Asociación Argentina
de Mecánica Computacional

Mecánica Computacional Vol XXXIX, págs. 167-176 (artículo completo)
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Bahía Blanca, 1-4 Noviembre 2022

DESIGN OF MAGNETORHEOLOGICAL DYNAMIC NEUTRALIZER FOR THE VIBRATION CONTROL UNDER BROADBAND EXCITATION AND DIFFERENT TEMPERATURES

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Keywords: Vibration Control, Viscoelastic Vibration Neutralizers, Magnetorheological Elastomers.

Abstract. Viscoelastic dynamic neutralizers can successfully reduce the vibration of a mechanical structure called primary system. However, the viscoelastic materials used on those devices are sensible to temperature changes and this can detune the dynamic neutralizer, reducing its vibration control capabilities. Therefore, this work proposes a magnetorheological dynamic neutralizer which is optimally designed to reduce the vibration of a multi degree of freedom primary system under broadband excitation. At the same time, by changing the magnetic field applied to the magnetorheological material it is possible to mitigate the temperature changes in the neutralizer and its detuning. The modal parameters and frequency response function of the system are computed using a finite element model. The optimum design of the magnetorheological neutralizer is achieved by reducing the frequency response due to the optimal neutralizer parameters, natural frequency and position of the neutralizer on the primary system. The detuning is analyzed by increasing the operating temperature and by changing the magnetic flux density on the magnetorheological elastomer. This is provided by an electromagnet whose magneto-electrostatic behavior is simulated using a finite element method also. The results show a significant reduction in the neutralizer natural frequency with the rise of the material temperature and an inverse effect with the increase of the magnetic field. Furthermore, the detuned frequency response of the compound system, primary and neutralizer, is partially corrected for a higher magnetic flux density.

1 INTRODUCTION

Smart materials are created to respond in a controlled way with changes in their properties by external excitation. One type of those intelligent structures are the magnetorheological materials which modify their rheological properties as a function of the applied magnetic field (Ahamed et al., 2018). According to the components of the mixture, rheological materials can be categorized into fluids, plastomers, polymeric gels and elastomers. Magnetorheological Elastomers (MRE) does not suffer as significantly from contamination and particulate sedimentation as the magnetorheological fluids, and can withstand greater absolute stiffness (Jaafar et al., 2021). The MREs are created by the combination of a rubber or an elastomer matrix with ferromagnetic particles. Also, they can be classified as isotropic, if the curing process occurred in the absence of a magnetic field, or anisotropic, if the sample was cured with an external magnetic field (Bastola & Hossain, 2020). The anisotropic MRE has the benefit of higher influence of the magnetic field on the dynamic characteristics of the elastomer which is also affected by the number of magnetic particles (Nam et al., 2021).

Viscoelastic materials such as elastomers presents high energy dissipation capabilities which enable the creation of viscoelastic dynamic neutralizers. Those devices introduce high mechanical impedance in the primary system in which is desirable to reduce the vibration. Den Hartog (1985) introduced a damped vibration neutralizer for a single-degree-of-freedom system with a broadband vibration attenuation. Further, Espíndola et al. (2010) proposed an optimum design of neutralizer based on the concept of generalized equivalent parameters and fractional derivatives for the parametric model of the viscoelastic material. Later, Bavastri et al. (2014) and Febbo et al. (2016) expanded the viscoelastic neutralizer design methodology for a cubic nonlinear single-degree-of-freedom system and presented the neutralizer resonant frequency modification with the operating temperature detuning. Moreover, an optimal design methodology of the vibration neutralizers for passive vibration control of multi-degrees-of-freedom (MDOF) structures of all shapes and sizes, for a given temperature, considering the neutralizer natural frequency and position was proposed by Silva and Bavastri (2019).

Magnetorheological elastomers have the advantage of modifying their mechanical properties with the applied magnetic field when compared to simple viscoelastic materials. The increase in current in the electromagnet induce a higher magnetic flux density on the MRE and, consequently, a greater resonance frequency. This feature allows the creation of adaptive tuned vibration absorbers such as those proposed by Deng and Gong (2008) and Komatsuzaki et al. (2016). Later, Choi and Wereley (2022) developed a MRE based absorber for propeller aircraft seat. Furthermore, the magnetorheological elastomer can be used to create a vibration isolator considering simultaneously the magnetic field and preloading effect (Bastola & Li, 2018). Leng et al. (2021) successfully designed a semi-active vibration control of offshore platform with MRE-base isolation system.

Despite those advances, the mechanical properties of elastomers are sensible to temperature variations and, consequently, can detune the magnetorheological neutralizer. Zhang et al (2011) demonstrated that the dynamic properties of MRE can modify significantly with the temperature. Also, the temperature influences the initial and the magnetic-induced modulus of the magnetorheological elastomers (Wen et al., 2020). Silva et al. (2021) characterized a MRE sample as a function of frequency, temperature, and magnetic field, demonstrating that the viscoelastic behavior with the magnetic dipole model can successfully predict the dynamic properties of the MRE.

This paper aims to apply the characterized magnetorheological elastomer by Silva et al. (2021) as a vibration neutralizer and study the influence of temperature and magnetic field in the vibration control. The optimum design methodology of the viscoelastic vibration neutralizer demonstrated in Silva and Bavastri (2019) for a multi-degree-of-freedom system is used to create a passive vibration control of a plate clamped on both ends with a magnetorheological absorber. Once the neutralizer optimal parameters are acquired (the neutralizer mass, natural frequency and position), the temperature and magnetic field are changed in order to identify their effect on the vibration control capabilities. The rigid mass of the magnetorheological neutralizer is formed by an electromagnet responsible to generate the magnetic flux density on the MRE which is simulated using a finite element method.

2 MAGNETORHEOLOGICAL ELASTOMERS

A magnetorheological elastomer (MRE) can be modeled by the combination of the behavior of a viscoelastic material and the changes caused by the application of a magnetic field on the elastomer. Also, the magnetorheological elastomers are sensible to frequency and temperature changes. Therefore, in order to model the frequency dependence of the mechanical properties of the viscoelastic material, the fractional-order-derivatives differential equations are an efficient method (Bagley & Torvik, 1983). The complex shear modulus of a viscoelastic element can be described by the three-parameter model as

$$G^*(\omega) = G_0 + G_1(\omega i)^\alpha \tag{1}$$

where G_0, G_1 and α are obtained experimentally, i and ω represent the imaginary unit and angular frequency respectively. According to Ferry (1980) the change in temperature induce a shift in the shear modulus curve as a function of frequency. Then, the influence of the temperature can be added by the reduced frequency variable $\omega_r = \omega\alpha_T(T)$. The parameter $\alpha_T(T)$ is defined as the shift factor and can be calculated by the WLF (William-Landel-Ferry) function as

$$\log \alpha_T(T) = -\theta_1 \frac{(T - T_0)}{(\theta_2 + T - T_0)} \tag{2}$$

where T and T_0 are the operating and reference temperature, θ_1 and θ_2 are material characteristic parameters experimentally determined. Therefore, the complex shear modulus can be written as

$$G^*(\omega, T) = G_0 + G_1[i\omega\alpha_T(T)]^\alpha \tag{3}$$

The magnetic field applied to the material increase the shear modulus of the MRE. Although, the relationship between the mechanical properties is nonlinear and can saturate, generating a minor change on the shear modulus. The magnetic dipole moment quantifies the contribution of the interaction between ferromagnetic particles in relation to the applied magnetic field. According to Shen et al. (2004), the dipole moment is given by

$$m(H) = \frac{4}{3}\pi r^3 \mu_0 \mu_r \chi H_0 \left[\frac{1}{1 - (4/3)\chi\zeta(r/d_0)^3}\right] \tag{4}$$

where d_0 is the initial distance between the particles forming the dipole, H_0 represents the intensity of the external magnetic field, ζ is the Apéry constant, χ is the magnetic susceptibility of magnetic particles and r is the particle mean radius. Also, φ is the magnetic particles volume in the elastomer matrix, μ_0 and μ_r are the magnetic permeability of free space and relative magnetic permeability of the MRE. The increment of the dynamic shear modulus as a function of an applied magnetic field is given by (Silva et al., 2021)

$$\Delta G(H_0) = \frac{9}{2}\frac{\varphi\zeta m^2}{d_0^3\pi^2 r^3\mu_0\mu_r} \tag{5}$$

Combining the complex shear modulus of a viscoelastic material with the increment induced by the magnetic field, the complex shear modulus of the MRE can be computed as

$$G_{MRE}^*(\omega,T) = G^*(\omega,T) + \Delta G \tag{6}$$

Assuming that $\Re$ and $\Im$ represents the real and imaginary part of the complex number and defining the dynamic shear modulus as $G_{MRE}(\omega,T) = \Re[G_{MRE}^*(\omega,T)]$ and the loss factor as $\eta_{MRE}(\omega,T) = \Im[G_{MRE}^*(\omega,T)]/\Re[G_{MRE}^*(\omega,T)]$ of MRE, the equation (6) can be written as a function of those parameters by

$$G_{MRE}^*(\omega,T) = G_{MRE}(\omega,T)[1 + i\eta_{MRE}(\omega,T)] \tag{7}$$

3 MAGNETORHEOLOGICAL DYNAMIC NEUTRALIZER

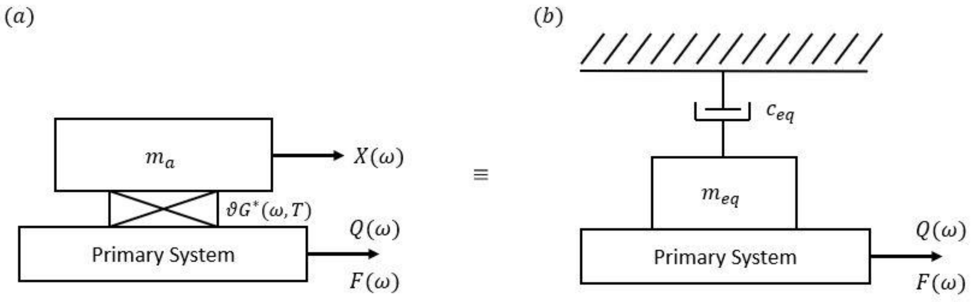

Figure 1: (a) Model of the magnetorheological neutralizer. (b) Dynamically equivalent model

A single degree of freedom magnetorheological dynamic neutralizer, as presented in Figure 1(a), is formed by the absorber mass m_a and the magnetorheological element connecting this mass to the primary system. The complex stiffness $K^*(\omega)$ of the magnetorheological element is described by

$$K^*(\omega,T) = \vartheta G_{MRE}^*(\omega,T) \tag{8}$$

where ϑ corresponds to the geometric factor. This factor is influenced by the geometry and the load type in the viscoelastic element (compression or shear). For a shear deformation the geometric factor of a uniform piece of the viscoelastic material is given by $\vartheta = A/h$ where A is the cross-sectional area and h is the height of the MRE. The dynamic stiffness in the base of the neutralizer is given by (Espíndola et al., 2010)

$$K_a(\omega,T) = \frac{F(\omega)}{Q(\omega)} = \frac{\omega^2 m_a \vartheta G^*(\omega,T)}{m_a\omega^2 - \vartheta G^*(\omega,T)} \tag{9}$$

Using the generalized equivalent parameters, the neutralizer can also be described according to the Figure 1(b). Assuming $r(\omega,T) = G(\omega,T)/G(\omega_a,T)$ and $\varepsilon_a = \omega/\omega_a$, the neutralizer dynamic stiffness can be described by the equivalent quantities as

$$K_a(\omega,T) = -\omega^2 m_{eq}(\omega,T) + i\omega c_{eq}(\omega,T) \tag{10}$$

where

$$m_{eq}(\omega,T) = \frac{-m_a r(\omega,T)\{\varepsilon_a^2 - r(\omega,T)[1+\eta^2(\omega,T)]\}}{[\varepsilon_a^2 - r(\omega,T)]^2 + [r(\omega,T)\eta(\omega,T)]^2}$$

$$c_{eq}(\omega,T) = \frac{m_a \omega_n r(\omega,T)\eta(\omega,T)\varepsilon_a^3}{[\varepsilon_a^2 - r(\omega,T)]^2 + [r(\omega,T)\eta(\omega,T)]^2} \tag{11}$$

The natural frequency ω_a of the neutralizer is defined as the frequency in which, in absence of damping, makes the denominator of the equation (9) equals to zero. Therefore,

$$\omega_a = \sqrt{\frac{\vartheta G_R(\omega,T)}{m_a}} \tag{12}$$

4 PASSIVE VIBRATION CONTROL FOR A MDOF SYSTEM

The equation of motion in the frequency domain for a multi-degree-of-freedom system with n generalized coordinates $\boldsymbol{Q}(\omega)$ driven by an external force $\boldsymbol{F}(\omega)$ with one inserted magnetorheological neutralizer is given by

$$\left[-\omega^2\widetilde{\boldsymbol{M}} + i\omega\widetilde{\boldsymbol{C}} + \boldsymbol{K}\right]\boldsymbol{Q}(\omega) = \boldsymbol{F}(\omega) \tag{13}$$

where $\boldsymbol{K}$ is the stiffness matrix of the primary system, $\widetilde{\boldsymbol{M}}$ and $\widetilde{\boldsymbol{C}}$ are the mass and damping matrices of the compound system defined as

$$\widetilde{\boldsymbol{M}} = \boldsymbol{M} + \boldsymbol{M_{eq}}$$

$$\widetilde{\boldsymbol{C}} = \boldsymbol{C} + \boldsymbol{C_{eq}} \tag{14}$$

where $\boldsymbol{M}$ and $\boldsymbol{C}$ represents the mass and damping matrices of the primary system and $\boldsymbol{M_{eq}}$ and $\boldsymbol{C_{eq}}$ are diagonal matrices containing the equivalent mass and damping of the neutralizer fixed in the r^{th} degree of freedom, for $r = 1, \ldots, n$. In order to reduce the computational effort, the neutralizer design is performed in the modal space by a transformation from the generalized coordinates to the principal coordinates, which can be represented by

$$\boldsymbol{Q}(\omega) = \widehat{\boldsymbol{\Phi}}\widehat{\boldsymbol{P}} \tag{15}$$

where $\widehat{\boldsymbol{\Phi}}$ is the $n\ x\ \hat{n}$ truncated normalized modal matrix for $\hat{n}$ as the number of modes in the frequency range of interest plus residues. Also, $\widehat{\boldsymbol{P}}$ is the principal generalized coordinates of the primary system. Then, the equation (13) can be rewritten as

$$\left(-\omega^2\left[\widehat{\boldsymbol{\Phi}}^{\boldsymbol{T}}\widetilde{\boldsymbol{M}}\widehat{\boldsymbol{\Phi}}\right] + i\omega\left[\widehat{\boldsymbol{\Phi}}^{\boldsymbol{T}}\widetilde{\boldsymbol{C}}\widehat{\boldsymbol{\Phi}}\right] + \left[\widehat{\boldsymbol{\Phi}}^{\boldsymbol{T}}\boldsymbol{K}\widehat{\boldsymbol{\Phi}}\right]\right)\widehat{\boldsymbol{P}} = \widehat{\boldsymbol{\Phi}}\boldsymbol{F}(\omega) \tag{16}$$

which can be further simplified by

$$\left(-\omega^2\left[\widehat{\boldsymbol{I}} + \widehat{\boldsymbol{\Phi}}^{\boldsymbol{T}}\boldsymbol{M_{eq}}\widehat{\boldsymbol{\Phi}}\right] + i\omega\left[\boldsymbol{\Gamma} + \widehat{\boldsymbol{\Phi}}^{\boldsymbol{T}}\boldsymbol{M_{eq}}\widehat{\boldsymbol{\Phi}}\right] + [\boldsymbol{\Lambda}]\right)\widehat{\boldsymbol{P}} = \widehat{\boldsymbol{\Phi}}\boldsymbol{F}(\omega) \tag{17}$$

where $\boldsymbol{\Gamma} = \widehat{\boldsymbol{\Phi}}^{\boldsymbol{T}}\boldsymbol{C}\widehat{\boldsymbol{\Phi}} = diag(2\xi_j\omega_j)$ for ξ_j as the modal damping ratio and ω_j the natural frequency of the j^{th} mode, with $j = 1, \ldots, \hat{n}$. Also, $\widehat{\boldsymbol{I}}$ represents the truncated identity matrix and $\boldsymbol{\Lambda} = diag(\omega_j^2)$ called the truncated spectral matrix. From equation (17), the compound system receptance can be defined as

$$\boldsymbol{H}(\boldsymbol{\omega}) = \frac{\boldsymbol{Q}(\omega)}{\boldsymbol{F}(\omega)} = \widehat{\boldsymbol{\Phi}}^{\boldsymbol{T}}\left(-\omega^2\left[\widehat{\boldsymbol{I}} + \widehat{\boldsymbol{\Phi}}^{\boldsymbol{T}}\boldsymbol{M}_{\boldsymbol{eq}}\widehat{\boldsymbol{\Phi}}\right] + i\omega\left[\boldsymbol{\Gamma} + \widehat{\boldsymbol{\Phi}}^{\boldsymbol{T}}\boldsymbol{M}_{\boldsymbol{eq}}\widehat{\boldsymbol{\Phi}}\right] + [\boldsymbol{\Lambda}]\right)^{-\mathbf{1}}\widehat{\boldsymbol{\Phi}} \tag{18}$$

Den Hartog (1980) introduced a parameter called mass ratio σ as the absorber mass divided by the primary system mass in order to define the optimum absorber mass. This value is usually assumed between 0.1 to 0.25 for a single-degree-of-freedom system. Further, Silva and Bavastri (2019) demonstrated that for a multi-degree-of-freedom system the mass of a single neutralizer can be computed by

$$m_a = \frac{1}{\hat{n}}\sum_{j=1}^{\hat{n}}\frac{\sigma_j m_j}{\widehat{\Phi}_{r_i j}^2} \tag{19}$$

where σ_j and m_j are the mass ratio and modal mass of the j^{th} vibration mode. Moreover, $\widehat{\Phi}_{r_i j}^2$ is the modal matrix value for the r_i^{th} degree of freedom in which the neutralizer is attached.

5 NEUTRALIZER OPTIMIZATION

The optimal neutralizer design is achieved by finding the neutralizer mass, equation (19), that minimizes the Euclidean norm of the principal coordinates $\widehat{\boldsymbol{P}}(\boldsymbol{\omega})$, equation (16), in the frequency band of interest. Hence, the optimization problem for the steady state solution of a single magnetorheological neutralizer has the form

$$f_{obj}(\boldsymbol{x}) = \left\| \max_{\omega_1<\omega<\omega_2} \left|\widehat{\boldsymbol{P}}(\boldsymbol{\omega},\boldsymbol{x})\right| \right\|_2 + w m_a \tag{20}$$

Subject to $\omega_a^L < \omega_a < \omega_a^U$. In which $\boldsymbol{x} = (\omega_a, \widetilde{\boldsymbol{x}})$ is the design vector, ω_a and $\tilde{x}$ are the absorber natural frequency and position. Also, ω_1 and ω_2 represents the lower and upper limits of the frequency band and the parameters ω_a^L and $\omega_a^U$are the minimum and maximum value of the neutralizer natural frequency. For a given primary system total mass m_{sp} , the weighting factor w, which gives both parts of the objective function the same order of magnitude, can be computed by

$$w = \frac{\left\| \max_{\omega_1<\omega<\omega_2} \left|\widehat{\boldsymbol{P}}(\boldsymbol{\omega},\boldsymbol{x})\right| \right\|_2}{0.02 m_{sp}} \tag{21}$$

The nonlinear optimization is performed by a differential evolution algorithm which can be classified as an evolutionary computation to find the global minimum of a multivariate function. The modal parameters of the primary system are extracted using the PyAnsys library and the optimization method is available at the SciPy library in the Python programing language (Virtanen, 2020).

6 NUMERICAL EXAMPLE

The primary system consists of a steel plate clamped at both ends. The plate dimensions are $0.3 \times 0.01 \times 0.6$ m with a total mass of $14{,}13$ kg. The modal analysis was performed in the ANSYS software using a 3D model with 20-node solid element, element size of 0.005 m and a total of 30 mode shapes. Furthermore, the passive control is performed to reduce a broadband excitation up to a frequency of 300 Hz. In this frequency band the system presents two mode shapes of frequencies 148.52 and 240.35 Hz, which can be visualized in Figure 2. In order to represent the primary system receptance, the excitation and responses points are chosen at $(0.01, 0.01, 0.3)$ and $(0.29, 0.01, 0.3)$ from the origin and a modal damping ratio of 0.01 for the selected modes.

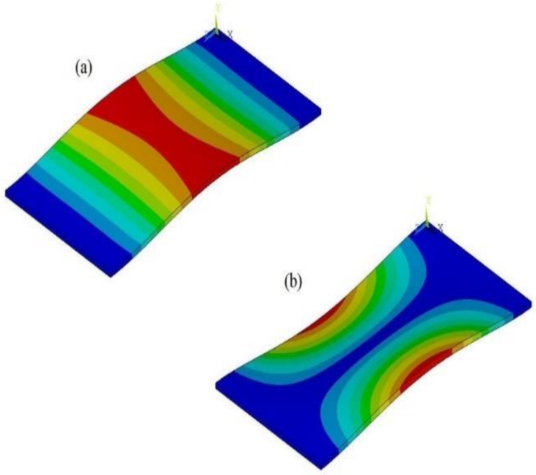

Figure 2: (a) First mode shape. (b) Second mode shapes of the primary system.

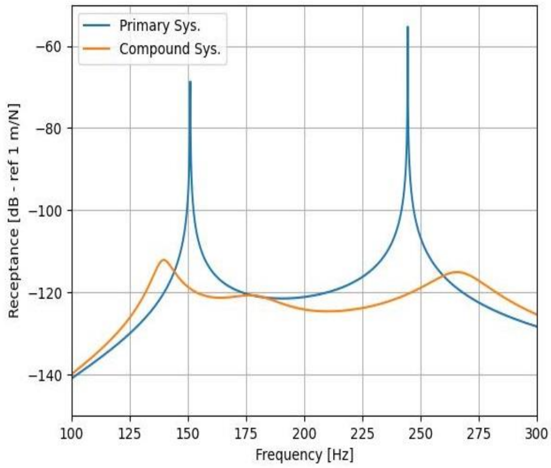

Figure 3: Primary and compound system receptance

The magnetorheological material selected for this analysis is an isotropic MRE with butyl rubber matrix whose properties are described by Silva et al (2021). The neutralizer natural frequency is restricted to the frequency band of 100 to 300 Hz and the design temperature is arbitrarily selected at 293 K. Once the modal parameters are extracted from the finite element model, the frequency response of the compound system is computed through an algorithm in Python language which is also used to optimize the magnetorheological neutralizer with the selected material and initial parameters.

The optimal results for the neutralizer mass, natural frequency and position are 0.349 kg, 179.29 Hz and $(0.3, 0.1, 0.2975)$, respectively. The frequency response function of the primary system and the compound system with the optimum magnetorheological neutralizer can be visualized in Figure 3. After the neutralizer optimum parameters are acquired, the geometric factor can be calculated using equation (12). Assuming a constant geometric factor, the temperature effect on the vibration control can be analyzed by calculating the modification on the neutralizer characteristic frequency for the given temperature with equation (12). The modified natural frequencies for the temperature $283, 288, 297, 303$ K are presented in Table 1. Moreover, the compound system receptance for those temperature can be observed in Figure 4.

Temperature [K]	283	288	293*	298	303
Characteristic Frequency [Hz]	205.25	190.51	179.29	170.68	164.03

*Design temperature and optimum frequency

Table 1: Neutralizer natural frequency at different temperatures

A higher magnetic flux density in the MRE induces an increase in the dynamic shear modulus and, consequently, a higher neutralizer natural frequency. Therefore, the application of the magnetic field can reduce the neutralizer detuning caused by the increase in temperature. Assuming a change in the operating temperature to 303 K, the characteristic frequency of the magnetorheological neutralizer can be increased up to 170.06 Hz with a magnetic flux density of $0.5\ T$ applied to the MRE. The compound system receptance for the different temperature and magnetic flux density are represented in figure 5. From this figure, it is clear that when the magnetic field is applied (green curve) it approaches to the blue curve, which represents a higher natural frequency of the neutralizer.

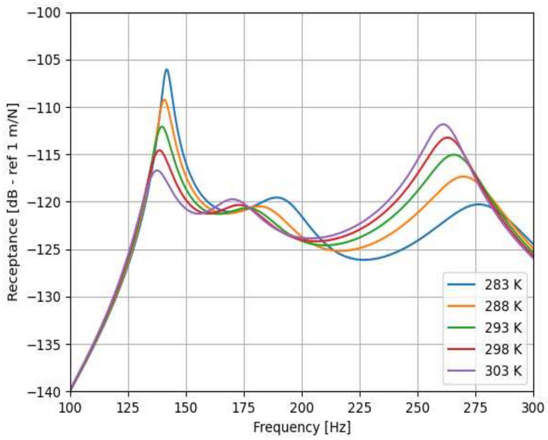

Figure 4: Compound system receptance for different temperature.

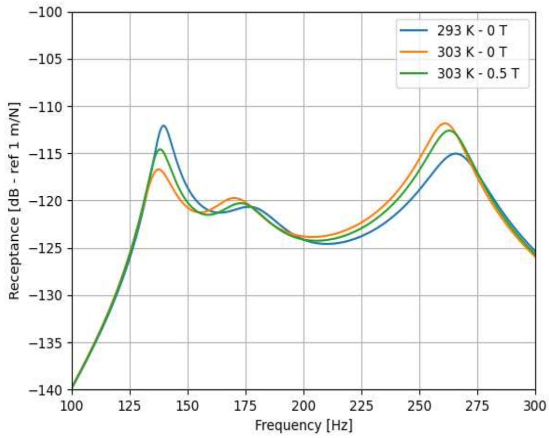

Figure 5: Receptance for the modified temperature and magnetic flux density.

The magnetic field on the MRE can be provided by an electromagnet which is also used as the absorber rigid mass. A simplified representation of the magnetorheological neutralizer, using the calculated mass and geometric factor, can be visualized in Figure 6. The device consists of a steel base, two magnetorheological elastomer pieces, steel core and coil.

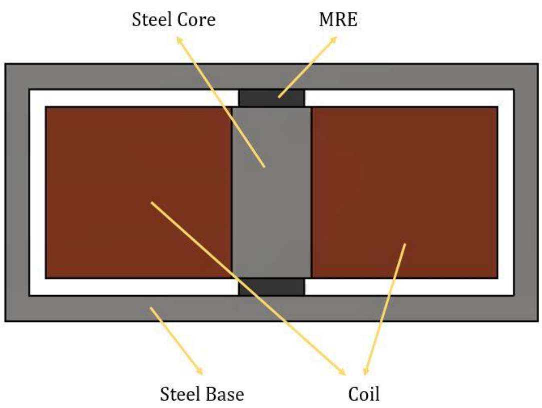

Figure 6: Magnetorheological Neutralizer

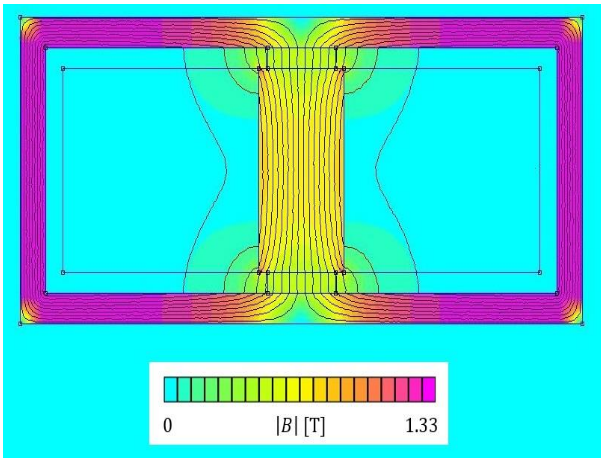

Figure 7: Magnetostatic Analysis

The magnetostatic simulation is performed using a finite element method in the software Finite Element Method Magnetics (FEMM) and the magnetic flux density distribution can be observed in Figure 7. The lower values of the magnetic flux density are represented in blue, the medium values around 0.7 T in yellow and the higher values of approximately 1.3 T in purple. Furthermore, the magnetic flux density simulated across the MRE is approximately 0.47 T. The results are generated using triangular elements with size of 0.5 mm, electric current of 1 A and a total of 1500 turns in the electromagnet.

7 CONCLUSIONS

In this work, a magnetorheological dynamic neutralizer in order to reduce the vibration of a multi degree of freedom system under broadband excitation was optimized. In this sense, we first present the characteristics of magnetorheological elastomers, the possibility to create a dynamic vibration neutralizer from it, and the mathematical framework of the multi-degree of freedom systems having magnetorheological dynamic neutralizers attached to them. Since these neutralizers can modify their characteristic frequencies by a change in the operating temperature, we tested different temperatures to observe the detuning of the natural frequency of the neutralizer, which can increase the maximum receptance value of the compound system. At the same time, a rising magnetic field increase the natural frequency of the MRE. In this sense, we prove that the detuning caused by an increment in temperature can be mitigated by the applied field on the MRE. However, the reduction of the maximum receptance caused by the electromagnetic field can be very limited since the size of the electromagnet is dependent on the neutralizer mass. The application of a different elastomer matrix or the utilization of anisotropic MRE can improve the results and the design of the magnetorheological neutralizer.

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