Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXV, pp. 2805-2815
Alberto Cardona, Norberto Nigro, Victorio Sonzogni, Mario Storti. (Eds.)
Santa Fe, Argentina, Noviembre 2006

APPLICATION OF THE METHOD OF FUNDAMENTAL SOLUTIONS AS A COUPLING PROCEDURE TO SOLVE OUTDOOR SOUND PROPAGATION PROBLEMS

Sebastian Hampel[a], Adrián P. Cisilino[b] and Sabine Langer[a]

[a]Institute of Applied Mechanics, Technical University Braunschweig, P.O. Box 3329, D-38023, Braunschweig, Germany, sebastian.hampel@tu-bs.de
[b]Welding and Fracture Division, Faculty of Engineering, University of Mar del Plata, Av. Juan B. Justo 4302 - 7600 Mar del Plata, Argentina, cisilino@fi.mdp.edu.ar

Keywords: BEM, method of fundamental solutions, coupling, ray method, sound propagation.

Abstract. In a computational model for outdoor sound propagation, the relevant propagation phenomena, among which are refraction and diffraction, must be implemented. All numerical methods applied in this field so far have disadvantages or limits. The Finite Element Method has to discretize the domain and hence is restricted to closed or at least moderate sized domains.
The Boundary Element Method can hardly consider inhomogeneous domains and the computation effort increases exponentially for large systems. Geometric acoustics algorithms like ray tracing consider sound as particles and are hence not able to represent wave phenomena.
It is the aim of this work to combine the advantages of the BEM and of the ray method: In the near-field where obstacles and complex geometries occur - and so diffraction and multiple reflection are expected - the model uses the BEM. Then, a ray model is coupled to compute the sound emission at large distances, because this model can take into account refraction resulting from wind or temperature profiles. The ray model requires point sources as input data. However, a boundary element calculation always delivers the pressure or its normal derivative along the boundary. Hence, for the coupling of both models it is necessary to convert the BEM results into equivalent point sources. The Method of Fundamental Solutions (MFS) is found suitable for this purpose.
To couple the BEM and ray model, the acoustic half-space is divided into a BEM domain and a ray domain by defining a virtual interface. Along this interface, the pressure is computed with the BEM. The idea behind the MFS is to place a number of sources with unknown intensities around the domain of interest. These intensities are then computed in order to fulfill prescribed boundary conditions at discrete points on the boundary of the domain. The MFS can be either applied with fixed source positions or with an optimization algorithm, which finds the optimal source positions by minimizing the residual along the boundary in a least-squares sense. Both types of the MFS are used in this work. The verification of this new coupling procedure is shown for a two-dimensional problem consisting of a of a noise barrier in a homogeneous atmosphere, for which a reference solution is known.

1 INTRODUCTION

The Method of Fundamental Solutions (MFS) has so far been applied to various acoustic problems. A review of the developments and application of the MFS for scattering and radiation problems is given by Fairweather et al (2003). The performance of the MFS for acoustic wave scattering is analyzed by Alves et al. (2005) In the literature, many different names have been used for this method, as e.g. multipole radiator synthesis or equivalent source method.

The advantages of the MFS are basically its properties as a meshless method: It does not require the discretization of the model and no integration has to be performed. Optionally, an optimization algorithm can be used to optimize the position of the sources in order to minimize the residual at the prescribed boundary points, see Fairweather et al (2003) and Cisilino et al. (2001). The Method of Fundamental Solutions can either be applied for fixed source positions or with an optimization algorithm to find the source position where the residual is minimal. The second approach is usually referred in the literature as the MFS with moving sources.

The methods of computational acoustics can basically be divided into two groups, the wave based methods and methods of geometrical acoustics. The first type consider the characteristic of sound propagation as traveling waves, and so include all wave phenomena like diffraction and interference. They are based on any kind of wave equation, which can be the scalar wave equation in the time domain or the Helmholtz equation in the frequency domain. These methods are usually implemented using the Finite Element Method (FEM) or the Boundary Element Method (BEM). Whereas in the geometrical acoustics approach the wave character is neglected and sound propagation is considered as propagation of sound particles. The travel path of a sound particle is called sound ray. Most of these methods require point sources as input data.

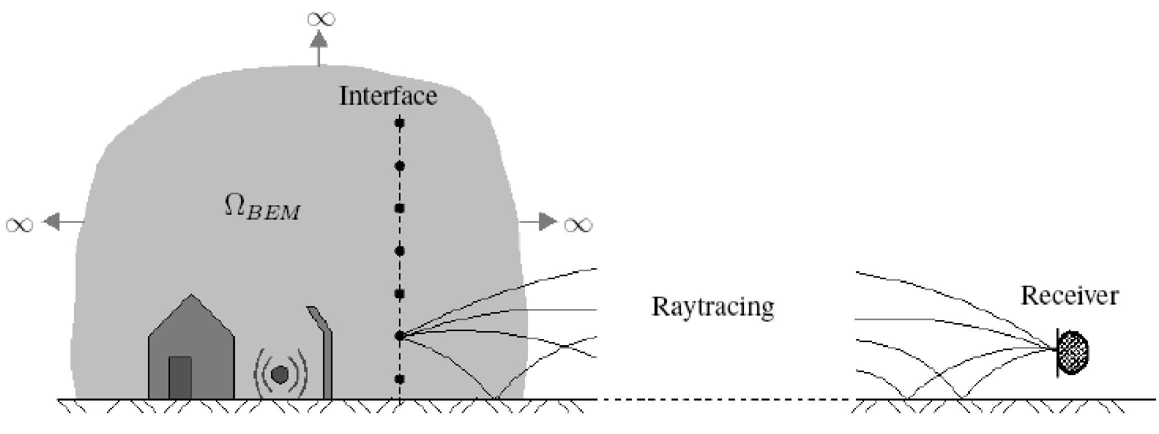


Figure 1: Sketch of the hybrid method (BEM-Ray tracing).

For an application with a noise barrier or a noise protection dam around the source and receivers at the far-field, the BEM is used for the near-field around the source, where the geometry might be complex and where diffraction and multiple reflections occur. For the far-field over large propagation range, a ray method is applied which includes the effect of refraction in the atmosphere due to a vertical profile of sound speed. This sound speed profile can either result from a temperature profile or - using the effective sound speed approach - also from a wind speed profile. Figure 1 shows the coupling scheme for the hybrid method consisting of BEM and ray tracing. Some typical example problems for this application are shown in figure 2. Thus, to make use of both kinds of methods in one problem calculation, it is necessary to transform the sound field values (e.g. pressure) as output from the wave-based method into equivalent point sources as input for the ray method. This will be done by using the MFS in the following.

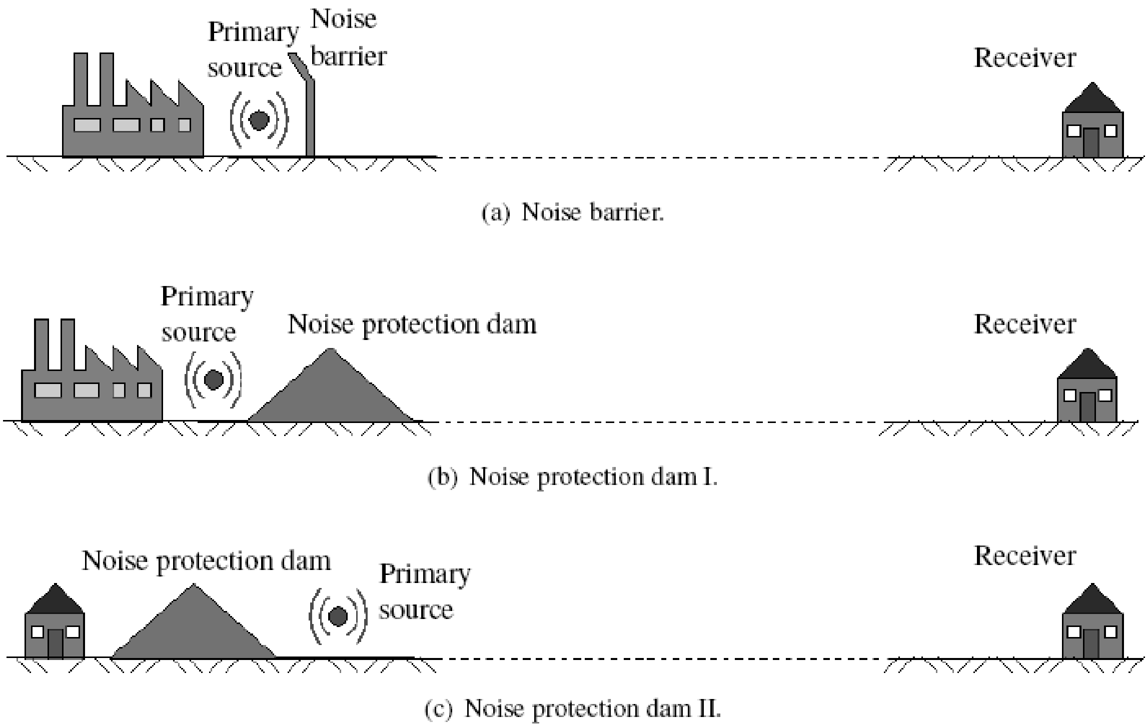


Figure 2: Typical problems considered in outdoor sound propagation.

2 THE MFS FOR ACOUSTIC PROBLEMS

Acoustic problems in the frequency domain are governed by the Helmholtz equation:

$$\Delta p + k^2 p = 0, \tag{1}$$

where p is the acoustic pressure and k is the wave number, which is the quotient of the angular frequency and the sound speed. In the following, 2D-problems are considered. A 2D fundamental solution for eq. (1) is known to be

$$G(x_i, \xi_j) = -\frac{i}{4} H_0^{(1)}(kr), \tag{2}$$

which describes the pressure at x_i caused by a unit source at ξ_j . $H_0^{(1)}$ is the Hankel function of zero order and first kind. Here, i denotes the imaginary unit (not to mix with the index i for the field points!), k is the wave number and r is the distance from the source point S_j at position ξ_j to the field point R_i at position x_i. If a half-space over rigid ground is considered, the fundamental solution changes into

$$G(x_i, \xi_j) = -\frac{i}{4} H_0^{(1)}(kr) - \frac{i}{4} H_0^{(1)}(kr'), \tag{3}$$

where *r'* is the distance of the mirror source S'_j to the field point R_i (see figure 3).

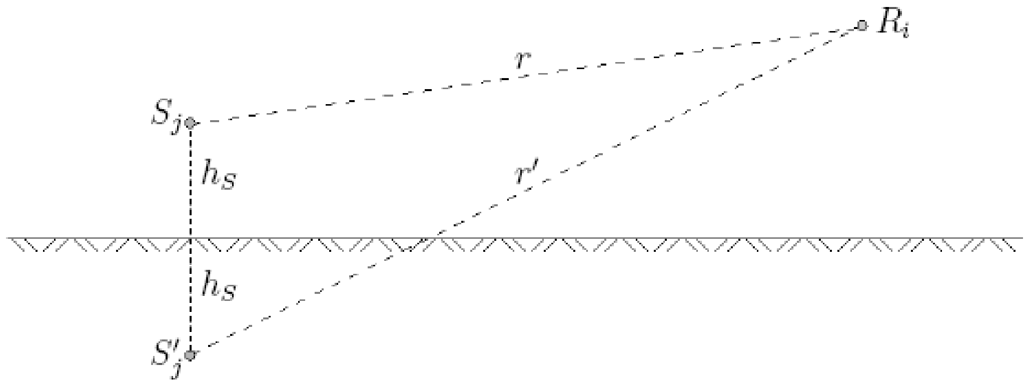


Figure 3: Geometry of source-receiver-distance in the case of a half-space.

The idea behind the MFS is to place a number of sources around the domain of interest Ω_{MFS}, with their positions and intensities set in order to fulfill the given boundary conditions along the boundary Γ_{MFS} of the domain MFS (see figure 4). Each source j at position ξ_j outside the domain MFS contributes a pressure field which is described by the fundamental solution $G\left(x_i,\xi_j\right)$. The approximate solution is yield by collocation on a number of points x_i on the boundary Γ_{MFS}. For a given boundary point x_i the pressure value $p(x_i)$ is given by the linear superposition of all contributions $j = 1, 2, \ldots, N$, weighted by the intensity coefficients a_j for each source:

$$\sum_{j=1}^{N} G(x_i,\xi_j)\cdot a_j = p(x_i), \quad x_i \in \Omega, \quad \xi_i \in \bar{\Omega}. \tag{4}$$

This equation is used as boundary conditions by inserting the known boundary values $\overline{p}_i$ at boundary points i on the right hand side. Doing this for all M boundary points results in a system of linear equations

$$\mathbf{A}\cdot\mathbf{x} = \mathbf{b}, \tag{5}$$

where the matrix entries A_{ij} consist of the fundamental solutions $G\left(x_i,\xi_j\right)$ at point i due to a source with unit intensity at point j, so

$$A_{ij} = G(x_i,\xi_j), \tag{6}$$

the solution vector **x** contains the unknown source intensity coefficients a_j , and vector **b** contains the known boundary values.

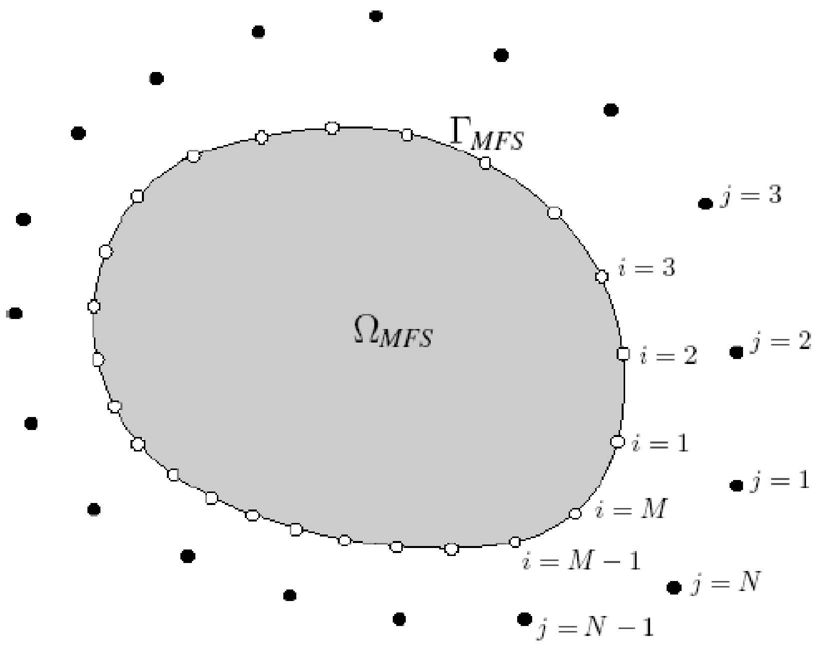


Figure 4: General sketch for interior MFS problem.

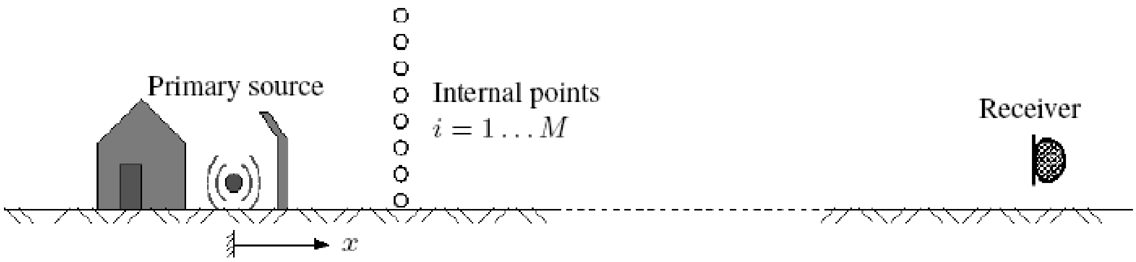
The number of sources *N* does not have to be equal to the number of prescribed boundary points *M*, but can also be smaller. In this case a non-square matrix arises from the equations shown above. This system of equations represents a linear least-squares problem which can be solved using a Single Value Decomposition (SVD) algorithm. For further information about SVD and its numerical implementation see e.g. Press et al (1992).

3 APPLICATION WITH FIXED SOURCE POSITIONS

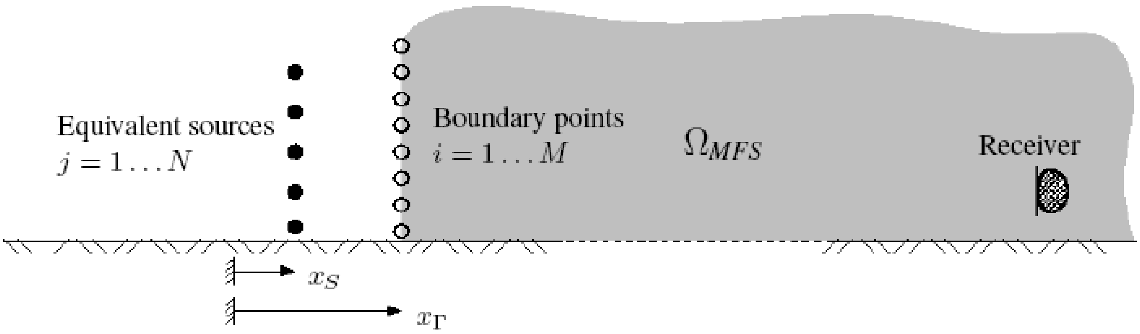
Here, the MFS with fixed source positions is applied to the described coupling problem. Figure 5(a) shows the configuration for the BEM calculation, with the primary source and obstacles in the near-field. The problem is solved using a two-dimensional approach. The near-field is approximated in the MFS by a number of equivalent (or secondary) sources (•), see Figure 5(b). The denotation in this figure is chosen according to Figure 4.

For the MFS the pressure at the boundary points (○) has to fulfill the pressure values yielded by the BEM calculation. The vertical line of boundary points can be considered as the left border of the MFS domain where the ray tracing calculation will be used. The x-position of these vertical lines is x_S for the sources and x_Γ for the boundary points, respectively.

The problem of the dam on the left hand side of the point source is considered (Figure 2(c)). Figure 6 shows the condition number of the matrix **A** versus the number of sources for three different source positions. The number of boundary points is fixed at M = 600, and they are placed in a vertical line at x_Γ = 10.5m and height up to 60m. Obtained results show that the positions of the sources x_S have a strong influence on the matrix condition. If the sources are too far away from the boundary points compared to the distance between two sources or two boundary points - which is the case for x_S = 5.0m -, the system will be ill-conditioned (i.e. high condition numbers). On the other hand, if the sources are too close to the boundary, the matrix entries will become infinite as the fundamental solution *G* in equation 6 is singular for $r \rightarrow 0$. In brief, the condition number of matrix **A** depends on the number of sources and their relative positions with respect to the boundary points.



(a) Configuration for BEM.



(b) Configuration for MFS.

Figure 5: Sketch of the configurations for the considered coupling problem with fixed source positions.

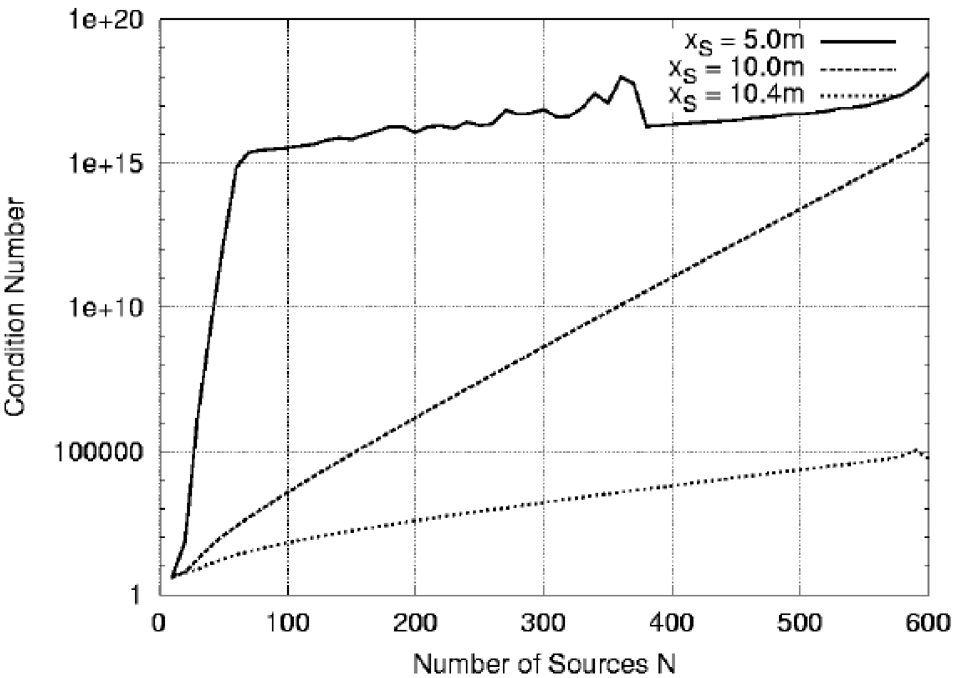


Figure 6: Condition number of the system matrix depending on the number of sources; Three different source positions x_s = 5.0m, 10.0m and 10.4m; x_Γ = 10.5m.

Figure 7 shows the relative error as a function of the receiver distance for the case of an homogeneous domain. The relative error was computed by comparing the pressure results obtained using the two-step methodology (BEM-ray method) to the reference solution given by a BEM calculation. The number of boundary points and the source positions are the same as in Figure 6 in order to evaluate the effect of the matrix condition number on the actual quality of the result. The number of source points was chosen to $N = 200$. It can be seen that the error curves associated to source positions close to the boundary (i.e. $x_S = 10.0$m and $x_S = 10.4$m) show a nice behavior. On the other hand, the error for the source position $x_S = 5.0$m diverges and delivers unusable results. This is a consequence of the very high condition number of about 10^{16} (see figure 6). Results in Figure 7 also show that the error is always zero at $x_\Gamma = 10.5$m, since the boundary conditions in the MFS were imposed in this position. Just beyond this position the error increases because the approximated pressure field is not smooth in the near-field. At some distance from the source, the pressure field smoothes for $x_S = 10.0$m and $x_S = 10.4$m and the proposed procedure approaches the reference solution very good. The error is almost constant over the whole range and even up to 10 km (not shown in Fig. 7) the relative error is lower than two percent.

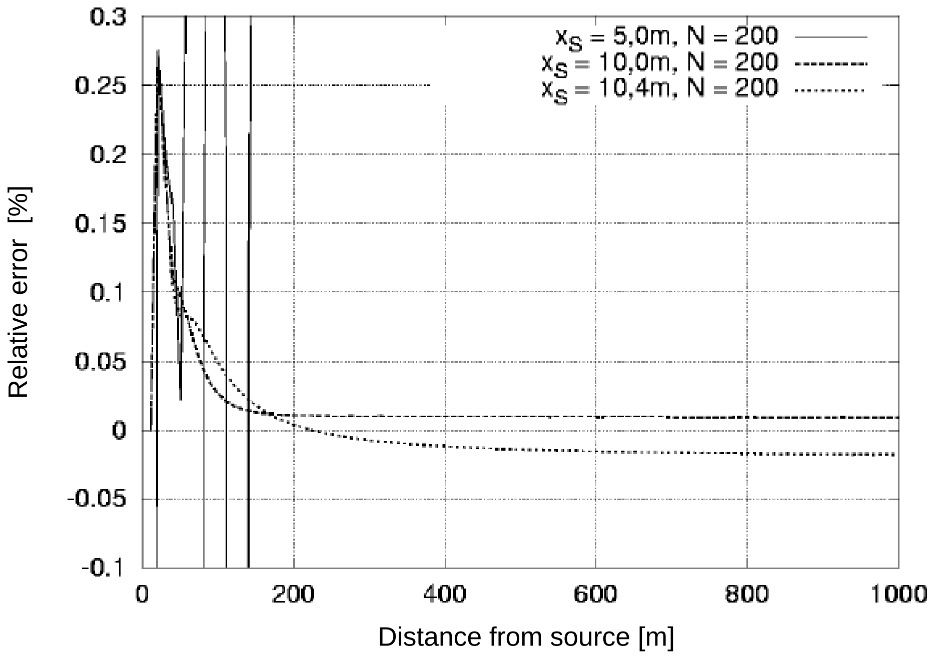


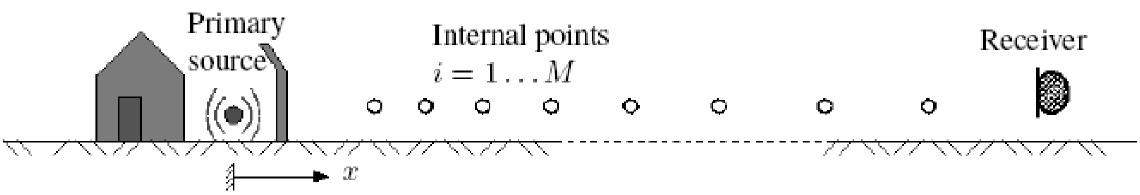
Figure 7: Relative error at internal points for the three different source positions $x_s = 5.0$m, 10.0m and 10.4m.

4 REDUCED MFS MODEL WITH OPTIMIZED SOURCE POSITIONS

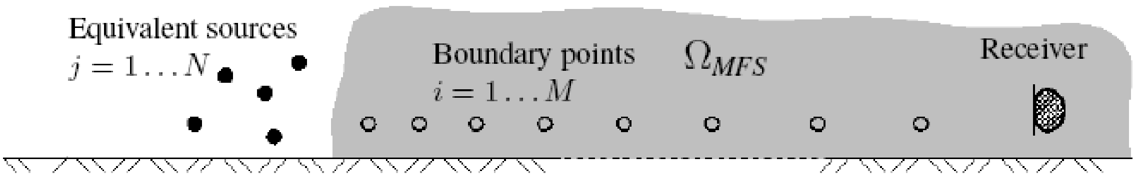
For an optimization algorithm it is usually very challenging to find the global minimum of a residual, if it has many local minima and maxima. The pressure field with complex pressure values and a non-trivial geometry shows to have a residual field which oscillates strongly in space. So for these cases a gradient-based optimization algorithm will fail, if the initial source positions are not chosen accidentally very close to the optimal positions.

To avoid these problems, an approach is proposed in the following, which separates the complex pressure signal into its amplitude and phase information. It is shown that with this separation the optimization algorithm can be successfully applied to find the optimal source position. By optimizing the solution with respect to the amplitude only, the residual becomes quite smooth and therefore the problem is much easier to solve for the optimization algorithm. The time-harmonic characteristic will be added and adjusted in a second step. To consider only the amplitude for optimization, the MFS algorithm is changed, so that the fundamental solution is replaced by its absolute value and as boundary condition the pressure amplitudes are used:

$$\sum_{j=1}^{N} |G(x_i, \xi_j)| \cdot a_j = |p(x_i)|, \quad x_i \in \Omega, \quad \xi_i \in \bar{\Omega}. \tag{7}$$



(a) Configuration for BEM.



(b) Configuration for MFS.

Figure 8: Sketch of the configurations for the considered coupling problem with optimization.

The optimization algorithm leads to the source positions corresponding to the minimum residual at the boundary points $i = 1 \dots M$ (Netlib Software Library, 2005).

Here, as boundary points of the MFS problem, points are chosen in the far-field of the primary source (see Figure 8). Since the coupling problem and both of the computational domains (BEM and ray tracing) do not have defined boundaries, these points are not boundary points in the classical sense. However, the declarations “boundary points” and “boundary conditions” are still used here according to Figure 4. The reason for placing these points in the far-field is that this is the actual area of interest.

As solution vector x, the real values of a_j are obtained. To get the missing phase information, the complex intensities $\hat{a}_j$ are written in the Eulerian form

$$\hat{a}_j = |a_j| e^{i\varphi_j}, \tag{8}$$

where φ_j is the phase angle. This phase angle follows from an additional complex boundary condition, e.g. at a point x_{BC} in the far-field, where the exact pressure p_{BC} for the homogeneous case is known from the BEM calculation.

$$\arg\{a_j\, e^{i\varphi_j}\, G(x_{BC}, \xi_j)\} = \varphi_{BC} = \arctan\left(\frac{\Im\{p_{BC}\}}{\Re\{p_{BC}\}}\right), \quad j = 1 \dots N. \tag{9}$$

This condition ensures that at the phase of the pressure contribution from source j coincides with the phase of the boundary condition at x_{BC}. From the phase angle φ_j, the complex intensity can be derived using eq. (8).

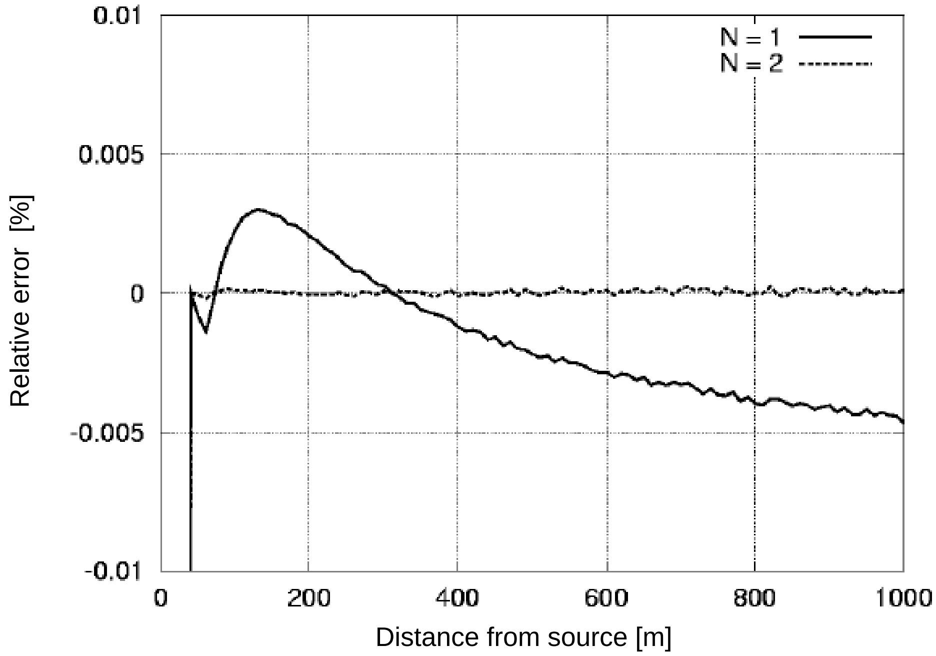


Figure 9: Relative error of the pressure amplitudes at internal points.

Figure 9 shows the relative error of the pressure amplitudes at internal points for the problem of a sound barrier of height h_B = 4m at x = 10.5m, a point source at x_S = 0 and h_S = 0.5m. Only five points were chosen to fit the pressure amplitude: At x_{BC} = 40m, 70m, 200m, 700m and 1000m. The first curve is for only one source point (N = 1), the second is for two sources (N = 2). For N = 1 it can be seen that the relative error at the boundary points increases with distance. This is because the residual was defined not in terms of relative, but absolute values. The points closer to the source have higher pressure values and therefore are 'weighted stronger' when considering the residual over the whole range. The optimal source position found by the optimization algorithm is at (x^* = 1.585m, z^* = 5.010m) for N = 1 and (x_1^* = 12.139m, z_1^* = 1.951m), (x_2^* = 8.618m, z_2^* = 2.511m) for N = 2. The total residuals are $3.841 \cdot 10^{-3}$ (N = 1) and $1.695 \cdot 10^{-11}$ (N = 2), respectively. To get to this residual it takes about 120 iterations and a computation time of a few seconds on an average personal computer. It was found that a good starting point for the source(s) when using optimization is on top of the barrier, in this case at x_B = 7.5m, h_B = 4m.

In Figure 10 the approximate solution received with the reduced separated model is compared to the reference solution. It can be seen that for short distances behind the barrier (Figure 10) there is a phase error, which disappears for increasing distances. This behavior arises when the difference between r and r' is not negligible compared to the wave length. So the distance range where the approximate results contain the phase shifts from the reference solution will be smaller for lower frequencies and lower source heights, where the area of destructive interference between direct and indirect sound path is closer to the source.

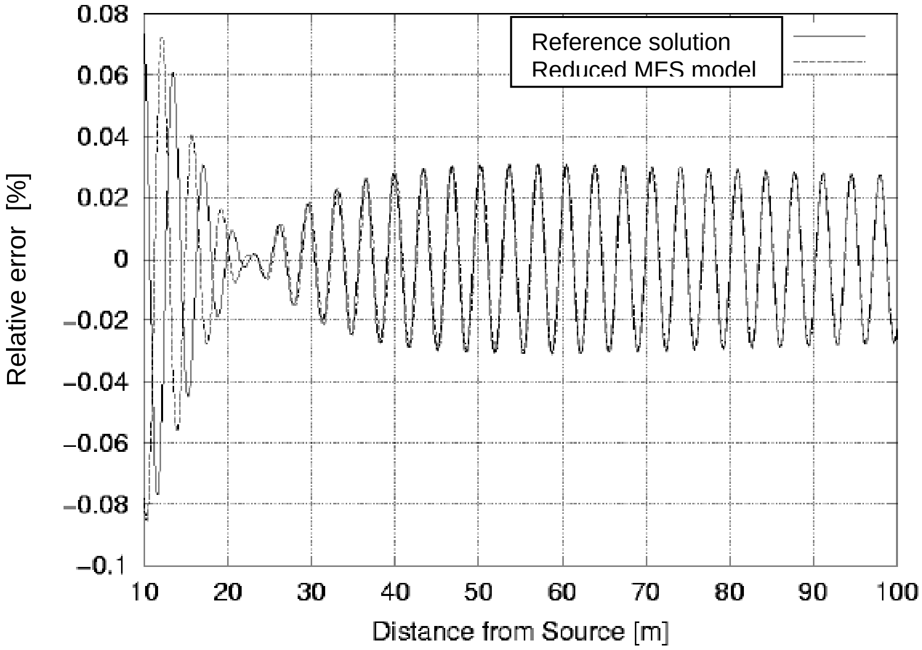


Figure 10: Comparison of the real part of the approximation (reduced MFS model) with the reference solution: 10 to 100m.

5 SUMMARY

It is shown in this paper that the MFS can be successfully applied to couple a wave-based method and ray tracing method for solving outdoor sound propagation. The pressure distribution, which is a result from a wave-based method, e.g. the BEM, can be well-approached by a number of equivalent point sources, which are required as input data for most ray methods. Two ways are presented to use the MFS: The first one assumes fixed source positions whereas the second variant applies an optimization algorithm to find the optimal positions for the equivalent sources. The verification is done for a problem two-dimensional problem for which a reference solution is known. Error analyses are performed for both MFS types and the results encourage using them for this coupling purposes.

The described coupling algorithm allows combining the advantages of the BEM and the ray tracing method: The BEM takes into account the diffraction at edges exactly, whereas the ray method can easily handle refraction in the atmosphere due to a sound speed profile. Hence, this study can hopefully be a step to a stronger use of numerical simulations in the design process of acoustic and noise protecting measures.

ACKNOWLODGEMENTS

The work reported in this paper was supported by a grant within the alfa-project "ELBENet" on Boundary Element Methods. The authors wish to thank the European Union for the financial support.

REFERENCES

C. J. S. Alves and S. S. Valtchev. Numerical comparison of two meshfree methods for acoustic wave scattering. *Engineering Analysis with Boundary Elements*, 29, 371–382 ,2005.

A. P. Cisilino and B. Sensale. Optimal placement of the source points for singular problems in the method of fundamental solutions. In *Advances in Boundary Element Techniques II*. Hoggar Press, 2001

G. Fairweather, A. Karageorghis, and P. A. Martin. The method of fundamental solutions for scattering and radiation problems. *Engineering Analysis with Boundary Elements*, 27, 759–769, 2003.

Netlib Software Library. Minpack. URL: http://www.netlib.org/minpack/, Chicago, USA, 2005.

W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling. *Numerical Recipes in Fortran*. Cambridge University Press, 1992.