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COMPARISON OF FAILURE CRITERIA OF A SLENDER WOOD STRUCTURE UNDER STOCHASTIC AND QUASI-STATIC WIND LOADS

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Abstract. A stochastic model of a wood pole under wind loads is addressed in order to study its failure under different criteria. In order to evaluate wind loads on a structure, the Argentine standard CIRSOC 102-2005 proposes simplifications that take into account the effects of shape, location and dynamics of a load variable in time, resulting in an equivalent quasi-static action, which simplifies the strength and deformations assessment of a given structure. However, if a temporal and spatial varying record of the wind load is considered in detail, the results should be comparable to those obtained with the standard load. The structural model consists of a wood utility pole with one end embedded in the ground and the other end free, with deterministic geometry and material properties values. The pole is subjected to a wind load along its length. The stochastic dynamic wind load as a temporal function derived from a Power Spectral Density Function using the Spectral Representation Method and taking into account spatial and temporal correlations. The structural model is discretized with FEM and the realizations are analyzed with Monte Carlo method. In the study, the maximum displacement at the tip is assumed as the failure threshold. This work seeks to explore results from different failure criteria for dynamic loads and to compare them with the results of a pole under quasi-static deterministic wind load. The criteria are the first passage, the dwell time above the failure threshold, the extreme values distribution, the crossing rate and the cumulative proportion of displacements above the threshold of all the realization. A comparison is carried out by means of the fragility curves to assess the suitability of each approach.

1 INTRODUCTION

The behavior of a utility wood pole with linearly varying diameter (truncated cone) under wind load is analyzed in order to assess its reliability. The case of a quasi-static wind load as prescribed by the Argentine standard (CIRSOC 102, 2005) is solved as a reference. Previous works by the authors have addressed different complexities both in the load and in the material properties. A pole under quasi-static wind load was analyzed considering the random geometric variables (Gonzalez de Paz and Rosales, 2015) and the study included the reliability analysis regarding strength and serviceability. Then, the modulus of elasticity of a wood pole was considered as a homogeneous random variable and as a stochastic field in Gonzalez de Paz et al. (2016); in this case, the first passage was evaluated. More complex models of the wood properties were addressed in Gonzalez de Paz et al. (2017). In effect, previous models were compared with the weak zone approach including the change of stiffness due to the knots. Confidence bands were calculated and the load was represented by a stochastic temporal function. The failure of the pole is calculated regarding to the top displacement serviceability condition which limit is proposed in the wood structures' Argentine standard (CIRSOC 601, 2016). Five failure criteria are employed in the present work: the first passage deals with the time when the response first crosses a given threshold (Benaroya et al., 2005; Shinozuka and Wu, 1988); the dwell time counts the total time that the structure is in the undesirable zone (Shinozuka and Wu, 1988); the extreme value distribution is the probability distribution function of the extreme values in a certain period of time (Melchers and Beck, 2018); the crossing rate measures the number of times in which the displacement crosses the limit in a certain time interval (Melchers and Hough, 2007) and the time integrated displacements measures the area of the displacement-time plot above the limit w.r.t. the total plot area (Melchers and Beck, 2018).

The stochastic wind load is found by the Spectral Representation Method Shinozuka and Deodatis (1991) starting from the Davenport's power spectrum (Dyrbye and Hansen, 1996). The resulting function includes the temporal and spatial correlations.

The fragility curves are a very useful means to evaluate the probability of damage of a structure as a function of one relevant variable (Palencia et al., 2008; Gonzalez de Paz et al., 2016).

In this study, the five failure criteria mentioned above are analyzed in order to adopt a failure level for each of them. Afterwards, the stochastic model (stochastic wind load) is run and the responses are statistically processed in order to obtain the fragility curves as a function of a range of wind reference velocities. The failure criteria results are compared with the quasi-static wind velocity case.

2 STRUCTURAL MODEL

The structure under study is a *Eucalyptus grandis* pole which is a vertical column which diameter varies linearly, embedded in the soil. The structural model is a beam clamped at the ground line subjected to transverse wind load. Two models of wind action are applied lengthwise the pole. One of them is the proposed by the Argentine standard CIRSOC 102 (2005) and the other is a dynamic stochastic load with accounts for spatial and temporal correlations. The wood properties parameters and the geometric quantities (height and ground and top diameters) were obtained from Torrán et al. (2009). The geometric quantities are depicted in Table 1.

The Modulus of Elasticity (MOE) value is a constant and equals to the deterministic value found by Torrán et al. (2009), MOE = 10.935 N/m^2. Damping in timber material is considered random with a uniform distribution, assuming values between 1% to 3% of the critical damping c_c. The mass is considered as a deterministic value equal to 707 kg/m^3, corresponding to the

Table 1: Geometric Data (Torrán et al., 2009)

GL-T distance	ground line diameter	top diameter
10.175 m	0.262 m	0.191 m

mean value presented in Torrán et al. (2009) for a mean moisture content of 45%.

3 WIND LOAD

The described model is subjected to two types of wind load: a quasi-static load calculated following the standard CIRSOC 102 (2005), variable only in height and a stochastic dynamic load, variable in height and time.

3.1 Stochastic Dynamic Wind Load

In order to calculate the dynamic stochastic wind load in the time domain, it is necessary to recreate a temporal record which is composed of two contributions: one is a deterministic mean value with position variation and the other, a fluctuating random field with variation in position and time.

The fluctuating wind velocity is obtained by the application of Spectral Representation Method (SRM) (Shinozuka and Deodatis, 1991). The method starts from a Power Spectral Density Function (PSDF) and a coherence function, to be chosen in accordance with the type of problem to be simulated. Then, the random signals are created as a superposition of harmonic functions with a random phase angle, weighed by coefficients that represent the importance of the value of frequency within the spectrum and the spatial correlation. The process can be simulated by the following:

$$f_j(t) = \sum_{k=1}^{3}\sum_{n=1}^{N} |H_{jk}(\omega_n)| \sqrt{2\Delta\omega}\cos[\hat{\omega}_n t + \Phi_{kn}] \tag{1}$$

where $\Delta\omega$ is the frequency interval with which the PSDF is discretized, $\omega_n = \Delta\omega(n-1)$, $\hat{\omega}_n = \omega_n + \psi_{kn}\Delta\omega$, ψ_{kn} is a random value uniformly distributed between 0 and 1, N is the amount of frequency ranges and, Φ_{kn} are the random independent phase angles uniformly distributed between 0 and 2π. More details can be found in Shinozuka and Deodatis (1991). The SRM requires of the implementation of different steps. The Davenport's PSDF is chosen (Dyrbye and Hansen (1996)):

$$R_N(z,\omega) = \frac{\omega S(z,\omega)}{\sigma^2(z)} = \frac{2}{3}\frac{f_L^2}{(1+f_L^2)^{4/3}} \tag{2}$$

where ω is the frequency in Hz, σ is the standard deviation and f_L is the non-dimensional frequency $f_L = \omega L_u/U(z)$. L_u is the length scale of turbulence (1200 m in Davenport's PSDF) and $U(z)$ is the wind mean velocity at height z. The expression for $U(z)$ correspond to the potential law adopted by the Argentinian standard (CIRSOC 102, 2005) $U(z) = 2.01V(z/z_g)^{2/\alpha}$ where V is the nominal wind velocity which, together with z_g and α, are values given by the standard code depending on the characteristics of the structure location.

Then, the assumed coherence function is

$$Coh(z_i, z_j, \omega) = \exp\left\{-2\omega\frac{C_z \mid z_i - z_j \mid}{U(z_i)+U(z_j)}\right\} \tag{3}$$

where z_i and z_j are the heights of two given points of the pole. Then, each S_{ij} of the $S(\omega)$ matrix, for a given value of frequency can be calculated as

$$S_{ij}(z_i, z_j, \omega) = \sqrt{S(z_i, \omega)S(z_j, \omega)}\; Coh(z_i, z_j, \omega) \tag{4}$$

Following this procedure, each value $S_{ij}(z_i, z_j, \omega)$ will be calculated, and then, for each frequency ω, $H(\omega)$ matrices will be found. Finally, it is possible to construct the temporal series given by

$$u(z_j, t) = \sum_{k=1}^{m}\sum_{n=1}^{N} H_{jk}(\omega_n)\sqrt{2\Delta\omega}\cos[2\pi\hat{\omega}_n t + \Phi_{kn}]. \tag{5}$$

Table 2: Adopted values employed in the calculation of the time dependent velocity field.

Coefficients	σ^2	L_u	C_z	ω_c	$\Delta\omega$	t	Δt	N	m
Value	38.77	1200 m	11.5	2.5Hz	0.004 Hz	300 s	0.3 s	625	10

Once the fluctuating component of the wind speed has been determined, the Argentinean Standard (CIRSOC 102, 2005), with some modifications carried out in order to take into account the dynamics of the wind, is employed in the calculation of the wind load. This standard defines the transversal wind force F as:

$$F = q_z\, GC_f A_f \tag{6}$$

where G is the gust-effect factor, C_f is the force coefficient which includes the effect of the shape of the structure, Af is the projected area normal to the wind. q_z is the dynamic velocity pressure evaluated at height z of the structure:

$$q_z = 0.613\, k_z\, k_{zt}\, k_d\, V^2 I \tag{7}$$

where k_z is the dynamic pressure exposure coefficient, k_{zt} is the topographic factor, k_d is the wind directionality factor, V is the basic wind speed and I is the importance factor. k_z is a function of the elevation z and the exposure category that described the ground surface roughness using information about the natural topography, vegetation and constructed facilities in the vicinity of the structure of interest.

For the determination of the fluctuating component of the wind velocity, the expression $V\,k_z = U(z)$ was used. Then, the Equation 7 becomes

$$\bar{q}_z = 0.613\, k_{zt}\, k_d(U(z) + u(z))V\, I \tag{8}$$

and the Equation 6 results:

$$F = \bar{q}_z\, GC_f A_f \tag{9}$$

Table 3 shows the adopted values for the mentioned coefficients.

4 FAILURE CRITERIA

In general, the failure of a structure is assumed when the demand S is larger or equal to a given limit R, *i.e.* the probability of failure is written as

$$p_f = P[G(R, S) \leq 0] \tag{10}$$

where $G(\cdot)$ is known as the 'limit state function' and the probability of failure is identical to the limit state violation. The types of failure studied is this work are operational, without collapse, analyzing maximum displacements due to wind load acting on the structure.

Table 3: Coefficients employed in the determination of the wind load according to CIRSOC 102 (2005).

Coefficients	G	C_f	I	k_d	k_{zt}	α	z_g
Value	1	2.0	1	0.85	1	9.5	274 m

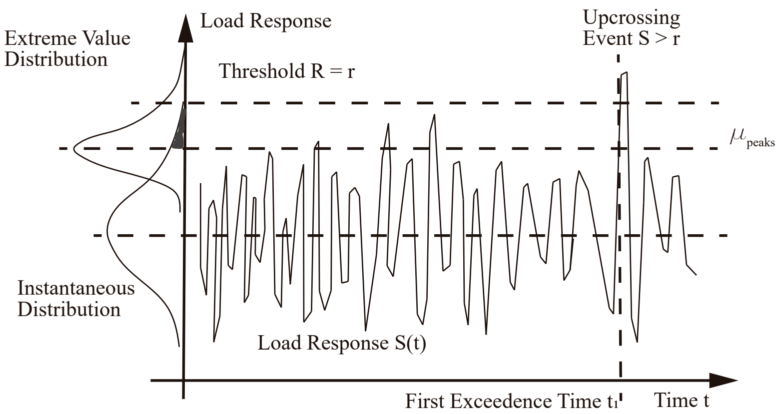


Figure 1: Typical realizations of load effect $S(t)$ and threshold R, with R a time-independent variable (Melchers and Hough, 2007).

4.1 Static Failure Criterion

In this study, the limit for the static load model is set when the displacement in the top of the pole (demand S) is equal or larger to a maximum displacement allowed for this structural type of $R = H/100 = 0.10175$ m (CIRSOC 601, 2016).

4.2 Dynamic Failure Criteria

Loads and their associated response can be expressed as stochastic processes. Figure 1 shows a realization of a load response S which fluctuates around a mean value. Also, an instantaneous probability distribution f_S can be described. Threshold $R = r$ represents the strength or limit state of a structural system. In this study, R takes a deterministic value, invariant in time.

4.2.1 First Passage

The first passage problem is highly related to the reliability of a structure under a dynamic load. It is also known as first excursion problem or first crossing problem (Benaroya et al., 2005). The term "first passage failure", however, does not necessarily indicate that a structure fails immediately following the first time response passes a given threshold level $R = r$.

$$p_f = P(S(t) > R) \tag{11}$$

where p_f is the probability of failure for the first passage, $S(t)$ a load response realization and R the defined failure threshold. R represents the limit state of the structure and can be a random variable, but in this study adopts a deterministic value equals to a displacement 1% height of the pole.
This threshold R separates the safe part of the unsafe part of the process. If a process exceeds the threshold at least once, it is said to fail for "first passage". In reliability, the probability of

failure for the first passage and the first time of occurrence are usually analyzed. Here, we will focus only on the first result.

4.2.2 Dwell time above the failure threshold

That the studied parameter of the structure exceeds a limit value and fails for the first passage, does not imply that the structure fails immediately after the event (Shinozuka and Wu, 1988). We can think that the failure is due to accumulation of damage, and a way to measure it is to count the total time that the structure is in the undesirable zone, i.e. the dwell time of a parameter over the defined failure threshold. However, this value does not provide information about how much a structure parameter exceeds the threshold.

4.2.3 Extreme Value Distribution

Sometimes only the maximum values of a parameter (load or response) are recorded in a certain period of time. The corresponding probability distribution is known as extreme value distribution, and a high level of response is particularly significant for the estimation of structural safety (Melchers and Beck, 2018). A local maximum of a stochastic process S is defined as the value of $S(t)$ such that $S'(t) = 0$ and $S''(t) < 0$. Local maxima are peaks in a typical realization of $S(t)$ and depend on the magnitude of these with respect to the limit state. In addition, they provide information on the magnitude of the damage.

4.2.4 Crossing Rate

Considering the limit state function $G(t) = R(t) - S(t)$, where $R(t)$ is the strength and $S(t)$ the value of a studied time variable parameter of the structure, we establish the probability $P(G(t) \leq 0)$ as crossing problem. The time in which G is zero for first time is called "failure time" and the event is a "failure for the first passage". Counting the number of up-crossings through a threshold "a" in a time interval Δt, the up-crossing rate ν_a^+ will be: $\nu_a^+ = \dfrac{up - crossing}{\Delta t}$.

As mentioned earlier, the collapse of the structure may not occur due to the first crossing event. However, it does have special significance in dynamic problems that involve fatigue.

4.2.5 Time integrated displacement

The classical approach is to consider the integration transferred to the load or load effect process, which is then assumed to be representable, over the total time period, by an extreme value distribution.This means the load effect has been turned from a process into a random variable. Also, the resistance is assumed to be time invariant. Then the problem becomes time-invariant, and it is possible to perform the analysis in a classical way (Melchers and Beck, 2018).

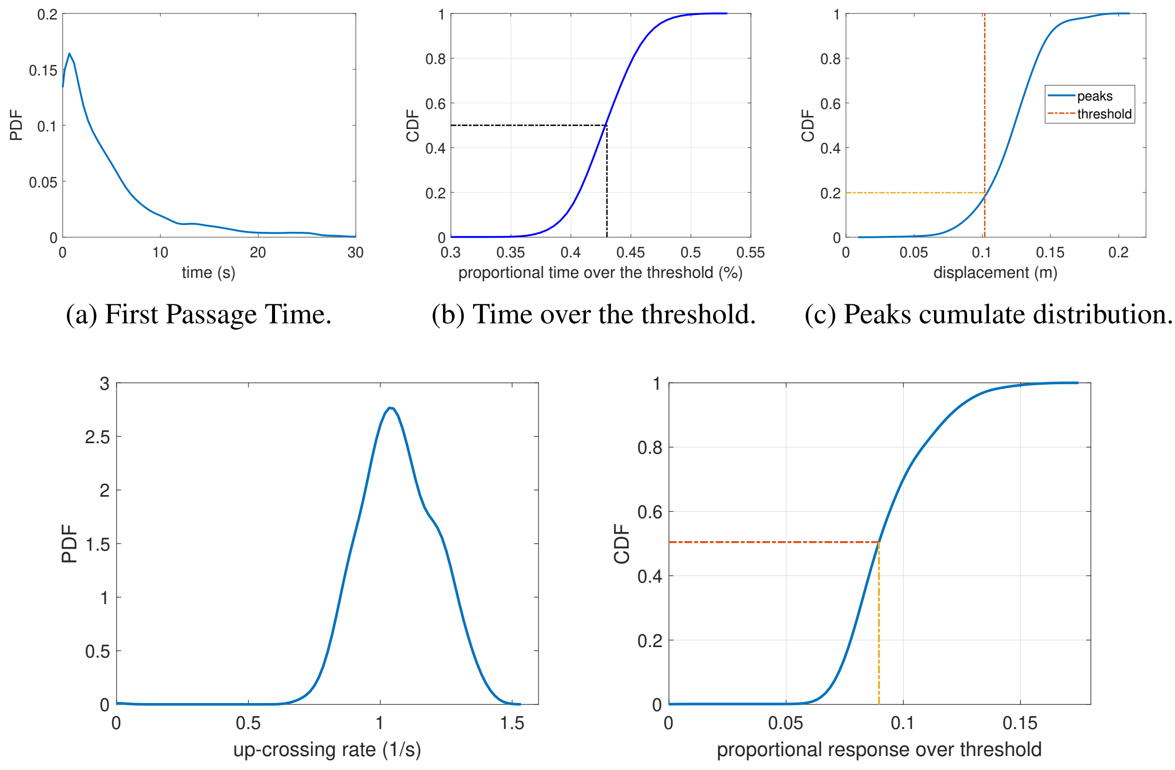
In the time-integrated approach, the whole lifetime [0, tL] of the structure is considered as a single unit, and all statistical properties of all random variables must relate to this lifetime. The probability distributions of interest are the distribution of maximum responses for loads and the minimum resistances in a lifetime structure. Simple comparison of the maximum load and the minimum resistance is, however, not appropriate since it is highly unlikely that the occurrence of the lifetime maximum load will coincide with the lifetime minimum resistance. The time-integrated approach is based on the concept of applying a loading system to the structure at regular intervals in time. In this case the probability of failure of the structure may be considered simply a function of the number N of statistically independent loading applications to cause failure.

5 RESULTS

From the described criteria, the limit state function $G(\cdot)$ was determined in each case. For this, for the deterministic failure wind velocity a dynamic stochastic wind load was performed and the displacement of the top of the pole was analyzed in a model under this dynamic load. If this displacement is greater than 1% of the value of the height of the pole H, it is a failure condition. With the Monte Carlo Method, the analysis of different parameters was made from the distribution of the displacements, and a $G(\cdot)$ function was defined for each criterion. Finally, these functions were applied to the results from the models under wind loads generated from a range of wind velocities and the different fragility curves were found.

5.1 Analysis for Deterministic Failure Wind Velocity

For the deterministic model with wind load from the standard, it was determined that the speed at which the limit state is reached is 27.6 m/s. A number of 1000 stochastic dynamic wind loads were generated with $V = 27.6$ m/s as basic wind velocity and then they were applied to the described pole model. Pole top displacements were obtained and analyzed based on the criteria listed. Figure 2 shows the different analysis.



(a) First Passage Time. (b) Time over the threshold. (c) Peaks cumulate distribution.

(d) Upcrossing rate (e) Proportional time integrated displacement.

Figure 2: Dynamic stochastic model with V =27.6 m/s. Analysis of different failure criteria.

Figure 2a shows the time distribution of the first passage, whose shape fits an exponential distribution $1-e^{-\nu t}$, as mentioned in Benaroya et al. (2005). Failure probability for first passage criterion for this wind velocity is high, and we conclude that it is a very strict criterion for the proposed failure definition. It also studied how much time the displacements dwelled above the threshold (Figure 2b) and the relationship with respect to the total simulation time was calculated. It was found that the dwell time above the failure threshold has an average of 43% and this value was adopted as the limit for this criterion. Figure 2c shows the cumulative distribution of extreme values or peaks of a realization. For 27.6 m/s, about 80 % of the extreme values are above the defined failure threshold, and this is the condition we adopt for this case. The up-crossing rate distribution for deterministic failure wind velocity is shown in Figure 2d. It can be observed that this value exhibits a approximate mean value of 1/s. However, this limit cannot be adopted as a single criterion given that for wind velocities higher than 27.6 m/s the displacement crossing rate is is lower and it is seen that most of the displacements remain above the threshold. To identify whether the displacements are above or below it, this criterion was combined with the extreme value distribution. It is proposed that if the mean of the extreme values is above the threshold, then the realization happens mostly in the region above the limit and then, it is in a failure condition. Finally, the distribution of the time integral of the displacements above the threshold with respect to the total displacements was analyzed. It was found that, for the studied velocity, the mean value is a 9% of this proportion and then, this is the adopted value for the failure criterion.

In what follows, the limit state function $G(\cdot)$ is listed for each criterion:

- $G_{first\,passage} : \delta_{threshold} - \delta(t)$
- $G_{dwell\,time} =: Time\,over\,threshold \geq 43\%\,total\,time$
- $G_{extreme\,values} =: P(\delta_{peaks}(t) \geq \delta_{threshold}) \geq 0.8$
- $G_{crossing\,rate} : \nu^{+} > 1/s \cap \mu_{peaks} > \delta_{threshold}$
- $G_{integrated\,displacement} : P\left(\frac{\int_t \delta_{up}(t)dt}{\int_t \delta(t)dt} \geq 0.09\right) > 0.5$

where $\delta(t)$ is the pole top displacement as a function of time, $\delta_{threshold}$ is the allowable maximum displacement for the failure of the static model, $\delta_{peaks}(t)$ is the extreme value of the displacement at time t, $\delta_{up}(t)$ is the displacement above $\delta_{threshold}$.

5.2 Fragility Curves

Based on the limits proposed above for each criterion, an analysis was performed por wind velocities in a range of 18 m/s to 31.5 m/s. The fragility curves were constructed for each case and are depicted in Figure 3. From the plot, it is apparent that the curve corresponding to the failure criterion of the first passage yields high probabilities for wind velocities less than 27.6 m/s, resulting in a quite conservative criterion. For the other criteria, the curves are around the line of deterministic failure being the curve of the crossing rate criterion the leftmost and with smoother slope. The criteria that account for the dwell time over the threshold, the extreme value distribution and the proportion of accumulated displacement in time, result in rather steep curves, the latter lying between the other two. This is due to the fact that the last criterion considers both the dwell time as the magnitude of the displacements that overpass the prescribed limit. Then, it is concluded that the criterion of the time integrated response is the most adequate

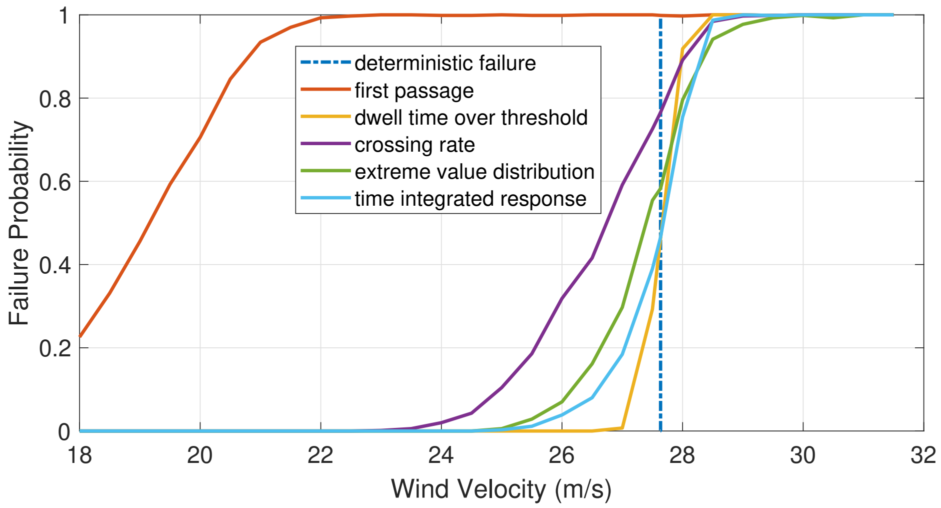


Figure 3: Fragility curves.

since it deals with more quantity of information and the fragility curve approximates better to the quasi-static model of the standard.

6 CONCLUSIONS

The standard (CIRSOC 102, 2005) suggests procedures to simplify the dynamic effect of the load by means of different coefficients. If the aim is to assess the reliability of the structure under the wind load, the stochastic nature of the response due to the random load cannot be disregarded. Thus, a failure criterion applied to a response of the structure under an equivalent load could be misleading. The equivalent criterion to the failure of a quasi-static model is the first passage, but, as was observed above, the probability of failure for the reference model is very close to 1 and consequently, high probability of failure is found for lower velocities. Other criteria were explored for the random wind load which are related to the fatigue phenomenon: dwell time, extreme value distribution, crossing rate and proportion of the response above the threshold accumulated in time. In order to define the corresponding limit state functions, the dynamic response at the top of a pole clamped at the ground under stochastic wind load was studied using the velocity in which the deterministic quasi-static model fails. This test was carried out using the Monte Carlo method with $N = 1000$ realizations to perform the statistical analysis. For the crossing rate criterion, that is, how many times per unit of time the realization values goes from a value below the limit to surpass it, it was found that it cannot be taken as an isolated parameter and it should be combined with other criterion. Not only it is relevant to know the crossing rate but also if the displacement remains above or below the prescribed threshold when the number of crossings is less than 1/s. To determine whether the realization crosses and stays above the limit, it was proposed the failure condition that assesses if the mean of the peak distribution mean surpasses that value. Once the limits were prescribed, these functions were applied to the dynamic response of the pole under stochastic wind in a range of reference velocities ranging from 18 m/s and 31.5 m/s, yielding the fragility curves for each criterion. It was verified that the curve found using the first passage criterion develops mainly in the range of velocities lower to 27.6 m/s while the remaining curves show variations around this velocity. The curve that accounts for the crossing rate exhibits higher probabilities for lower wind velocities and then, the results can be regarded as more conservative. For the other fragility curves, the criterion that considers the time integrated response lies between the dwell time and the extreme values distribution curves.

It could be concluded that such curve combines the dwell time and the extreme values, then being the most representative among all the analyzed criteria and the most adequate to describe the probability of the proposed service failure.

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