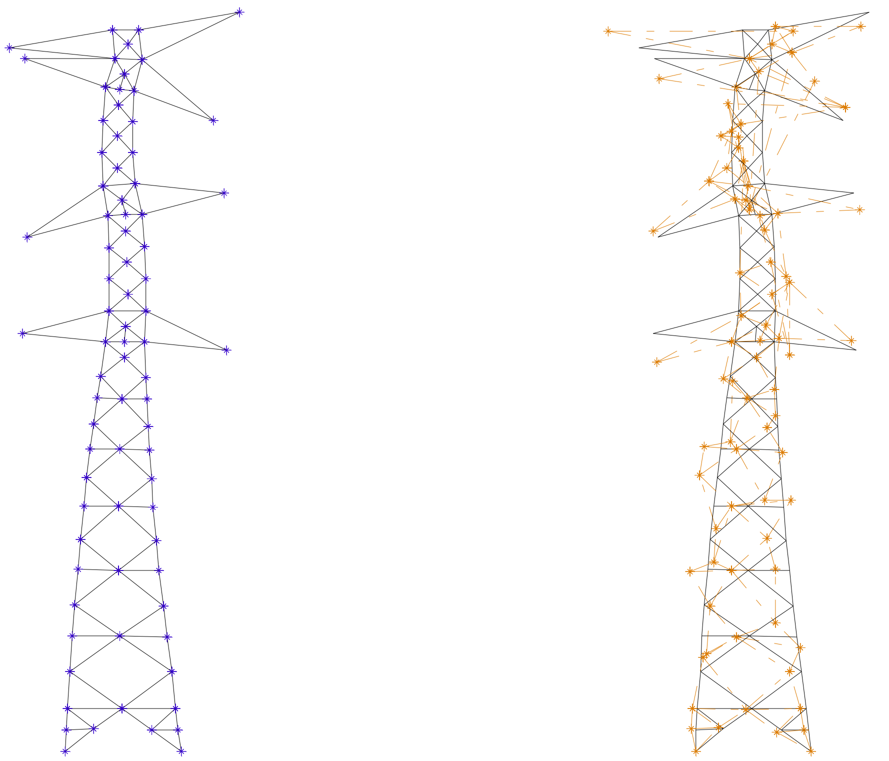


Figure 5: Fourth Mode Expansion: (a) SEREP, (b) Guyan.



(a) (b)

Figure 6: Fifth Mode Expansion: (a) SEREP, (b) Guyan.



(a) (b)

Figure 7: Sixth Mode Expansion: (a) SEREP, (b) Guyan.

4 CONCLUSIONS

In this paper the behavior of two mode shape expansion techniques when modal data are obtained from system identification procedures were analyzed. Mode shape initial values were provided through a numerically simulated dynamic test of a transmission line tower, conducted by Fadel Miguel et al. (2006).

It was shown that the so-called Guyan Reduction/Expansion may present very poor results if the measured degrees of freedom are not enough to describe this system mass inertia and special care may be taken into account with this methodology, avoiding its use in a blind fashion to expand the mode shapes.

On the other hand, when the identified mode shapes are well correlated with the theoretical model the System Equivalent Reduction Expansion Process (SEREP) may provide very accurate results, even considering the low ratio between measured nodes and total nodes in the example. As other SEREP advantages may be considered the easiness to program and the not high computer time required.

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