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IDENTIFICATION OF RIGID SUBCHAINS IN GRAPHS OF MECHANISMS OBTAINED FROM CONTRACTED GRAPHS

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Abstract. The enumeration of graphs of kinematic chains of mechanisms is of utmost importance in the design of new mechanical devices. A database of graphs of kinematic chains of mechanisms can be used in the conceptual design stage to search, by using a computer, mechanisms with given structural properties: number of degrees of freedom, loops, joints and links, as well as particular characteristics of links and joints of the mechanisms, such as the input joints, the ground and the output links, among others. Any state-of-the-art database of graphs of mechanisms contains millions of mechanisms. Each graph must not be and must not contain a rigid subgraph as a substructure. The correctness of the algorithm to identify degenerated graphs is critical to define the correctness of the database of mechanisms. This work firstly reviews several approaches to identify degenerated mechanisms. Secondly, a rigid-chain identification procedure for graphs of kinematic chains obtained from contracted graphs by the assignment of binary chains is described. The results are validated with those found in the literature.

1 INTRODUCTION

The enumeration of graphs of kinematic chains of mechanisms is of utmost importance in the design of new mechanical devices using computed-aided engineering software. Any state-of-the-art database of graphs of mechanisms contains a number of items that ranges from some few, when the number of kinematic loops is one or two, to thousand or millions of mechanisms as the number of loop increases to five or six.

The database of graphs of kinematic chains of mechanisms is used in the conceptual design stage to search, by using a computer, mechanisms with given structural properties: number of degrees of freedom, loops, joints and links, as well as particular characteristics of links and joints of the mechanisms, such as the input joints, the ground and the output links, among others. A database with kinematic chains with one and two degrees-of-freedom and up to ten links were used by the authors to design aeronautical mechanisms (Pucheta, 2008; Pucheta and Cardona, 2008, 2007, 2010, 2013) and low-voltage circuit breakers (Pucheta et al., 2012).

There are several methods to generate kinematic chains that have been developed in the last fifty years using computer programs (Woo, 1967; Tsai, 2001). Most approaches start satisfying the Gruebler-Kutzbach-Chevychev (GKC) mobility formula and use Graphs Theory based methods because of the correspondence existing between kinematic chains and graphs, where links in the kinematic chains are identified with vertices of the graph and joints with edges. Another class of kinematic chains can be obtained by the union by a link or joint of two kinematic chains. These are called fractionated kinematic chains and the graphs of these chains contain a edge cut or vertex cut (Tsai, 2001). This work assumes that all kinematic chains in the data base are non-fractionated. Besides, the minimum degree considered in the graphs is two, such that only closed chain are studied, excluding subchains that are open chains. The graphs of kinematic chains satisfying the GKC formula and a given mobility must not contain a rigid subgraph as a substructure with zero or negative mobility. The kinematic chain containing a rigid subgraph is called a degenerated chain. The correctness of the algorithm to identify degenerated graphs is critical to define the correctness of the database of mechanisms.

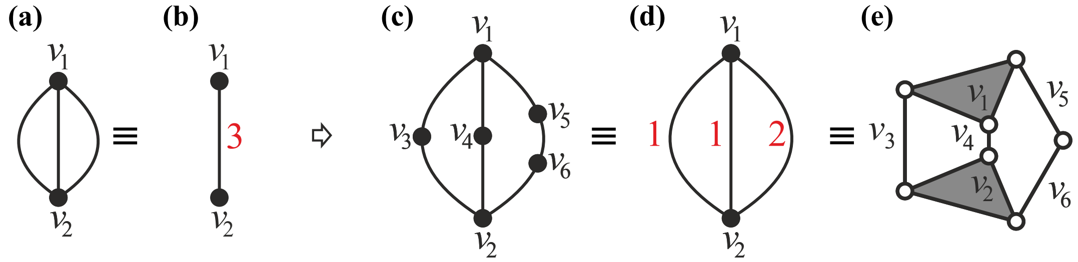

Figure 1: A contracted graph represented by a multigraph (a) or by labeled simple graph (b) where labels indicated edge multiplicity. The contracted graph is used as a basis to assign binary strings (c) and (d) to form a kinematic chain (e)

Graphs of non-fractionated kinematic chains are generated by using two main approaches: (i) non-isomorphic graph generators satisfying the GKC formula (Martins and Carboni, 2008; Sunkari and Schmidt, 2005, 2006), or (ii) by the assignment of binary chains to contracted graphs (Lee and Yoon, 1992; Tuttle, 1996; Tsai, 2001; Ding et al., 2011, 2012), i.e., to graphs where any vertex degree is higher than 2 and the graphs satisfy certain relationships derived from the GKC and Euler formula for loops; see for example, a valid assignment process in Fig. 1. After using one of these methods a degenerated subchain algorithm needs to be executed as the last step of the enumeration process.

This work firstly reviews several computational approaches to identify degenerated kinematic chains (Lee and Yoon, 1992; Tuttle, 1996; Martins and Carboni, 2008; Sunkari and Schmidt, 2005, 2006; Ding et al., 2010; Xia et al., 2012). Then, it presents a rigid-chain identification procedure for graphs of kinematic chains obtained from contracted graphs by the assignment of binary chains. The results are validated with those found in the literature.

2 MAIN APPROACHES FOR DEGENERATED CHAIN IDENTIFICATION

The locked subchains for 1-DOF planar kinematic chains can be originated in the enumeration of these kinematic chains and becomes the chain unfeasible. Some simple examples are shown in Figure 2. Then, the main approaches to identify them are described.

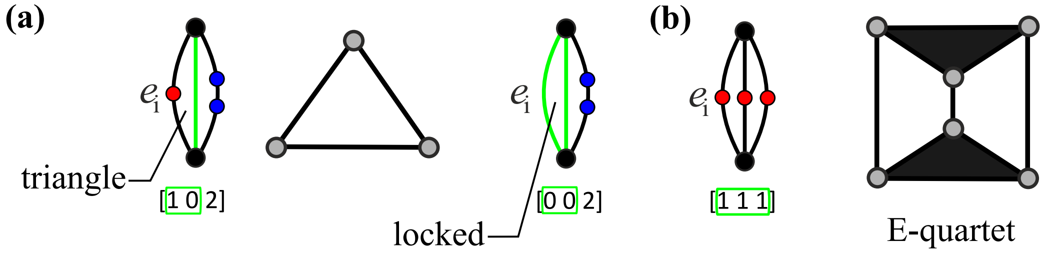

Figure 2: One- and two-loop locked 1-DOF kinematic chains. Triangles can be identified from the assignment vector of binary chains(a) whereas E-quartets can be identified from assignment vectors: more complex E-quartets can appears as subgraphs and require the listing of paths of length 2 between two higher-links. Rigid subchains with 3 or more loops can be included in the tests presented in this paper.

Sunkari and Schmidt (2005) reviewed several approaches to identify degenerated subchains in kinematic chains. They detected that some prior approaches contained errors when assuming that the graphs of kinematic chains were all planar graphs. That assumptions lead to differences in the number of kinematic chains enumerated. Among the several degenerated subchain identification reviewed, they highlighted that the algorithm by Lee and Yoon (1992) was the most efficient and valid for planar and non-planar kinematic chains. Lee and Yoon (1992) proposed an algorithm based on the recursive reduction of the binary chain strings. Sunkari and Schmidt (2006) gave a formal proof of the correctness of the Lee and Yoon's degeneracy testing algorithm and developed an improved version of the algorithm. This was used to successfully enumerate a large database of non-isomorphic graphs of kinematic chains. The graphs were firstly generated by using a McKay-type algorithm and satisfying the GKC formula. A similar approach was conducted by Simoni et al. (2009) but using a different degeneracy testing algorithm.

Martins and Carboni (2008) proposed an exhaustive algorithm for computing several structural properties, like connectivity, degree of control, redundancy and variety; as a remarkable contribution, all these properties were given in terms of the order of the screw system. Such algorithm is also useful for degeneracy testing because the mobility is intensively computed for the graph and any biconnected subgraph (Simoni et al., 2009). Using a minimal loop basis for the graph of a kinematic chain, all loops of the kinematic chains are generated, then they are combined without repetitions to form all biconnected components of the graph and the GKC formula is computed for each of them. The loops were expressed as binary strings indexed by the edges of the graphs so that the operation of composition can be defined by modulo-2 sums. The computational cost to generate all biconnected components, from the ν loops of the fundamental basis, is expensive ($\mathcal{O}(2^{\nu})$) for kinematic chains with more than five loops. Despite its cost, the algorithm is simple and correct and can be taken as reference to design and test other, more optimal, degeneracy testing algorithms.

Ding et al. (2008) developed a decomposition of the structure of a kinematic chain into a base loop followed by open paths, where the loops and operators for subtraction and symmetric difference of the loops were given in terms of binary vectors indexed by the vertices and the operators used the vertices degrees. Later, Ding et al. (2010) expressed the loops as binary vectors indexed by the edges the edges of the graphs and modified the operators accordingly. They used this algorithm for identifying rigid sub-chain detection as well as for driving pair identification. The rigid sub-chain identification algorithm was improved by contraction of particularly chosen loops (Ding et al., 2012) and was used for enumeration of several databases of kinematic chains; see Ding et al. (2016) and the self-citation therein.

One of the common pattern of the approaches in the previous paragraphs is the starting point, which is the computation of the number of independent loops. The loop-based approach is the most promising one for degeneracy testing because the computational complexity is given in terms of the number of independent loops.

There are other approaches that deserve to be mentioned. An algorithm based on the simulation of percolation physics, known as Pebble game, was proposed by Slojka et al. (2011). Because its cost is $\mathcal{O}(|V|\,|E|)$, it is fast for rigidity detection and also for decomposition of a kinematic chain into Assur chains but it was not tested on a complete database of graphs.

3 GRAPHS OF MECHANISMS OBTAINED FROM CONTRACTED GRAPHS

Contracted graphs are graphs and multigraphs (graphs with multiple edges) with vertices with degree higher than 2. They are bases for the generation of kinematic chains by means of the distribution of a number of binary vertices onto the edges of the contracted graphs in such a way contracted graph edges are converted into binary paths, i.e., a sequence of binary vertices and edges.

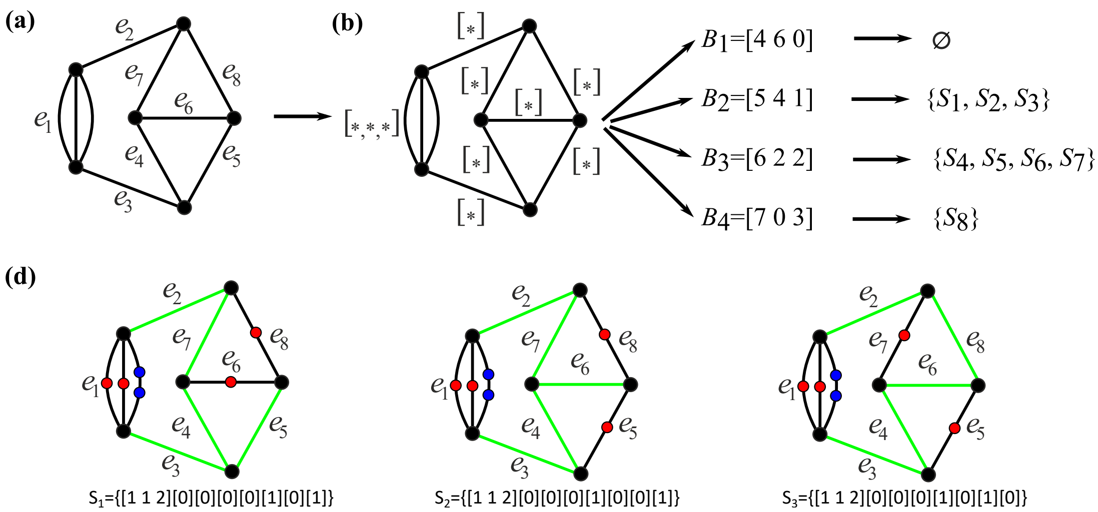

Figure 3: The multigraph of a contracted graph is labelled (a) and their indices are used to sort the vectors of lengths of binary strings (b) that are assigned to each multiple edge. The assignment process compute the number of binary chains with null length (direct connection by an edge), length 1 and length 2. Below in (d) the assignment of 5 direct connections (shown in green color), 4 binary chains of length 1 (shown in red color) and 1 binary chain of length 2 (shown in blue color) are shown; 3 cases are feasible results. Below each graph, the list of assignment vectors for each edge, sorted as in (a) are shown.

The distribution must be made without generating repetitions. In this sense, two approaches were followed in the literature:

1. The non-isomorphic generation where all combinatorial assignments are different.

2. The generation subjected to several constraints that prevents some cases of rigidity followed by isomorphism identification.

After the generation stage, both approaches require to execute a rigidity identification algorithm.

The generation is orderly performed as shown, for example, in Figure 3. The first contracted graph for the benchmark case is shown in Fig. 3(a). It has 4 vertices with degree 3 and 2 vertices with degree 4. A number of 8 feasible kinematic chains are obtained after passing the rigidity test and the first three are shown in Fig. 3(d).

4 RESULTS

The enumeration used as benchmark is the list of kinematic chains of planar mechanisms with 1 to 6 DOF and 5 independent loops. For this number of loops, there are 118 contracted graphs that are used as basis for the binary chain assignments. The pairs of links and joints for 1 to 6 GDL are respectively:

$$(n, j) = \{(12, 16), (13, 17), (14, 18), (15, 19), (16, 20), (17, 21)\}$$

The numerical experiments consist in the division of the rigidity identification in three consecutive steps:

1. Identification of triangles by using simple rules applied on assignment vectors of multiedges. This consists in finding the 00 and 01 cases in the assignments (see Figure 2).

2. Identification of E-quartets as proposed by Tuttle (1996).

3. Identification of higher order rigid subchains by using the loop-based approach proposed by Ding et al. (2010).

The computation times for the numerical experiments are shown in Fig. 4. The lower curve shown the generation of kinematic chains using a group theory based approach. The other three curves shown the generation followed by the rigidity testing. It clearly reveals that the computation time elapsed for rigid identification ranges from 3 to 10 times the generation time, the latter occurs when a loop based approach is used. The experiment with labelled as *Multiedges* considers the rules for the identification of triangles and E-quartets. It is fast but it is not correct because it fails to detect three-loop rigid subchains. The rigid detection based on loop decomposition gives the right results when validated again the literature for most cases. The results computed are in the list

$$|G| = \{6856, 27496, 83547, 216291, 504599, 1086579\}$$

and are graphically shown in Fig. 5. They matches those found in the literature but differs in one graph for the last result, which should be 1086580 according to Ding (2015).

Future works will be focused on identifying that difference and on optimizing the loop computation algorithms. Another aspect to improve is the generation stage. In the last experiment shown in Fig. 5, $(17, 21)$, it can be seen that the number of generated graphs duplicates the number of feasible graphs. This becomes more relevant for 6 loop enumeration. Therefore, some graph theory based rules and more insight on the problem is needed to reduce the number of generated graphs.

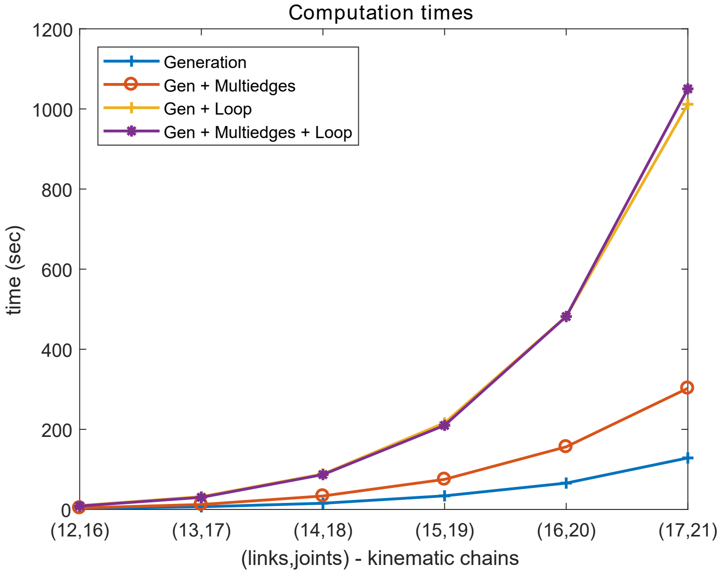

Figure 4: Computation times for the generation and the combination of two rigidity testing experiments: identification of assignments for multiedges and a loop-based approach.

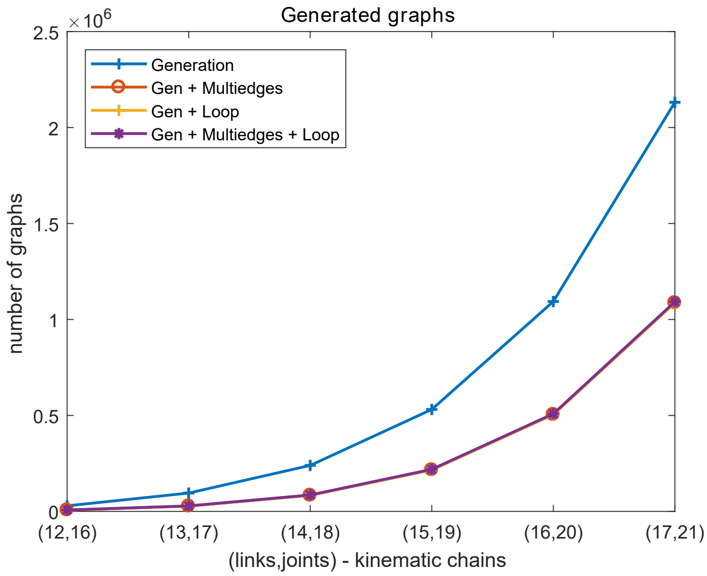

Figure 5: Number of feasible graphs enumerated. There is no appreciated difference in the graphs. However, the *Multiedges* experiment is not correct.

5 CONCLUSIONS

This paper described an ongoing research on rigidity testing using contracted graphs, binary chain assignment and a loop-based approach. The experiment shown that some rules for rigidity identification can be applied very fast but lead to a incomplete enumeration. Also, the loop-based approach needs always to be applied to identify three-loop rigid subchain to reach right results.

Next research directions towards a more efficient enumeration will be focused on several loop-based approaches for both the generation and the rigid subchain identification.

Computing times do not include the time to canonically code the graphs. As a final stage, the graph of the kinematic chain is encoded. For instance, the *degree code*, the adjacency matrix or the *graph6* format can be used. The whole database will be shared with the mechanism and machine community for comparison purposes and use.

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