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THERMODYNAMIC-CONSISTENT MICROPLANE THEORY FOR QUASI BRITTLE MATERIALS AT HIGH TEMPERATURES

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Abstract. In this work, a temperature-dependent thermodynamically consistent Microplane Theory is developed at material level to simulate the mechanical failure behavior of quasi-brittle materials like concrete exposed to high temperature fields under dominant tensile stresses. As it is well known, the combination of mechanical and thermal phenomena strongly affects the material brittleness. A temperature-dependent failure criterion based on the smeared cracked approach depending on normal and shear micro-stresses acting on each microplane is implemented. Finally, with the aim to validate the developed constitutive model, obtained numerical results are contrasted against experimental examples from the bibliography.

1 INTRODUCTION

When quasi brittle materials like concrete are subjected to high temperatures in long term exposures, two effects turn evident. On the one hand, and as a result of the dehydration process of the cement paste, there is an irreversible degradation of two fundamental material properties: the elastic stiffness (thermal damage) and the material strength (thermal decohesion). On the other hand, a particular failure or fracture mode develops, the so-called concrete spalling, characterized by fracture planes parallel to the heated surface and perpendicular to the temperature flux. These events lead to severe degradations in the mechanical properties (cohesion, friction, strength and stiffness) and changes in their failure mechanisms.

While in tensile and low confinement regime quasi-brittle materials evidence deficiencies such as low strength and brittle response, compression and high confinement regimes are characterized by diffuse failure modes.

The Microplanes Theory seems to be adequate to predict numerical and macroscopically the failure behavior of these materials, with the advantage of incorporating microstructure information.

On the other hand, thermodynamically consistent theories are required to accurately simulate the complex behavior of quasi brittle materials. A well-established thermodynamically consistent approach of the Micropane Theory has been described by Carol et al. (2001) and Kuhl et al. (2001). This criterion is followed in this work to develop a thermodynamically consistent Microplane Theory for quasi brittle materials like concrete able to predict their failure behavior under high temperatures fields.

In Section 2, general basis of the proposed Microplane Theory are presented, in Section 3 the applied failure criterion and loading surface are described. Section 4 covers the numerical analysis with the proposed constitutive model. Finally, Section 5 states the conclusions.

2 THERMODYNAMICALLY CONSISTENT MICROPLANE THEORY

The Microplane approach originally proposed by Bažant and Oh (1983) consists in the formulation of constitutive laws at microplane level defining the mechanical behavior of planes (the microplanes) generically orientated. Then, the macroscopic response shall be achieved through the consideration of appropriated thermodynamically consistent homogenization process over the responses in all microplanes.

2.1 Kinematic assumptions

Assuming kinematic constraints, the normal and tangential strains at microplane level (ε_N and $\boldsymbol{\varepsilon}_T$, respectively) are computed by means the following relationships

$$\varepsilon_N = \boldsymbol{N} : \boldsymbol{\varepsilon}^{mac} \quad , \quad \boldsymbol{\varepsilon}_T = \boldsymbol{T} : \boldsymbol{\varepsilon}^{mac} \tag{1}$$

being $\boldsymbol{\varepsilon}^{mac}$ the macroscopic strain tensor projected on a microplane of normal direction $\boldsymbol{n}$. The projection tensors are defined as

$$\boldsymbol{N} = \boldsymbol{n} \otimes \boldsymbol{n} \quad , \quad \boldsymbol{T} = \boldsymbol{n} \cdot \boldsymbol{I}^{sym} - \boldsymbol{n} \otimes \boldsymbol{n} \otimes \boldsymbol{n} \tag{2}$$

being $\boldsymbol{I}^{sym}$ the symmetric part of the fourth-order identity tensor.

In the elasto-plastic regime and assuming small strains, both macro- and microscopic strains are computed according to the Prandtl–Reuss additive decomposition. Particularly, at microplane level, normal and tangential strain rates are obtained as

$$\dot{\varepsilon}_N = \dot{\varepsilon}^e_N + \dot{\varepsilon}^p_N \quad , \quad \dot{\boldsymbol{\varepsilon}}_T = \dot{\boldsymbol{\varepsilon}}^e_T + \dot{\boldsymbol{\varepsilon}}^p_T \tag{3}$$

where the supra-indexes e and p denote elastic and plastic components, respectively.

2.2 Thermodynamically consistent homogenizations

This section is aimed at describing the fracture energy-based plasticity formulation which relates the normal and tangential micro-stress components with the corresponding microstrains. Assuming the macro free-energy potential as the integral of the micro free-energy on a spherical region of unit volume Ω, the following micro-macro free-energy relationship is proposed

$$\psi^{mac} = \frac{3}{4\pi} \int_{\Omega} \psi^{mic} \mathrm{d}\Omega \tag{4}$$

being ψ^{mic}=$\psi^{mic}(\varepsilon_N^e, \boldsymbol{\varepsilon}_T^e, \kappa)$ the free-energy potential at microplane level, expressed in terms of the strain components and the scalar internal variable κ. Its evolution law, regarding the kinematic projection of Eq. (1), is given by

$$\dot{\psi}^{mic} = \left(\boldsymbol{N}\sigma_N + \boldsymbol{T}^T \cdot \boldsymbol{\sigma}_T\right) : \dot{\boldsymbol{\varepsilon}}^{mac} - D \tag{5}$$

with the constitutive micro-stresses computed as

$$\sigma_N := \frac{\partial \psi^{mic}}{\partial \varepsilon_N} \quad , \quad \boldsymbol{\sigma}_T := \frac{\partial \psi^{mic}}{\partial \boldsymbol{\varepsilon_T}} \; , \tag{6}$$

and the microscopic dissipation D, that satisfies the following condition

$$D = -\phi^{mic}\dot{\kappa} \geq 0 \quad , \quad \phi^{mic} = \frac{\partial \psi^{mic}}{\partial \kappa} \quad \rightarrow \quad \dot{\phi}^{mic} = \bar{H}\dot{\kappa} \tag{7}$$

being ϕ^{mic} the dissipative stress and $\bar{H}$ the hardening/softening modulus.

According to Coleman and Noll Coleman and Noll (1963) and Coleman and Gurtin (1967), the Clausius–Duhem inequality, the homogenization of the microplanes energy leads to the definition of the macroscopic stress tensor in terms of the microscopic stress components

$$\boldsymbol{\sigma} = \frac{\partial \psi^{mac}}{\partial \boldsymbol{\varepsilon}} = \frac{3}{4\pi} \int_{\Omega} \boldsymbol{N}\sigma_N + \boldsymbol{T}^T \cdot \boldsymbol{\sigma}_T d\Omega \, . \tag{8}$$

To avoid the difficulties involved in the analytical solution of Eq. (8), Bažant and Oh (1985) proposed integration techniques, which dealt with the numerical solution of the integral over all possible spatial directions by a weighted sum over a finite number of microplanes

$$\boldsymbol{\sigma} \approx \sum_{I=1}^{n_{mp}} \left[\boldsymbol{N}^I {\sigma_N}^I + \boldsymbol{T}^{T,I} \cdot {\sigma_T}^I\right] w^I \tag{9}$$

being n_{mp} the adopted number of microplanes and w^I the corresponding weight coefficients.

2.3 Microplane elasto-plastic constitutive formulation

Assuming a decoupled form of the microscopic free-energy potential corresponding to the elasto-plastic regime, it can be expressed as the sum of both elastic and plastic counterparts

$$\psi^{mic} = \psi^e \left(\varepsilon_N^e, \boldsymbol{\varepsilon}_N^e\right) + \psi^p \left(\kappa\right) \; . \tag{10}$$

The elastic contribution is computed as

$$\psi^e = \frac{1}{2} E_N \left(\varepsilon_N^e\right)^2 + \frac{1}{2} E_T \boldsymbol{\varepsilon}_T^e \cdot \boldsymbol{\varepsilon}_T^e \tag{11}$$

where the elastic normal and tangential micro-modules, E_N and E_T, are defined according to Leukart (2005) as

$$E_N = 3K \quad , \quad E_T = \frac{10}{3G} - 2K \tag{12}$$

being K and G the bulk and shear macroscopic moduli, respectively. Regarding Eqs. (6) and (11), the micro-strains result

$$\sigma_N = E_N \varepsilon_N^e \quad , \quad \boldsymbol{\sigma}_T = E_T \boldsymbol{\varepsilon}_T^e \,. \tag{13}$$

The evolution laws of the plastic strain components and the internal variable, under consideration of a convex yield function Φ and a plastic potential Φ^*, result

$$\dot{\varepsilon}_N^p = \dot{\lambda} \frac{\partial \Phi^*}{\partial \sigma_N} \quad , \quad \dot{\boldsymbol{\varepsilon}}_T^p = \dot{\lambda} \frac{\partial \Phi^*}{\partial \boldsymbol{\sigma}_T} \quad , \quad \dot{\kappa} = \dot{\lambda} \frac{\partial \Phi^*}{\partial \phi^{mic}} \tag{14}$$

being $\dot{\lambda}$ the plastic multiplier rate. Complementary, the classical Kuhn–Tucker loading/unloading and the consistency conditions must be considered

$$\Phi \leq 0 \quad , \quad \dot{\lambda} \leq 0 \quad , \quad \Phi \dot{\lambda} = 0 \quad , \quad \dot{\Phi} \dot{\lambda} = 0 \,. \tag{15}$$

3 MICROPLANE MODEL FOR CONCRETE UNDER HIGH TEMPERATURE

In this section a temperature-dependent Microplane constitutive model is proposed based on the Microplane Theory for plain concrete at ambient temperature given in Vrech et al. (2016).

The proposed mathematical expression of the parabolic failure criterion for plain concrete dependent on high temperature effects in terms of the normal and tangential microstresses σ_N and $\boldsymbol{\sigma}_T$, respectively, is

$$\Phi = \alpha(T) \left[\frac{3\left(f_c' + f_t'\right)}{8 f_t' f_c'^2} \right] \|\sigma_T\|^2 + \sigma_N - \beta(T) f_t' = 0 \tag{16}$$

with f_t' and f_c', the uniaxial tensile and compressive strengths, respectively, and T the absolute temperature value that strongly degrades the comparison stress f_t'. $\alpha(T)$ and $\beta(T)$ are the temperature-dependent functions computed according to

$$\alpha(T) = 1 + \gamma_1 \left(T - 20\right) \,, \tag{17}$$

$$\beta(T) = 1 - \gamma_2 \left(T - 20\right) \,. \tag{18}$$

In the post-peak regime, the yield criterion softens according to

$$\Phi_s = \alpha(T) \left[\frac{3\left(f_c' + f_t'\right)}{8 f_t' f_c'^2} \right] \|\sigma_T\|^2 + \sigma_N - \beta(T) \phi^{mic} = 0 \,. \tag{19}$$

The softening dissipative stress ϕ^{mic} is expressed in terms of the internal state variable κ and T. The strength degradation during post-peak processes is controlled by ϕ^{mic}. Figure (1) represents the variation of the maximum failure curve with increasing temperatures, from 20 to 600°C, represented in the stress space of coordinates σ_N-σ_T.

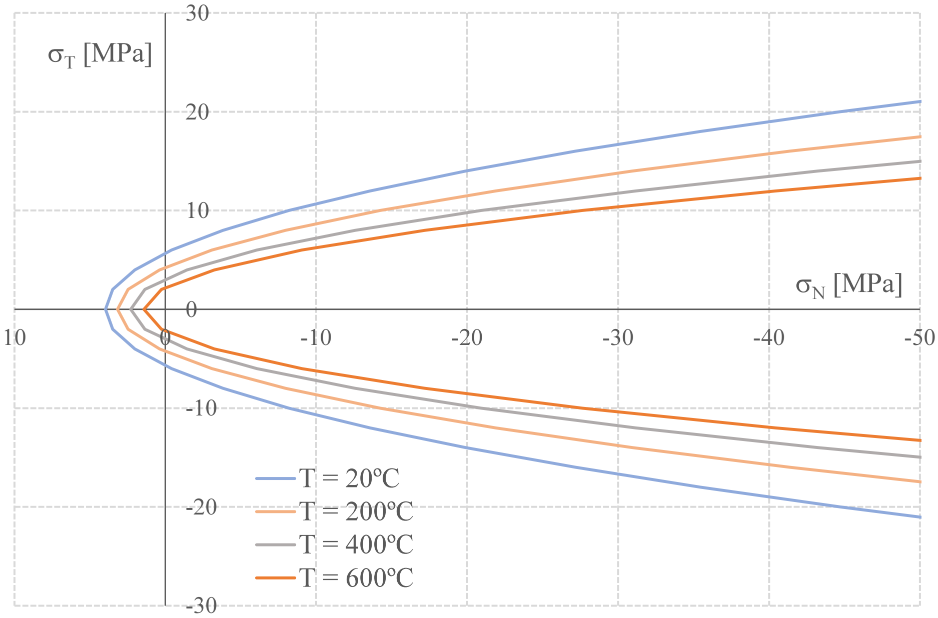

Figure 1: Microplane failure criterion degradation with increasing temperature

The dissipative stress, based on the Fracture Energy Theory, is obtained by

$$\phi^{mic} = f_t' \, \exp\left(\frac{-5h_T}{u_r \, G_I}\dot{\kappa}\right) \tag{20}$$

being G_I the fracture energy in mode I type of failure, u_r the maximum crack opening displacement and h_T the corresponding characteristic length, that depends on the absolute temperature T according to

$$h_T = h \exp\left[-0.00308(T-20)\right] \tag{21}$$

where h corresponds to the material characteristic length at 20°C. Analysing Eqs. (19) and (20), it should be noted that the comparison strength decreases due to both, mechanical and thermal effects in softening regime.

3.1 Temperature dependent elastic properties

As it was proposed by Ripani et al. (2014), based in a large experimental database, the following expressions for the Young modulus E and Poisson coefficient ν should be applied

$$E = E_0 \left(1 - \alpha_E T\right) \quad , \quad \nu = \nu_0 \left(1 - \alpha_\nu T\right) \tag{22}$$

being E_0 and ν_0 the elasticity modulus and Poisson coefficient at 20°C, respectively. Whereas α_E= 0.0014 and α_ν= 0.0010 are degradation parameters. The approximations results for E and ν varying with T, are show in Figures 2 and 3, respectively.

4 NUMERICAL ANALYSIS

This section illustrates the main features and capabilities of the proposed formulation comparing some numerical results against experimental data available in the literature. Uniaxial tensile tests were selected to evaluate the predictive capabilities of the proposed model in mode

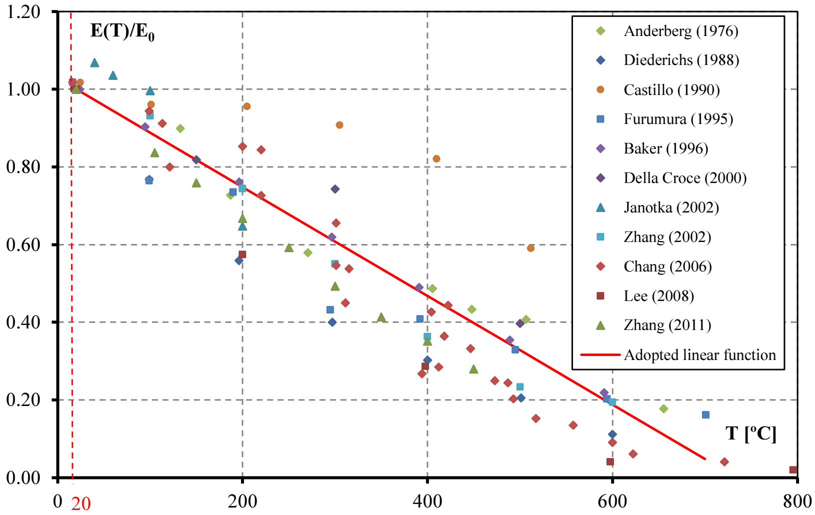

Figure 2: Experimental data and approximated concrete elasticity modulus function, depending on the acting temperature (Ripani et al., 2014).

I type of fracture. Figure 4 shows the considered boundary conditions of the tests pursued in this work.

The considered tests by Li et al. (1998) involve prismatic samples specimens with the dimensions 500 x 100 x 20 mm^3 under plane strain conditions. The two-dimensional formulation by Park and Kim (2003) instead of the spherical one was considered for the microplane distribution, see Figure 5.

The model parameters and material properties are given in Table 1

Table 1: Model parameters and concrete properties by Li et al. (1998).

γ_1	0.0025
γ_2	0.0011
Elasticity modulus $-$ E [GPa]	39.5
Poisson modulus $-$ ν	0.2
Compressive strengh $-$ f'_c [MPa]	35
Tensile strengh $-$ f'_t [MPa]	3.8
Tensile rupture displacement $-$ u_r [mm]	0.151
Crack spacing $-$ h [mm]	100

The results of model predictions and experimental data for the uniaxial tensile test in terms of stress–strain diagrams are reported in Figure 6. In first place, plane concrete at 20°C has been analysed. As can be observed, the comparison with the experimental results shows very good agreement regarding peak and residual strengths, as well as pre- and post-peak behaviors. Subsequently, the numerical response is studied for temperatures of 200, 400 and 600°C. σ-ε curves are consistent with the expected results are obtained. The lack of experimental results makes their comparison impossible. The material degradations due to coupled thermo-mechanical effects causes an evident decrease in the tensile strengths and a more extended softening regimen. In the present numerical analysis, 84 microplanes were implemented. With the aim to understand the thermo-mechanical behavior of each one of them and their contribution to the global response, Figure 7 shows σ-ε curves corresponding to one microplane located perpendicular to the tensile displacement (called MP1) and one tangential to it (designated MP21). At room tem-

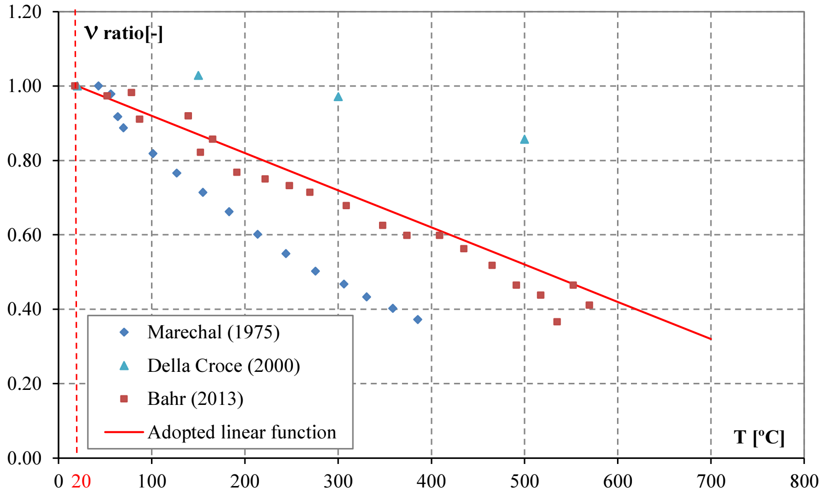

Figure 3: Experimental data and approximated Poisson ratio function, depending on the acting temperature (Ripani et al., 2014).

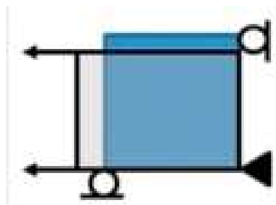

Figure 4: Load configuration and restraint conditions for uniaxial tension test

perature, the notorious collaboration of MP1 is demonstrated against the scarce participation of the tangent one. As expected, as the temperature increases, its contribution tends to increase.

5 CONCLUSIONS

In this work a thermodynamically consistent temperature-dependent Microplane constitutive model for quasi-brittle materials like concrete, has been proposed. The model formulation, founded on a macroscopic smeared crack approach and on the full thermodynamic consistency, considers a parabolic failure criterion that reproduces the degradation of the material properties and the softening behavior caused by the application of high temperature fields.

The results in this work illustrate the capabilities of the thermodynamically consistent Microplane plasticity to reproduce the macroscopic behavior of concrete under high temperature fields in terms of Stress-Strain curves.

This work consists of a first approximation stage and it is subsequently planned to consider the directional temperature variation regarding microplane directions and carrying out the comparison with experimental results, also covering all mechanical load spectra.

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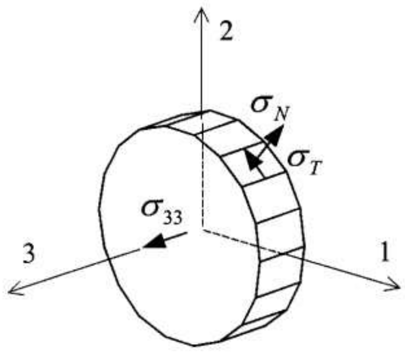

Figure 5: Two-dimensional Microplane model proposed by Park and Kim (2003)

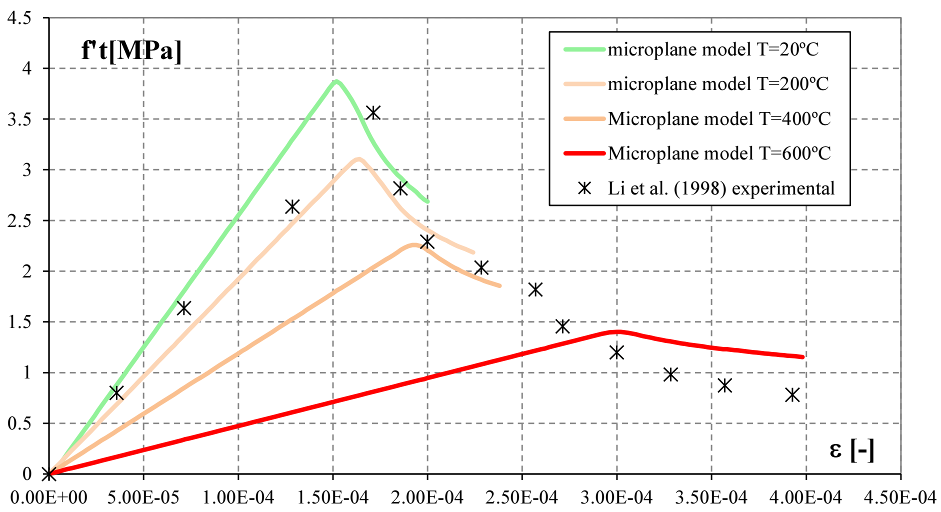

Figure 6: Tensile tests: verification with experimental data Li et al. (1998)

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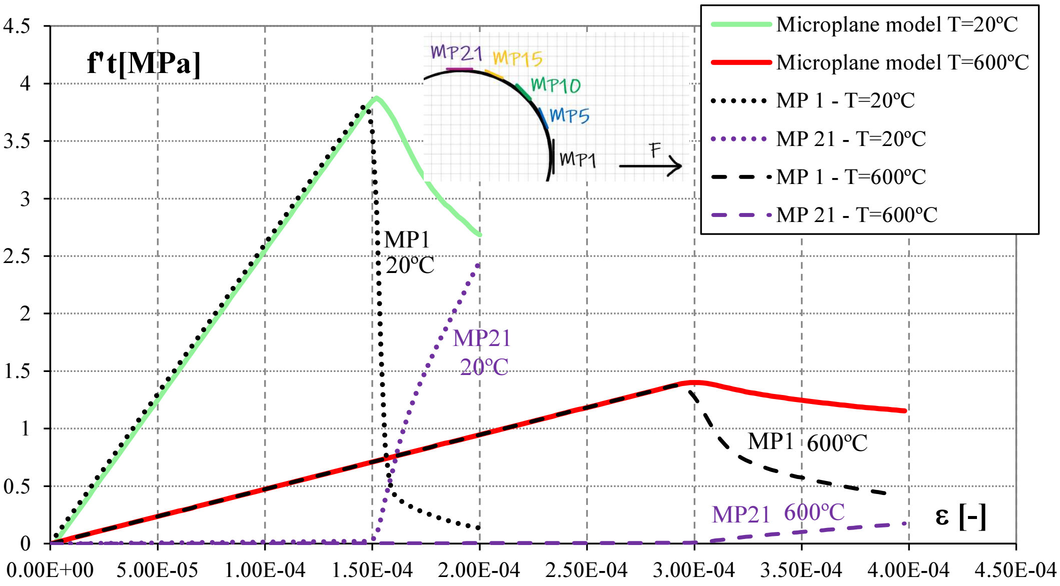

Figure 7: Microplanes contribution to the global response for tensile test

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